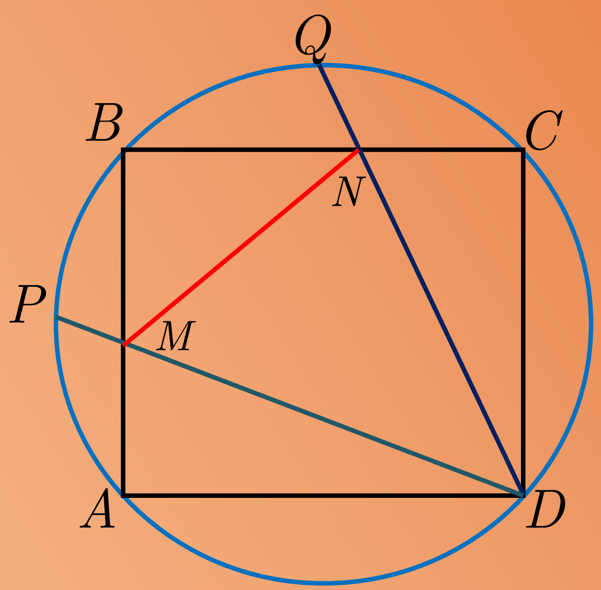
R.M.MADRAHIMOV, N.B.KAMALOV, B.B.YUSUPOV, S.A.BEKMETOVA

TALABALAR MATEMATIKA OLIMPIADASI MASALALARI



O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

AL-XORAZMIY NOMLI URGANCH DAVLAT UNIVERSITETI

Ro‘zimboy Madrahimov, Ne’matjon Kamalov
Baxtiyor Yusupov, Sadoqat Bekmetova

TALABALAR MATEMATIKA OLIMPIADASI MASALALARI

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Ushbu kitob ko'p yillardan beri o'tkazilib kelinayotgan talabalar o'rtasidagi ichki va respublika matematika fan olimpiadalarida berilgan, jami 338 ta misol va masalalardan tashkil topgan. Kitobda berilgan misol va masalalarning 198 tasi yechilib, to'la isboti bilan berilgan. Qolgan 140 tasi mustaqil yechish uchun tavsiya etilgan. Bulardan tashqari, olimpiadalarda taklif etilgan jami 550 ta test materiallari kalitlari bilan berilgan.

Kitob olimpiadaga tayyorgarlik ko'rayotgan talabalar va ularning ustozlari uchun tayyor material hisoblanadi.

Kitob oliy o'quv yurtlari talabalari uchun mo'ljallangan.

Mas'ul muharrir **Alimardon Atamuratov**, f.m.f.n.

Taqrizchilar **Qurol Navruzov**, f.-m.f.d., prof.,
Yo'ldosh Saidov, f.-m.f.n.

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M U N D A R I J A

"Har bir kuzatuvchi (olim) diqqat bilan o'lchasin, hamisha o'z ishlaridan qoniqmasin, o'z ishlarini qayta-qayta tekshirib tursin, mumkin qadar, kamroq g'ururlansin, tobora tirishqoqlik bilan ishlasin va mehnatdan hech zerikmasin".

Abu Rayhon Beruniy

SO'ZBOSHI

Respublikamiz ta'lim-tarbiya tizimida qator islohiy o'zgarishlar amalga oshirilgan bo'lib, ularning asosiy maqsadi yosh avlodni layoqati, qobiliyati, iqtidorini aniqlash, ochish va ularning rivojlanishi uchun shartsharoit va imkoniyatlarni yaratishdan iboratdir.

Bular o'quvchilarning fanga bo'lgan qiziqishini shakllantirish, faolligini oshirish va ularni rag'batlantirish bilan bog'liqdir. Olimpiada ishtirokchilarning ijodiy faolligini o'stirishda, tafakkur jarayonlarini shakllantirishda mantiqiy fikrlash, matematikadan nostandart masalalarni yechish alohida ahamiyat kasb etadi.

Iqtidorli talabalarning fanlardan egallagan bilimlari qanchalik chuqur va mustahkamligini, ularning ijodiy fikrlash doirasining kengligini, quvvaiy hofizasining kuchliligini aniqlovchi mezonlardan biri olimpiadadir.

Olimpiadada qatnashishni orzu qilgan har bir talaba chuqur mas'uliyatni his qilgan holda, oliy o'quv yurtida olgan bilimlari bilan cheklanib qolmay, maxsus adabiyotlardan dastlabki tayyorgarlikni o'tashi kerak.

Ushbu kitob matematika olimpiadasiga tayyorgarlik ko'rayotgan talabalarga mo'ljallangan.

"... dalil va aniqlik haqiqiy ilmga xosdir"

Abu Rayhon Beruniy

1-§. Olimpiada masalalari

1. Agar $x_1, x_2, ..., x_n > 0$ sonlari $x_1 \cdot x_2^2 \cdot ... \cdot x_n^n = 1$ shartni qanoatlantirsa, quyidagi tengsizlikni isbotlang.

$$\frac{1}{x_1} + \frac{2}{x_2} + ... + \frac{n}{x_n} \geq \frac{n(n+1)}{2}$$

2. Agar $n \geq 2$ va $n \in \mathbb{N}$ bo'lsa, quyidagi tengsizlikni isbotlang.

$$\frac{1}{2^2} + \frac{1}{3^2} + ... + \frac{1}{n^2} \leq n\left(1 - \sqrt[n]{\frac{1}{2}\left(1 + \frac{1}{n}\right)}\right)$$

3. $n \geq 3$ $(n \in \mathbb{N})$ larda $(n+1)^n < n^{(n+1)}$ ni isbotlang.

4. Agar $P(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n-1} x + 1$ ko'phad n ta haqiqiy ildizga ega bo'lsa, $P(n) \geq 3^n$ ekanini isbotlang. Bu yerda $a_1, a_2, ..., a_n \geq 0$.

5. $a, b \geq 0$ shartlarni qanoatlantiruvchi $\forall a, b$ lar uchun $ab \leq e^{a-1} + b \ln b$ tengsizlikni isbotlang.

6. Bizga haqiqiy koeffitsiyentli $Q(x) = x^m + b_1 x^{m-1} + ... + b_m$ va $P(x) = x^n + a_1 x^{n-1} + ... + a_n$ ko'phadlar berilgan. Agar $P(x)$ ko'phad turli xil $x_1, x_2, ..., x_n$, n ta ildizga ega bo'lsa, u holda $\sum_{j=1}^{n} \frac{Q(x_j)}{P'(x_j)}$ ni toping.

7. Agar uchburchak tomonlari uchun $a^2 + b^2 = c^2$ tenglik bajarilsa, $\forall n \geq 3$ $(n \in \mathbb{N})$ soni uchun $a^2 + b^2 < c^2$ tengsizlikning o'rinli ekanini isbotlang.

8. Agar $n \geq 2$, $|x| < 1$ bo'lsa, $(1-x)^n + (1+x)^n < 2^n$ tengsizlikni isbotlang.

9. $a > 0, b > 0, c > 0$ bo'lsa, quyidagi tengliksizlikni isbotlang.

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{a^8 + b^8 + c^8}{a^3 b^3 c^3}$$

10. Agar $a_1 > 0, a_2 > 0, ..., a_n > 0$ bo'lsa, $(a_1^{a_1} \cdot a_2^{a_2} \cdot ... \cdot a_n^{a_n})^n \geq (a_1 a_2 ... a_n)^{a_1 + a_2 + ... + a_n}$ tengsizlikni isbotlang.

11. $x, y, z, p, q, r > 0$ sonlari uchun $p + q + r = 1, x^p y^q z^r = 1$ shartlar o'rinli bo'lsa, quyidagi tengsizlikni isbotlang.

$$\frac{p^2 x^2}{qy + rz} + \frac{q^2 y^2}{px + rz} + \frac{r^2 z^2}{px + qy} \geq \frac{1}{2}$$

12. $f(x)$ funksiya quyidagi shartlarni bajarsin:

1) $f(x) \in C^{n-1}[x_0, x_n]$;

2) (x_0, x_n) intervalda n-darajali hosilaga ega;

3) $x_0 < x_1 < ... < x_n$ uchun $f(x_0) = f(x_1) = ... = f(x_n)$ tenglik o‘rinli.

U holda $\exists \varepsilon (\varepsilon \in (x_0, x_n))$ topilishini isbotlangki, $f^{(n)}(\varepsilon) = 0$ bo‘lsin.

13. Agar $f(x)$ funksiya $[a,b]$ da uzluksiz hamda (a,b) intervalda chekli hosilaga ega bo‘lib, chiziqli funksiya bo‘lmasa, u holda (a,b) da $\exists c \in (a,b)$ topiladiki, $|f'(c)| > \left|\frac{f(b) - f(a)}{b - a}\right|$ tengsizlik o‘rinli bo‘ladi. Shuni isbotlang.

14. Agar $f(x)$ funksiya $[a,b]$ segmentda 2-tartibli hosilaga ega bo‘lib, $f'(a) = f'(b) = 0$ bo‘lsa, u holda $\exists c \in (a,b)$ topilishini isbotlangki, quyidagi munosabat o‘rinli bo‘lsin:

$$|f''(c)| \geq \frac{4}{(b-a)^2}|f(b) - f(a)|$$

15. ABC uchburchakning ichida ixtiyoriy M nuqta olingan va bu nuqtadan AM, BM, CM to‘g‘ri chiziqlar o‘tkazilgan. Bu to‘g‘ri chiziqlar uchburchak tomonlarini, mos ravishda A_1, B_1, C_1 nuqtalarda kesib o‘tadi. Quyidagi tengsizlikni isbotlang:

$$\frac{AM}{A_1M} + \frac{BM}{B_1M} + \frac{CM}{C_1M} \geq 6$$

16. Agar $|x| < 1$ bo‘lsa, $1 + 2x + 3x^2 + 4x^3 + ...$ yig‘indining qiymati nimaga teng?

17. $f(x) \in C^1[a,b]$ bo‘lib, $a \leq x_n \leq b$ va $|f'(x)| \leq q < 1$ $(q \in \mathbb{R})$ bo‘lsin. Agar $x_n = f(x_{n-1})$ (bu yerda $f(x_0) \in [a,b]$) bo‘lsa, x_n yaqinlashuvchi ketma-ketlik ekanini isbotlang.

18. $f(x)$ funksiya $[0,+\infty)$ da monoton o‘suvchi bo‘lsa, $\int_1^n f(x)dx$ integralni yuqoridan va quyidan $f(1), f(2), ..., f(n)$ lar yordamida baholang.

19. $A = (a_{ij})_{i,j=1}^n$ matritsa uchun $|a_{ii}| \geq \sum_{i,j=1}^n |a_{ij}|$ bo‘lsa, A matritsa teskarilanuvchi ekanini isbotlang.

20. Quyidagi funksiyalarning berilgan oraliqda tekis uzluksiz emasligini isbotlang:

a) $f(x) = \frac{1}{x}$ (0.1) da;

b) $f(x) = x\sin x$ $[0,\infty)$ da;

c) $f(x) = \sin\frac{1}{x}$ (0.1) da.

21. ABC uchburchakning ichidagi M nuqta qanday joylashganda ushbu $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}$ yig'indi eng kichik qiymat qabul qiladi. Bu yerda a,b,c uchburchak tomonlarining uzunliklari va x,y,z lar, mos ravishda, BC,AC,AB tomonlargacha bo'lgan masofalar.

22. $\forall n > 1,\ (n \in \mathbb{N})$ da quyidagi tenglikni isbotlang.

$$\sin\frac{\pi}{n}\cdot\sin\frac{2\pi}{n}\cdot\ldots\cdot\sin\frac{(n-1)\pi}{n} = \frac{n}{2^{n-1}}$$

23. $f(x)$ funksiya $(0,1)$ da differensiallanuvchi. Agar $f(0)=0$, $f(1)=1$ bo'lsa, u holda $\exists a,b \in (0,1)\ (a \neq b)$ topilishini isbotlangki $f'(a)f'(b)=1$ bo'lsin.

24. ABC uchburchak ichida ixtiyoriy O nuqta olingan va bu nuqtadan uchburchak tomonlariga parallel to'g'ri chiziqlar o'tkazilgan. $AB \parallel DE$, $BC \parallel MN$, $AC \parallel FK$. Bu yerda $F,M \in AB$, $E,K \in BC$, $D,N \in AC$. U holda quyidagi tenglikni isbotlang: $\frac{AF}{AB}+\frac{BE}{BC}+\frac{CN}{AC}=1$

25. $\forall n \geq 2$, $a_1,a_2,\ldots,a_n \geq 0$ sonlari uchun quyidagi tengsizlikni isbotlang:

$$a_1+a_2+\ldots+a_n - n\sqrt[n]{a_1a_2\ldots a_n} \geq \left(\sqrt{a_1}-\sqrt{a_2}\right)^2$$

26. $x+\frac{1}{x}=2\cos\alpha$ bo'lsa, $x^n+\frac{1}{x^n}$ nimaga teng?

27. Agar $a \geq 0$, $b \geq 0$, $c \geq 0$ bo'lsa, quyidagi tengsizlikni isbotlang.

$$a^3+b^3+c^3+6abc \geq \frac{1}{4}(a+b+c)^3$$

28. $x^y+1=z$ tenglamani tub sonlarda yeching.

29. Agar $f(x)=\frac{1}{1-x^2}$ $|x| \neq 1$ bo'lsa, u holda $f^{(n)}(0)$ ning qiymatini hisoblang.

30. $a_i > 0,\ b_i > 0\ (i=1,2,\ldots,n)$ sonlari uchun quyidagi tengsizlikni keltirib chiqaring:

$$\sqrt{(a_1 + a_2 + ... + a_n)^2 + (b_1 + b_2 + ... + b_n)^2} \le \sqrt{a_1^2 + a_1^2} + \sqrt{a_2^2 + a_2^2} + ... + \sqrt{a_n^2 + a_n^2}$$

31. Agar $f(x)$ funksiya $[x_1, x_2]$ $(0 < x_1 < x_2)$ segmentda differensiallanuvchi bo‘lsa, u holda $\exists\xi \in (x_1, x_2)$ nuqta topiladiki, $\frac{1}{x_2 - x_1}\begin{vmatrix} x_1 & x_2 \\ f(x_1) & f(x_2) \end{vmatrix} = f(\xi) - \xi f'(\xi)$ tenglik o‘rinli bo‘ladi. Shuni isbot qiling.

32. Agar $f(x)$ funksiya ikkinchi tartibli differensiallanuvchi bo‘lsa, u holda ixtiyoriy simmetrik oraliqda $\exists\xi \in (x_0 - r, x_0 + r)$ topilishini isbotlangki, quyidagi tenglik o‘rinli bo‘lsin:

$$f''(\xi) = \frac{3}{r^3}\int_{x_0 - r}^{x_0 + r}\big(f(x) - f(x_0)\big)dx$$

33. Isbotlang. $\overline{abc} \cdot \overline{bca} \cdot \overline{cab} \ge \overline{aaa} \cdot \overline{bbb} \cdot \overline{ccc}$ bu yerda $\overline{xyz}$ –uch xonali son.

34. $\{x_n\}$ ketma-ketlik quyidagi shartlarni qanoatlantiradi. $x_1 = 2012$, $x_{n+1} = \frac{1}{4 - 3x_n}$ u holda $\lim\limits_{n\to\infty} x_n$ ni hisoblang.

35. Agar $f(x)$ funksiya $[a, b]$ segmentda uzluksiz differensiallanuvchi bo‘lib, $f(a) = f(b) = 0$ bo‘lsa, u holda $\exists\xi \in (a, b)$ topilishini isbotlangki, quyidagi tengsizlik o‘rinli bo‘lsin. $\left|f'(\xi)\right| \ge \frac{4}{(b-a)^2}\int_0^1 \left|f(x)\right|dx$.

36. $\forall n \ge 2$ $(n \in \mathbb{N})$ da $\log_n(n+1) > \log_{n+1}(n+2)$ tengsizlikni isbotlang.

37. Qanday uchburchakda $\frac{a\cos\alpha + b\cos\beta + c\cos\gamma}{a\sin\beta + b\sin\gamma + c\sin\alpha} = \frac{P}{9R}$ tenglik bajariladi. Bu yerda a, b, c lar uchburchak tomonlari. α, β, γ lar tomonlarga mos burchaklar. P perimetr va R uchburchakka tashqi chizilgan aylana radiusi.

38. Ixtiyoriy uchburchak uchun quyidagi tengsizlikni isbotlang: $\frac{2R}{r} \ge \frac{1}{\sin\frac{\alpha}{2}\left(1 - \sin\frac{\alpha}{2}\right)}$ bu yerda R va r lar, mos ravishda, uchburchakka tashqi va ichki chizilgan aylana radiuslari. α uchburchakning biror burchagi.

39. Tenglamani yeching. $\begin{vmatrix} x & c_1 & c_2 \dots c_n \\ c_1 & x & c_2 \dots c_n \\ \dots & \dots & \dots \\ c_1 & c_2 & c_3 \dots x \end{vmatrix} = 0$

40. Tenglamani natural sonlarda yeching. $x^y = y^x$ bu yerda $x \neq y$.

41. Fibonachchi sonlari deb 1,2 dan boshlanuvchi shunday sonlar qatoriga aytiladiki, har bir keyingi had oldingi ikkita hadning yig'indisiga teng. 1,2,3,5,8,13,... Fibonachchi sonlarining n-hadi quyidagi determinantga tengligini isbotlang:

$$\begin{vmatrix} 1 & 1 & 0 & 0 \dots 0 & 0 \\ -1 & 1 & 1 & 0 \dots 0 & 0 \\ 0 & -1 & 1 & 1 \dots 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 \dots -1 & 1 \end{vmatrix}$$

42. Agar $f(x)$ funksiya a nuqtada chekli hosilaga ega bo'lsa, $\lim\limits_{n\to\infty}\left(\dfrac{f\left(a+\dfrac{1}{n}\right)}{f(a)}\right)^n$ limitni hisoblang.

43. Agar $a, b \in \mathbb{R}$ va $n \in \mathbb{N}$ bo'lsa, $\lim\limits_{n\to\infty}\int\limits_0^{2\pi}\left|a\cos nx - b\sin nx\right|dx$ ni hisoblang.

44. Agar $f(x)$ funksiya $[a, b]$ segmentda aniqlangan va uzluksiz, shu bilan birga, qavariq bo'lsa, u holda

$$(b-a)\frac{f(a)+f(b)}{2} \le \int\limits_a^b f(x)dx \le (b-a)f\left(\frac{a+b}{2}\right)$$

tengsizlikni isbotlang.

45. a, b, c lar to'g'ri burchakli uchburchakning tomonlari bo'lsa, (c gipotenuza) $ab(a+b+c) < \frac{5}{4}c^3$ tengsizlikni isbotlang.

46. Agar $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$ bo'lsa, e^A ni hisoblang.

47. a_{ij} koeffitsiyentlar butun son bo'lsa, quyidagi tenglamalar sistemasi yagona yechimga ega bo'lishini ko'rsating:

$$\begin{cases} \frac{1}{2}x_1 = a_{11}x_1 + ... + a_{1n}x_n \\ \frac{1}{2}x_2 = a_{21}x_1 + ... + a_{2n}x_n \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{1}{2}x_n = a_{n1}x_1 + ... + a_{nn}x_n \end{cases}$$

48. $\left(1+\frac{1}{n}\right)^{n+1} > \left(1+\frac{1}{n+1}\right)^{n+2}$, $\forall\, n \in \mathbb{N}$ tengsizlikni isbotlang.

49. $\int \sin(\ln x)dx$ aniqmas integralni hisoblang.

50. $\int e^x \cdot \sin x\, dx$ aniqmas integralni hisoblang.

51. $x_{n+1} = \frac{1}{2}\left(x_n + \frac{1}{x_n}\right)$ bo'lsin, bu yerda $x_0 = 2$. $\{x_n\}$ ketma-ketlik yaqinlashuvchi ekanini isbotlang.

52. $n! < \left(\frac{n+1}{2}\right)^n$ $(n \geq 2)$ tengsizlikni isbotlang.

53. $(a+b)^n = \sum_{k=0}^{n} C_n^k a^{n-k} b^k$ Nyuton binomi formulasini isbotlang.

54. Agar $f(x)$ funksiya [0,1] da differensiallanuvchi bo'lib, $f'(0)=1, f'(1)=0$ bo'lsa, $\exists\, c \in (0,1)$ uchun $f'(c)=c$ ekanini isbotlang.

55. $f(x) \in [0,1]$ va $(0,1)$ da differensiallanuvchi. Agar $f(0) = f(1) = 0$ bo'lsa, $\exists\, x \in (0,1)$ uchun $f'(x) = f(x)$ ekanini isbotlang.

56. $P(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ... + \frac{x^n}{n!}$ ko'phad karrali ildizga ega emasligini isbotlang.

57. $a_1 = 0,\ a_n = \frac{a_{n-1}+3}{4}$ ketma-ketlikning yaqinlashuvchi ekanini isbotlang va uning limitini toping.

58. $[x] + [2x] + [3x] = 6$ tenglamani yeching, bu yerda $[x]$ - x sonning butun qismi.

59. $\int \frac{\sin x}{\sin x + \cos x + \sqrt{2}} dx$ aniqmas integralni hisoblang.

60. Quyidagi integrallarni hisoblang.

a) $\int \sqrt{\operatorname{tg} x}\, dx$; b) $\int \frac{dx}{\sin x}$; c) $\int_0^1 x f(x^2) dx$ bu yerda $\int_0^1 f(x) dx = a$

61. $f(x) = x^3 - 3x^2 + 1$ tenglamaning ildizi $0,6 < x_0 < 0,7$ oraliqda ekanini isbotlang.

62. $f(x) = x^2 - 8x + 1$ tenglamaning ildizi $x_0 \in (0,1)$ bo'lsa, bu ildizni verguldan keyin o'ndan bir aniqlikda toping.

63. Ixtiyoriy $x_1, x_2, ..., x_n \in \mathbb{R}$ uchun $|x_1 + x_2 + ... + x_n| \leq |x_1| + |x_2| + ... + |x_n|$ tengsizlikni isbotlang.

64. 2^{nx} va 5^{nx} sonlar ketma-ket yozilsa hosil bo'lgan sonning raqamlari soni $nx + 1$ ekanini isbotlang. Bu yerda $n \in \mathbb{N}$ shartni qanoatlantiruvchi ixtiyoriy natural son.

65. $f(x) \in C[a,b]$ bo'lsin. Agar $x_1, x_2, x_3 \in [a,b]$ bo'lsa, $\exists \xi \in (a,b)$ topiladiki, $f(\xi) = \frac{f(x_1) + f(x_2) + f(x_3)}{3}$ tenglik o'rinli bo'ladi. Shuni isbotlang.

66. $f(x) \in C^1[0,\infty)$, $|f^1(x)| \leq M$ bo'lsin. U holda $f(x)$ funksiya $[0,\infty)$ da tekis uzluksiz bo'lishini isbotlang.

67. i^i ni hisoblang. Bu yerda $i^2 + 1 = 0$.

68. $\forall z_1, z_2 \in \mathbb{C}$ kompleks sonlar uchun $|z_1 + z_2| \leq |z_1| + |z_2|$ tengsizlikni isbotlang.

69. $\forall z_1, z_2, ..., z_n \in \mathbb{C}$ kompleks sonlar uchun

$$|z_1 + z_2 + ... + z_n| \leq |z_1| + |z_2| + ... + |z_n|$$

tengsizlikni isbotlang.

70. $\int_0^{+\infty} e^{x^2} dx$ xosmas integralni hisoblang.

71. Faqat 2013 ta nuqtada uzluksiz va qolgan nuqtalarda uzilishga ega bo'lgan funksiya quring.

72. $f(x)$ va $g(x)$ funksiyalar berilgan bo'lsin. Ular integrallanuvchi bo'lmasa, ularning yig'indisi integrallanuvchi bo'ladimi? Integrallanuvchi bo'lsa, shu funksiyalarni toping.

73. Agar $f'(\sin^2 x) = 1 + \cos^2 x$ bo'lsa $f(x)$ ni toping.

74. $\lim\limits_{t\to 0}\int\limits_{-1}^{1}\frac{t}{t^2+x^2}\cos x\,dx$ limitni hisoblang.

75. $f_1(x)=x^2$ va $f_2(x)=x-1$ funksiyalar grafiklari orasidagi eng qisqa masofani toping.

76. Musbat x,y,z sonlar $\frac{1}{\sqrt{2}}\le z<\frac{1}{2}\min\{x\sqrt{2},y\sqrt{3}\}$, $x+z\sqrt{3}\ge\sqrt{6}$, $y\sqrt{3}+z\sqrt{10}\ge 2\sqrt{5}$ shartlarni qanoatlantirsa, $P(x,y,z)=\frac{1}{x^2}+\frac{2}{y^2}+\frac{3}{z^3}$ ifodaning eng katta qiymatini toping.

77. $n\in\mathbb{N},\ n>1$ uchun $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n}<n-n^{\frac{n-1}{n}}$ tengsizlikni isbotlang.

78. $x_1,x_2,\ldots,x_{2011}$ musbat sonlari $\sum\limits_{i=1}^{2011}\frac{1}{1+x_{i_i}^2}=1$ shartni qanoatlantirsa, $x_1\cdot x_2\cdot\ldots\cdot x_n\ge 2010^{2011}$ tengsizlikning o'rinli ekanini isbotlang.

79. $x_1,x_2,\ldots,x_n$ $(n\ge 2)$ musbat sonlari

$$\frac{1}{x_1+2011}+\frac{1}{x_2+2011}+\ldots+\frac{1}{x_n+2011}=\frac{1}{2011}$$

tenglikni qanoatlantirsa, $\frac{\sqrt[n]{x_1x_2\ldots x_n}}{n-1}\ge 2011$ tengsizlikni isbotlang.

80. Agar $x_i\in\left(0;\frac{\pi}{2}\right)$, $(i=1,2,\ldots,n)$ sonlari $\sum\limits_{i=1}^{n}\mathrm{tg}x_i\le n$ shartni qanoatlantirsa, $\sin x_1\cdot\sin x_2\cdot\ldots\cdot\sin x_n\le 2^{-\frac{n}{2}}$ tengsizlikni isbotlang.

81. Istalgan natural n soni $(n\ge 2)$ uchun

$$\frac{1}{n+1}\left(1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-1}\right)>\frac{1}{n}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots+\frac{1}{2n}\right)$$

tengsizlikni isbotlang.

82. a) agar a,b,c sonlari uchburchak tomonlarining uzunliklari bo'lib, $a+b+c=1$ shartni qanoatlantirsa, $n\in\mathbb{N}$, $n\ge 2$ uchun $\sqrt[n]{a^n+b^n}+\sqrt[n]{b^n+c^n}+\sqrt[n]{c^n+a^n}<1+\frac{\sqrt{2}}{2}$ tengsizlikni isbotlang;

b) agar musbat x,y,z sonlari $xyz=x+y+z+2$ tenglikni qanoatlantirsa, $5(x+y+z)+18\ge 8(\sqrt{xy}+\sqrt{yz}+\sqrt{zx})$ tengsizlik o'rinli bo'lishini isbotlang.

83. $(a^n - b^n) = (a-b)(a^{n-1} + a^{n-2} + ... + b^{n-1})$ tenglikni isbotlang.

84. $\left|\sin^n x\right| \le \left|\sin x\right|$, $\forall n \in \mathbb{N}, x \in (-\infty, +\infty)$ tengsizlikni isbotlang.

85. Ixtiyoriy n natural son uchun quyidagi tengsizlikni isbotlang:

$$\left|\sin nx\right| \le n\left|\sin x\right|$$

86. Ixtiyoriy n natural sonlarda $\frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{3n+1} > 1$ tengsizlikni isbotlang.

87. Quyidagi tengsizlikni isbotlang. $\frac{5^n}{n!} \le \frac{5^5}{5!}\left(\frac{5}{6}\right)^{n-5}$, $\forall\, n \in \mathbb{N}, n \ge 6$.

88. $S_n = arctg\frac{1}{2} + arctg\frac{1}{8} + ... + arctg\frac{1}{2n^2}$, $\forall n \in \mathbb{N}$ yig'indini toping.

89. Tengsizlikni isbotlang. $\ln(n+1) < 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} < \ln n + 1$, $\forall n \in \mathbb{N}$, $n \ge 2$

90. Ixtiyoriy juft n natural son uchun quyidagi tengsizlikni isbotlang:

$$x - \frac{x^n}{3!} + ... - \frac{x^{2n-1}}{(2n-1)!} \le \sin x \le x - \frac{x^3}{3!} + ... + \frac{x^{2n+1}}{(2n+1)!}, \quad 0 \le x \le \frac{\pi}{2}.$$

91. Ixtiyoriy $n$natural son uchun $(10^n + 10^{n-1} + ... + 1)\cdot(10^{n+1} + 5) + 1$ soni to'la kvadrat ekanligini isbotlang.

92. Ixtiyoriy natural m, n, p larda $x^{3m} + x^{3n+1} = x^{3p+2}$ ko'phad $x^2 + x + 1$ ko'phadga qoldiqsiz bo'lishini isbotlang.

93. Qanday natural m, n, p larda $x^{3m} - x^{3n+1} = x^{3p+2}$ ko'phad $x^2 - x + 1$ ko'phadga qoldiqsiz bo'linadi.

94. Qanday natural m, n, p larda larda $x^{3m} + x^{3n+1} = x^{3p+2}$ ko'phad $x^4 + x^2 + 1$ ko'phadga qoldiqsiz bo'linadi.

95. m ning qanday qiymatlarida $x^{2m} + x^m + 1$ ko'phad $x^2 + x + 1$ ko'phadga qoldiqsiz bo'linadi.

97. m ning qanday natural qiymatlarida $(x+1)^m + x^m + 1$ ko'phad $(x^2 + x + 1)^2$ ko'phadga qoldiqsiz bo'linadi.

96. Ixtiyoriy natural k va $a_1, a_2, ..., a_k$ larda $x^{ka_1} + x^{ka_2+1} + ... + x^{ka_k+k-1}$ ko'phadni $x^{k-1} + x^{k-2} + ... + 1$ ko'phadga qoldiqsiz bo'linishini isbotlang.

98. m ning qanday natural qiymatlarida $(x+1)^m + x^m + 1$ ko‘phad $x^2 + x + 1$ ko‘phadga qoldiqsiz bo‘linadi.

99. m ning qanday natural qiymatlarida $(x+1)^m - x^m - 1$ ko‘phad $(x^2 + x + 1)^2$ ko‘phadga qoldiqsiz bo‘linadi.

100. $a, b, c > 0$ va $a^2 + b^2 + c^2 = \frac{5}{3}$ bo‘lsa, $\frac{1}{a} + \frac{1}{b} - \frac{1}{c} < \frac{1}{abc}$ tengsizlikni isbotlang.

101. $\forall n \in \mathbb{N}$ lar uchun quyidagi tengsizlikni isbotlang.

$$\frac{e}{2n+1} < e - \left(1 + \frac{1}{n}\right)^n$$

102. (*Tyoplitsa teoremasi*) Agar $\lim\limits_{n\to\infty} x_n = a$ bo‘lib, $\sum\limits_{k=1}^{n} P_k = 1,\ P_k \geq 0,\ k = \overline{1,n}$ bo‘lsa, $\sum\limits_{k=1}^{n} P_k x_k$ ketma-ketlik yaqinlashuvhi va $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} P_k x_k = a$ bo‘lishini isbotlang.

103. Agar $a, b > 0$ va $x_i \in [a,b],\ (i = \overline{1,n})$ bo‘lsa, quyidagi tengsizlikni isbotlang: $(x_1 + x_2 + ... + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n}\right) \leq \frac{b}{a} \cdot n^2$

104. $\lim\limits_{x\to 0} \frac{1 - \cos x \cos 2x \cos 3x ... \cos nx}{x^2}$ limitni hisoblang.

105. Taqqoslang. e^{π} va π^{e}

106. Agar $a, b > 0$ va m butun son bo‘lsa, $\left(1 + \frac{a}{b}\right)^m + \left(1 + \frac{b}{a}\right)^m \geq 2^{m+1}$ tengsizlikni isbotlang.

107. $\forall n \in \mathbb{N}$ uchun $\frac{\pi^2}{6}\left(1 - \frac{6n+1}{(2n+1)^2}\right) < 1 + \frac{1}{2^2} + ... + \frac{1}{n^2} < \frac{\pi^2}{6}\left(1 - \frac{1}{(2n+1)^2}\right)$ qo‘sh tengsizlikning bajarilishini isbotlang.

108. Tenglikni isbotlang. $\sum\limits_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

109. O‘zaro kollinear bo‘lmagan $\vec{a}, \vec{b}, \vec{c}$ vektorlar berilgan. Agar $\vec{a} + \vec{b}$ vektor $\vec{c}$ vektorga va $\vec{b} + \vec{c}$ vektor $\vec{a}$ ga kollinear bo‘lsa, $\vec{a} + \vec{b} + \vec{c}$ vektorning uzunligini toping.

110. Tenglamaning haqiqiy ildizini toping. $xe^{-x} + e^{-x} + \frac{x^2}{2} - 1 = 0$.

111. Agar barcha haqiqiy x lar uchun

$$f(2x^3 - x) - 4x^2 f(x^2 - x - 1) = 8x^5 - 8x^3 - 11x^2 + 2$$

tenglik oʻrinli boʻlsa, $y = f(x)$ juft funksiyaga $x_0 = 1$ nuqtada oʻtkazilgan urinma tenglamasini toping.

112. Limitni hisoblang. $\lim\limits_{x\to\infty}\left(tg\dfrac{\pi x}{2x+1}\right)^{\frac{1}{x}}$

113. Agar $x \in [-\pi, \pi]$ boʻlsa, $\cos^2 x \cdot \sin x > -\dfrac{2}{3}$ tengsizlikni isbotlang.

114. Quyidagi limitni hisoblang:

$$\lim_{x\to+\infty}[(x+2)\ln(x+2) - 2(x+1)\ln(x+1) + x\ln x]$$

115. Ixtiyoriy ABC uchburchakda $\left(\overrightarrow{AB},\overrightarrow{BC}\right) + \left(\overrightarrow{BC},\overrightarrow{CA}\right) + \left(\overrightarrow{CA},\overrightarrow{AB}\right) < 0$ tengsizlikning bajarilishini isbotlang.

116. Hisoblang. $\lim\limits_{n\to\infty}(1 + a^n)^{\frac{1}{n}}, \quad a > 0$

117. Limitni hisoblang. $\lim\limits_{n\to\infty}\cos\dfrac{\pi}{4}\cos\dfrac{\pi}{8}\ldots\cos\dfrac{\pi}{2^{n+1}}$.

118. A va B lar nol boʻlmagan kvadrat matritsalar boʻlsin. Agar $AB = O$ (O-nol matritsa) boʻlsa, $\det A = \det B = 0$ ekanini isbotlang.

119. Tenglamalar sistemasini yeching.

$$\begin{cases} x_1 + 2x_2 + \ldots + 9x_9 + 10x_{10} = 55 \\ x_2 + 2x_3 + \ldots + 9x_{10} + 10x_1 = 55 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_{10} + 2x_1 + \ldots + 9x_8 + 10x_9 = 55 \end{cases}$$

120. Limitni hisoblang. $\lim\limits_{x\to\infty}\dfrac{(ax+1)^n}{x^n + b}$

121. x ni toping. $\lim\limits_{n\to\infty}\dfrac{n^x - (n-1)^x}{(n+1)^{x-1} + (n+2)^{x-1}} = 2013$

122. x ni toping. $\lim\limits_{n\to\infty}\left((1+x)(1+x^2)(1+x^4)\ldots(1+x^{2n})\right) = 2013$

123. Limitni hisoblang. $\lim\limits_{n\to\infty}\dfrac{(x+a+b)^{2x+a+b}}{(x+a)^{x+a}(x+b)^{x+b}}$

124. Funksiyaning eng katta va eng kichik qiymatini toping.

$$f(x) = (\arcsin x)^3 + (\arccos x)^3$$

125. (*Shtols teoremasi*) Bizga $\{x_n\}$ va $\{y_n\}$ sonli ketma-ketliklar berilgan. Ular quyidagi shartlarni qanoatlantirsin:

1) $y_{n+1} > y_n \ (n \in \mathbb{N})$; 2) $\lim\limits_{n\to\infty} y_n = +\infty, \quad \lim\limits_{n\to\infty} x_n = +\infty$;

3) $\lim\limits_{n\to\infty} \frac{x_{n+1} - x_n}{y_{n+1} - y_n} = l$ (l-chekli yoki cheksiz).

U holda $\lim\limits_{n\to\infty} \frac{x_n}{y_n} = l$ tenglikni isbotlang.

126. Agar α, β va γ ixtiyoriy uchburchakning ichki burchaklari bo‘lsa, u holda quyidagi tengsizlikni isbotlang: $\sin^2\alpha + \sin^2\beta + \sin^2\gamma \leq \frac{9}{4}$

127. Ushbu $a^2 + b^2 + c^2 \leq 9R^2$ tengsizlikni isbotlang. Bu yerda a, b, c lar uchburchak tomonlari va R tashqi chizilgan aylana radiusi.

128. ABC uchburchakda O nuqta tashqi chizilgan aylana markazi va H-balandliklar kesishgan nuqta bo‘lsa, quyidagi tenglikni isbotlang:

$$\overrightarrow{OH} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$

129. Agar $x^2 + y^2 + z^2 = 1$ bo‘lsa, $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} < 1$ tengsizlikni isbotlang.

130. Ixtiyoriy natural n uchun $\{n\sqrt{2}\} > \frac{1}{2n\sqrt{2}}$ tengsizlikning bajarilishini isbotlang. Bu yerda $\{a\}$- ifoda a sonning kasr qismini ifodalaydi.

131. Quyidagi yig‘indilarni toping:

1) $C_n^0 + C_n^2 + C_n^4 + C_n^6 + ...$;

2) $C_n^1 + C_n^3 + C_n^5 + C_n^7 + ...$;

132. Quyidagi yig‘indilarni toping:

1) $C_n^0 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - C_n^{10} + ...$;

2) $C_n^1 - C_n^3 + C_n^5 - C_n^7 + C_n^9 - C_n^{11} + ...$

133. Quyidagi yig‘indilarni hisoblang:

a) $C_n^0 + C_n^3 + C_n^6 + ...$;

b) $C_n^2 + C_n^5 + C_n^8 + ...$;

c) $C_n^1 + C_n^4 + C_n^7 + ...$

134. Quyidagi yig'indilarni hisoblang:

a) $C_n^0 + C_n^4 + C_n^8 + ...;$

b) $C_n^1 + C_n^5 + C_n^9 + ...;$

c) $C_n^2 + C_n^6 + C_n^{10} + ...;$

d) $C_n^3 + C_n^7 + C_n^{11} + ...$

135. Quyidagi yig'indini hisoblang:

a) $C_n^1 - \frac{1}{3}C_n^3 + \frac{1}{9}C_n^5 - \frac{1}{27}C_n^7 - ...;$ b) $C_n^0 - \frac{1}{3}C_n^2 + \frac{1}{9}C_n^4 - \frac{1}{27}C_n^6 + ...$

136. Limitni hisoblang: $\lim_{n\to\infty} \frac{1 + 2\sqrt{2} + 3\sqrt{3} + ... + n\sqrt{n}}{n^2\sqrt{n}}$

137. Agar $p \in \mathbb{N}$ bo'lsa, $\lim_{n\to\infty} \frac{1^p + 2^p + 3^p + ... + n^p}{n^{p+1}}$ limitni hisoblang:

138. Ayniyatni isbotlang.

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{2n-1} - \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n}$$

va $n \to \infty$ dagi limitini hisoblang.

139. $a, b, c \in \mathbb{R}^+,\quad a^{\log_3 7} = 27,\quad b^{\log_7 11} = 49,\quad c^{\log_{11} 25} = \sqrt{11}$ bo'lsa,

$a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}$ ni toping.

140. $arctg\frac{1}{3} + arctg\frac{1}{4} + arctg\frac{1}{5} + arctg\frac{1}{n} = \frac{\pi}{4},\quad n \in \mathbb{N}$?

141. $N = 100^2 + 99^2 - 98^2 - 27^2 + ... + 4^2 + 3^2 - 2^2 - 1^2$ va $N = x(\mathrm{mod}\,1000)$

bo'lsa, $x = ?$

142. r, s, t sonlar $8x^3 + 1001x + 2008 = 0$ tenglama ildizlari.

$$(r+s)^3 + (s+t)^3 + (t+r)^3\ ?$$

143. $a_{n+1} = \frac{n}{n+1}(a_n + 1), a_1 = 0, n \in \mathbb{N}$ bo'lsa a_{2014} ni toping.

144. Taqqoslang: $\left(3^5\right)^{\left(5^3\right)}$ va $\left(5^3\right)^{\left(3^5\right)}$.

145. $a_1, a_2, ...$ sonlar ketma-ketligi uchun da $a_{n+1} - 2a_n + a_{n-1} = 1$ tenglik bajariladi. $\{a_n\}$ ni $a_1, a_2,\ n$ lar orqali ifodalang.

146. $9 \cdot 99 \cdot 999 \cdot ... \cdot \underbrace{999...9}_{9999ta} = x(\mathrm{mod}\,1000)$ bo'lsa, $x = ?$

147. $x, y, z > 0,\quad xyz = 10^{81},\ (\lg x)(\lg yz) + (\lg y)(\lg z) = 468,$ bo'lsa,

$$\sqrt{\lg^2 x + \lg^2 y + \lg^2 z} - ?$$

148. $\{a_n\}$ ketma-ketlikda $a_1 = 1, a_2 = 1, a_{n+2} = 5a_{n+1} - 6a_n$ munosabat berilgan, bunda $n \geq 1$. Shu ketma-ketlikning umumiy hadini n orqali ifodalang.

149. $\{a_n\}$-ketma-ketlik ixtiyoriy $n \in \mathbb{N}$, $n \geq 1$ uchun $a_1 = 1$ va $a_{n+1} = \dfrac{a_n}{1 + na_n}$ tenglikni qanoatlantiradi. U holda a_{2014} ni hisoblang.

150. $\{a_n\}$ ketma ketlikda $a_1 = 2$, $a_2 = 3$, $a_{n+2} = \dfrac{a_{n+1}}{a_n}$, $n \in \mathbb{N}$ shartlar o'rinli bo'lsa, a_{2014} ni toping.

151. $\{a_n\}$ ketma-ketlik $a_1 = 1$ va $a_{n+1} = a_n + \dfrac{1}{a_n^2}$ shartlar bilan berilgan. $a_{9000} > 30$ tengsizlikni isbotlang.

152. Tenglamani yeching. $x^2 + 2 + \dfrac{4}{x^2 - 2x + 2} = 2x + \sqrt{12 - x^2 + 4x}$

153. Ifodaning eng katta qiymatini toping. $\sin^2 x \cdot \cos^4 x \cdot (2 - \sin^2 x)$.

154. Tengsizlikni isbotlang. $\left(2\sqrt[4]{a} + 2\sqrt[4]{b} + 2\sqrt[4]{c}\right) - (\sqrt{a} + \sqrt{b} + \sqrt{c}) \leq 3$

155. $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n!$ yig'indini toping.

156. $a_0, a_1, a_2, ..., a_{100}$ - natural sonlar va $a_1 > a_o$, $a_2 = 3a_1 - 2a_0$, $a_3 = 3a_2 - 2a_1, ..., a_{100} = 3a_{99} - 2a_{98}$ ekani ma'lum bo'lsa, $a_{100} > 2^{99}$ ekanini isbotlang.

157. Quyidagi ifodaning butun qismini toping.

$$\sqrt{2014 + \sqrt{2014 + \sqrt{2014 + ... + \sqrt{2014 + \sqrt{2014}}}}}$$

(bu yerda ildizlar soni cheksiz ko'p).

158. $a_n = \dfrac{1}{(n+1)\sqrt{n} + n\sqrt{n+1}}$ bo'lsa, $a_1 + a_2 + a_3 + ... + a_{99}$ ni hisoblang.

2-§. Olimpiada masalalari uchun yechimlar va ko‘rsatmalar

1. Mashhur Koshi tengsizligini qo‘llaymiz,

$$\frac{1}{x_1}+\frac{2}{x_2}+...+\frac{n}{x_n}=\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_3}+\frac{1}{x_3}+...+\underbrace{\frac{1}{x_n}+\frac{1}{x_n}+...+\frac{1}{x_n}}_{n\,marta}\geq$$

$$\geq\frac{n(n+1)}{2}\cdot\sqrt[\frac{n(n+1)}{2}]{\frac{1}{x_1x_2{}^2x_3{}^3...x_n{}^n}}=\frac{n(n+1)}{2}$$

isbot tugadi.

Natija. Agar $a>0, b>0, c>0$ sonlari uchun $ab^2c^3=1$ tenglik o‘rinli bo‘lsa, $\frac{1}{a}+\frac{2}{b^2}+\frac{3}{c^3}\geq 6$ tengsizlikni isbotlang. Bu tengsizlik yuqoridagi tengsizlikning $n=3$ holiga tushadi.

2. Buning uchun Koshi tengsizligini bir marta qo‘llash kifoya, ya’ni

$$n-\left(\frac{1}{2^2}+\frac{1}{3^2}+...+\frac{1}{n^2}\right)=\left(1-\frac{1}{2^2}\right)+\left(1-\frac{1}{3^2}\right)+...+\left(1-\frac{1}{n^2}\right)\geq$$

$$\geq n\sqrt[n]{\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)...\left(1-\frac{1}{n^2}\right)}=n\sqrt[n]{\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)...\left(1-\frac{1}{n^2}\right)}=$$

$$=n\sqrt[n]{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot...\cdot\frac{n-1}{n}\cdot\frac{n+1}{n}}=n\sqrt[n]{\frac{1}{2}\left(1+\frac{1}{n}\right)}$$

bundan berilgan tengsizlik kelib chiqadi.

3. Biz $x\geq 3$ da $f(x)=\frac{\ln x}{x}$ funksiyani qaraylik.

$$f'(x)=\frac{1-\ln x}{x^2}<0 \quad (x\geq 3)$$

Bu esa funksiyaning $(x\geq 3)$ oraliqda kamayuvchi ekanini bildiradi. U holda $\forall n, n+1\in[3,+\infty)$ sonlariga funksiyani ta’sir ettirsak:

$$n<n+1 \Rightarrow f(n)>f(n+1) \Rightarrow \frac{\ln n}{n}>\frac{\ln(n+1)}{n+1}\Rightarrow$$

$(n+1)\ln n>n\ln(n+1)\Rightarrow n^{n+1}>(n+1)^n$ ekanligi kelib chiqadi.

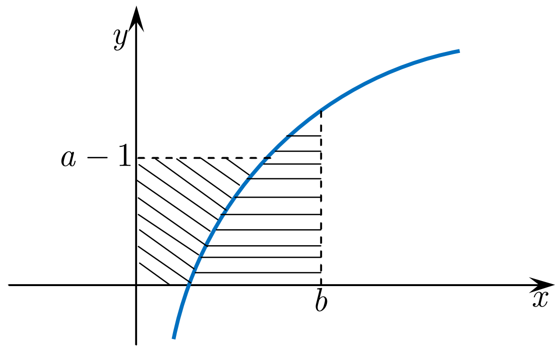
4. Uning ildizlarini $\alpha_1,\alpha_2,...,\alpha_n$ deb belgilaylik. U holda $P(x)=(x-\alpha_1)(x-\alpha_2)...(x-\alpha_n)$ tenglik o‘rinli. $\alpha_i=-\beta_i$ almashtirish bajarsak, $(i=\overline{1,n})$

$$P(x)=(x+\beta_1)(x+\beta_2)...(x+\beta_n)\Rightarrow P(2)=(2+\beta_1)(2+\beta_2)...(2+\beta_n)=$$
$$=(1+1+\beta_1)(1+1+\beta_2)...(1+1+\beta_n)\geq 3\sqrt[3]{\beta_1}\cdot 3\sqrt[3]{\beta_2}\cdot...\cdot 3\sqrt[3]{\beta_n}=3^n\sqrt[3]{\beta_1\beta_2...\beta_n}=3^n$$

ya’ni $P(n)\geq 3^n$. Chunki Viet teoremasiga ko‘ra, $\beta_1\beta_2...\beta_n=1$.

5. $y=\ln x \quad (x>0)$ funksiyani qaraymiz.

$y = \ln x \Rightarrow x = e^y$



$$(a-1)b \le \int_0^{a-1} e^y dy + \int_1^b \ln x dx = e^y\Big|_0^{a-1} + (x\ln x - x)\Big|_1^b = e^{a-1} - 1 + b\ln b - b \Rightarrow$$

$$\Rightarrow \quad ab \le e^{a-1} + b\ln b$$

6. $P(x)$ ko'phad n turli ildizga ega ekanligidan uni quyidagicha ko'rinishda yozish mumkin:

$$P(x) = (x - x_1)(x - x_2)\dots(x - x_n)$$

$$\forall j \in \overline{1,n} \quad P(x_j) = 0$$

Endi $m < n$ ekanligidan $\dfrac{Q(x)}{P(x)}$ ni oddiy kasrlarga yoyamiz:

$$\frac{Q(x)}{P(x)} = \sum_{k=1}^{n} \frac{A_k}{x - x_k} \qquad (*)$$

Bu oxirgi tenglikni ikkala tarafini ham $[P(x) - P(x_j)]$ ga ko'paytiramiz, u holda quyidagi ifodaga kelamiz:

$$Q(x) = \sum_{k=1}^{n} \frac{A_k[P(x) - P(x_j)]}{x - x_j} \qquad (**)$$

Endi oxirgi tenglikda $x \to x_j$ *da* limitga o'tsak hamda $\lim\limits_{x\to x_j} \dfrac{P(x) - P(x_j)}{x - x_j} = P'(x_j)$ ekanligini e'tiborga olsak,

$$\lim_{x\to x_j} Q(x) = \lim_{x\to x_j} \sum_{k=1}^{n} \frac{A_k[P(x) - P(x_j)]}{x - x_j} \quad \Rightarrow \quad Q(x_j) = A_j P'(x_j) \Rightarrow A_j = \frac{Q(x_j)}{P'(x_j)}$$

munosabatlarga ega bo'lamiz. Bularni $(*)$ ga olib borib qo'yasak va shu $(*)$ ifodaning ikkala tarafini ham x ga ko'paytirsak,

$$\frac{xQ(x)}{P(x)} = \sum_{k=1}^{n} \frac{xQ(x_k)}{P'(x_k)(x - x_k)} \qquad (***)$$

Bu $(***)$ ifodada $x \to \infty$ da limitga o'tsak, unda biz quyidagi ikki holga ega bo'lishimiz mumkin: 1-hol. $m = n - 1$ bo'lganda

$$\lim_{x\to\infty} \frac{xQ(x)}{P(x)} = \lim_{x\to\infty} \sum_{k=1}^{n} \frac{xQ(x_k)}{P'(x_k)(x - x_k)} \quad \Rightarrow 1 = \sum_{k=1}^{n} \frac{Q(x_k)}{P'(x_k)}$$

2-hol. $m < n-1$ bo‘lsa,

$$\lim_{x\to\infty}\frac{xQ(x)}{P(x)}=\lim_{x\to\infty}\sum_{k=1}^{n}\frac{xQ(x_k)}{P'(x_k)(x-x_k)} \Rightarrow 0=\sum_{k=1}^{n}\frac{Q(x_k)}{P'(x_k)} \text{ bo‘ladi.}$$

7. $a < c \Rightarrow a^{n-2} < c^{n-2}, \quad b < c \Rightarrow b^{n-2} < c^{n-2}$

$$a^n+b^n=a^2\cdot a^{n-2}+b^2\cdot b^{n-2}<a^2\cdot c^{n-2}+b^2\cdot c^{n-2}=c^{n-2}\cdot\left(a^2+b^2\right)=c^{n-2}\cdot c^2=c^2$$

$$a^n+b^n<c^n$$

Natija: $n\ge 3$ da $(\sin\alpha)^n+(\cos\alpha)^n<1$.

8. $|x|<1$ bo‘lgani uchun $x=\cos\alpha$ deb belgilab olamiz. Bu yerda $\cos\alpha=1, \cos\alpha=-1$ qiymatlarni qabul qilmaydi deb olamiz. U holda

$$(1-\cos\alpha)^n+(1+\cos\alpha)^n=2^n\left(\left(\sin\frac{\alpha}{2}\right)^{2n}+\left(\cos\frac{\alpha}{2}\right)^{2n}\right)<2^n$$ chunki 7-misolning natijasiga ko‘ra, $2n\ge 4$ bo‘lgani uchun $\left(\sin\frac{\alpha}{2}\right)^{2n}+\left(\cos\frac{\alpha}{2}\right)^{2n}<1$. Bundan esa berilgan tengsizlikning isboti kelib chiqadi.

9. Buning uchun $x>0, y>0, z>0$ sonlariga $x^2+y^2+z^2\ge xy+xz+yz$ tengsizlikni 3 marta qo‘llasak berilgan tengsizlik hosil bo‘ladi.

$$\frac{a^8+b^8+c^8}{a^3b^3c^3}=\frac{a^5}{b^3c^3}+\frac{b^5}{a^3c^3}+\frac{c^5}{a^3b^3}\ge\sqrt{\frac{a^5b^5}{a^3b^3c^6}}+\sqrt{\frac{a^5c^5}{a^3b^6c^3}}+\sqrt{\frac{b^5c^5}{a^6b^3c^3}}=\frac{ab}{c^3}+\frac{ac}{b^3}+\frac{bc}{a^3}\ge$$

$$\ge\sqrt{\frac{a^2bc}{b^3c^3}}+\sqrt{\frac{ab^2c}{a^3c^3}}+\sqrt{\frac{abc^2}{a^3b^3}}=\frac{a}{bc}+\frac{b}{ac}+\frac{c}{ab}\ge\sqrt{\frac{ab}{abc^2}}+\sqrt{\frac{ac}{ab^2c}}+\sqrt{\frac{bc}{a^2bc}}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}.$$

10. Biz $x>0$ oraliqda $f(x)=\ln x^x$ funksiyani qaraylik. $f'(x)=\ln x+1$ $\quad f''(x)=\frac{1}{x}>0 \quad (x>0)$ $f(x)$ ga Iensen tengsizligini qo‘llaymiz, ya’ni $a_1>0, a_2>0, \ldots a_n>0$ va $p_1\ge 0, p_2\ge 0, \ldots, p_n\ge 0$ $p_1+p_2+\ldots+p_n=1$ shartlarni qanoatlantiruvchi $a_i, p_i \left(i=\overline{1,n}\right)$ lar uchun $f''(x)\ge 0$ bo‘lsa, $f(p_1a_1+\ldots+p_na_n)\le p_1f(a_1)+\ldots+p_nf(a_n)$ o‘rinli, bundan

$$\ln(p_1a_1+\ldots+p_na_n)^{(p_1a_1+\ldots+p_na_n)}\le p_1\ln a_1^{a_1}+\ldots+p_n\ln a_n^{a_n}=$$

$$=\ln a_1^{a_1p_1}+\ldots+\ln a_n^{a_np_n}=\ln\left(a_1^{a_1p_1}\cdot\ldots\cdot a_n^{a_np_n}\right)\Rightarrow$$

$\Rightarrow \quad (p_1a_1+\ldots+p_na_n)^{(p_1a_1+\ldots+p_na_n)}\le a_1^{a_1p_1}\cdot\ldots\cdot a_n^{a_np_n}$ endi $p_1=p_2=\ldots=p_n=\frac{1}{n}$ deb olsak,

$$\left(a_1^{a_1}a_2^{a_2}\ldots a_n^{a_n}\right)^{\frac{1}{n}}\ge\left(\frac{a_1+a_2+\ldots+a_n}{n}\right)^{\frac{a_1+a_2+\ldots+a_n}{n}}$$ Koshi tengsizligiga ko‘ra,

$$\left(a_1^{a_1}a_2^{a_2}...a_n^{a_n}\right)^{\frac{1}{n}} \geq \left(\frac{a_1+a_2+...+a_n}{n}\right)^{\frac{a_1+a_2+...+a_n}{n}} \geq \left(\left(a_1^{a_1}a_2^{a_2}...a_n^{a_n}\right)^{\frac{1}{n}}\right)^{\frac{a_1+a_2+...+a_n}{n}} \Rightarrow$$

$$\Rightarrow \left(a_1^{a_1}\cdot a_2^{a_2}\cdot ...\cdot a_n^{a_n}\right)^n \geq \left(a_1a_2...a_n\right)^{a_1+a_2+...+a_n}.$$

11. $\forall x>0$ uchun $x \geq \ln x + 1$ ekanidan foydalanamiz. $x^py^qz^r=1$ ning ikkala tomonini natural logarifm ostiga olamiz. Natijada quyidagiga ega bo'lamiz:

$$p\ln x + q\ln y + r\ln z = 0 \qquad p\ln x + q\ln y + r\ln z \leq p(x-1)+q(y-1)+r(z-1) =$$
$$= px+qy+rz-(p+q+r) \quad \Rightarrow \; px+qy+rz \geq p+q+r=1$$

End

i $a,b,c,d>0$ sonlari uchun $\frac{a^2}{b}+\frac{c^2}{d} \geq \frac{(a+c)^2}{b+d}$ ekanini ko'rsatamiz.

$$(a^2d+c^2b)(b+d) \geq bd(a+c)^2 \;\Rightarrow\; a^2bd+a^2d^2+c^2b^2+c^2bd \geq bda^2+2abcd+bdc^2 \Rightarrow$$
$$\Rightarrow\; (ad-bc)^2 \geq 0$$

Shunga ko'ra,

$$\frac{p^2x^2}{qy+rz}+\frac{q^2y^2}{px+rz}+\frac{r^2z^2}{px+qy} \geq \frac{(px+qy)^2}{px+2rz+qy}+\frac{r^2z^2}{px+qy} \geq \frac{(px+qy+rz)^2}{2(px+qy+rz)} = \frac{px+qy+rz}{2} \geq \frac{1}{2}.$$

12. $[x_k,x_{k+1}], \forall k=\overline{0,n-1}$ segmentda $f(x)$ funksiya Roll teoremasini qanoatlantiryapti, u holda $\exists \xi_k \in (x_k,x_{k+1})$ topiladiki, $f'(\xi_k)=0$ bo'ladi. Bu fikr har bir segment uchun o'rinli. Ya'ni, $f'(\xi_1)=f'(\xi_2)=...=f'(\xi_k)=0$. Endi $[\xi_0,\xi_1],[\xi_1,\xi_2],...,[\xi_i,\xi_{i+1}]...$ $i=\overline{0.k-1}$ segmetlar uchun esa $f'(x)$ funksiya Roll teoremasining barcha shartlarini qanoatlantiradi. U holda $\exists \eta_i \in (\xi_i,\xi_{i+1})$ topiladiki, $f''(\eta_i)=0$ bo'ladi. $n-2$ qadamdan keyin biror $[\varsigma_1,\varsigma_2]$ segmentda $f^{(n-1)}(x)$ funksiya Roll teoremasini qanoatlantirishini matematik induksiya yordamida ko'rish qiyin emas. U holda $f^{(n-1)}(\varsigma_1)=f^{(n-1)}(\varsigma_2)=0 \Rightarrow \exists \varepsilon \in (\varsigma_1,\varsigma_2)$ topilib, $f^{(n)}(\varepsilon)=0$ tenglik o'rinli bo'ladi.

13. Biz $[a,b]$ ni n ta bo'lakka bo'lamiz. $a=x_0<x_1<...<x_n=b$ har bir $[x_k,x_{k+1}], \forall k=\overline{0,n-1}$ segmentda $f(x)$ funksiya Lagranj teoremasini qanoatlantiyapti. U holda $\exists \xi_k \in (x_k,x_{k+1})$ topiladiki, $f'(\xi_k)=\frac{f(x_{k+1})-f(x_k)}{x_{k+1}-x_k}$ tenglik o'rinli. $x_{k+1}-x_k=\Delta x_k$ deb belgilasak,

$$f(b)-f(a)=\sum_{k=0}^{n-1}(f(x_{k+1})-f(x_k))=\sum_{k=0}^{n-1}f'(\xi_k)\Delta x_k. \qquad (1)$$

Endi $|f'(c)| = \max_k\{f'(\xi_k)\}, \ c \in (a,b)$ deb belgilash kiritib va (1) dan

$$|f(b)-f(a)| = \left|\sum_{k=0}^{n-1} f'(\xi_k)\Delta x_k\right| \le \sum_{k=0}^{n-1}|f'(\xi_k)|\Delta x_k \le |f'(c)|\sum_{k=0}^{n-1}\Delta x_k = \\ = |f'(c)||b-a| \Rightarrow |f'(c)| \ge \left|\frac{f(b)-f(a)}{b-a}\right|$$

kelib chiqadi.

14. 1-hol. $f(x) = const$ bo‘lsin. U holda $\forall c \in (a,b)$ uchun tenglik bajariladi.

2-hol. $f(x)$ chiziqli funksiya bo‘lsin. U holda $f'(a) = f'(b) = 0$ shart o‘rinli bo‘lmay qoladi.

3-hol. $[a,b]$ segmentni teng ikkiga bo‘lamiz. $[a,\frac{a+b}{2}]$ segmentda $\varphi(x) = \frac{(x-a)^2}{2}$ va $[\frac{a+b}{2},b]$ segmentda $\psi(x) = \frac{(x-b)^2}{2}$ yordamchi funksiyalarni qaraymiz. $f(x)$ va $\phi(x)$ funksiyalar $[a,\frac{a+b}{2}]$ segmentda, $f(x)$ va $\psi(x)$ funksiyalar esa $[\frac{a+b}{2},b]$ sgmentda Koshi teoremasining barcha shartlarini qanoatlantiradi. Ya’ni $\xi_1 \in \left(a,\frac{a+b}{2}\right)$ topiladiki,

$$\frac{f'(\xi_1)}{\varphi'(\xi_1)} = \frac{f\left(\frac{a+b}{2}\right) - f(a)}{\varphi\left(\frac{a+b}{2}\right) - \varphi(a)} = \frac{8\left(f\left(\frac{a+b}{2}\right) - f(a)\right)}{(b-a)^2}$$

hamda $\xi_2 \in \left(\frac{a+b}{2},b\right)$ topiladiki,

$$\frac{f'(\xi_2)}{\psi'(\xi_2)} = \frac{f(b) - f\left(\frac{a+b}{2}\right)}{\psi(b) - \psi\left(\frac{a+b}{2}\right)} = \frac{8\left(f(b) - f\left(\frac{a+b}{2}\right)\right)}{(b-a)^2}$$

tengliklar o‘rinli bo‘ladi. $\varphi'(\xi_1)$ va $\psi'(\xi_2)$ larga qiymatini qo‘yamiz:

$$+\begin{cases} \dfrac{f'(\xi_1)}{\xi_1 - a} = \dfrac{8\left(f\left(\frac{a+b}{2}\right) - f(a)\right)}{(b-a)^2} \\ \dfrac{f'(\xi_2)}{\xi_2 - b} = \dfrac{8\left(f(b) - f\left(\frac{a+b}{2}\right)\right)}{(b-a)^2} \end{cases} \Rightarrow \begin{array}{l} \dfrac{8(f(b)-f(a))}{(b-a)^2} = \dfrac{f'(\xi_1)}{\xi_1 - a} - \dfrac{f'(\xi_2)}{\xi_2 - b} = \\ = \dfrac{f'(\xi_1) - f'(a)}{\xi_1 - a} - \dfrac{f'(\xi_2) - f'(b)}{\xi_2 - b} \end{array}$$

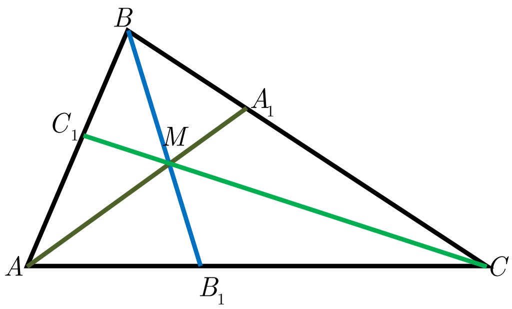
Bu ifodalardan $a < \xi_1 < \frac{a+b}{2}$ ekanini hisobga olsak, $f(x)$ funksiya $[a,\xi_1]$ da Lagranj teoremasini qanoatlantiryapti. Xuddi shunga o‘xshash $\frac{a+b}{2} < \xi_2 < b$ dan $[\xi_2,b]$ segment uchun ham. U holda $\exists c_1 \in (a,\xi_1)$ topiladiki,

$f''(c_1) = \dfrac{f'(\xi_1) - f'(a)}{\xi_1 - a}$ va $\exists c_2 \in (\xi_2, b)$ topiladiki, $f''(c_2) = \dfrac{f'(\xi_2) - f'(b)}{\xi_2 - b}$ lar o'rinli. Agar $f''(c) = \max\{f''(c_1); f''(c_2)\}$ deb olsak,

$$\left| \frac{8(f(b) - f(a))}{(b-a)^2} \right| = \left| f''(c_1) + f''(c_2) \right| \le \left| f''(c_1) \right| + \left| f''(c_2) \right| \le 2\left| f''(c) \right| \Rightarrow$$

$$\Rightarrow \quad \left| f''(c) \right| \ge \frac{4}{(b-a)^2} \left| f(b) - f(a) \right|.$$

15. Umumiylikka zarar yetkazmasdan tengsizlikning har ikkala tomoniga 3 ni qo'shamiz:



$$\frac{AM}{A_1M} + 1 + \frac{BM}{B_1M} + 1 + \frac{CM}{C_1M} + 1 \ge 6 + 3$$

$$\Rightarrow \frac{AA_1}{A_1M} + \frac{BB_1}{B_1M} + \frac{CC_1}{C_1M} \ge 9$$

ni isbotlash kifoya.

$S_{BMC} = \dfrac{1}{2} A_1M \cdot BC \cdot \sin MA_1B$, $S_{ABC} = \dfrac{1}{2} AA_1 \cdot BC \cdot \sin MA_1B \Rightarrow \dfrac{AA_1}{A_1M} = \dfrac{S_{ABC}}{S_{AMC}}$ xuddi shunga o'xshash $\dfrac{BB_1}{B_1M} = \dfrac{S_{ABC}}{S_{AMC}}, \dfrac{CC_1}{C_1M} = \dfrac{S_{ABC}}{S_{AMB}}$, endi

$$\frac{AA_1}{A_1M} + \frac{BB_1}{B_1M} + \frac{CC_1}{C_1M} = S_{ABC}\left(\frac{1}{S_{AMC}} + \frac{1}{S_{BMC}} + \frac{1}{S_{AMB}}\right) =$$
$$= (S_{AMC} + S_{AMB} + S_{BMC})\left(\frac{1}{S_{AMC}} + \frac{1}{S_{BMC}} + \frac{1}{S_{AMB}}\right) \ge 9$$

oxirgi tengsizlik har bir qavs ichiga Koshi tengsizligining $n = 3$ holini qo'llash orqali hosil qilinadi. Isbot tugadi.

16. Bu yig'indini S deb belgilab olamiz va tenglikning har ikkala tomonini x ga ko'paytirib, quyidagilarga ega bo'lamiz:

$$\begin{cases} S = 1 + 2x + 3x^2 + 4x^3 + \dots \\ Sx = x + 2x^2 + 3x^3 + 4x^4 + \dots \end{cases}$$

1-ifodadan 2-ni ayiramiz:

$$S(1-x) = 1 + x + x^2 + x^3 + x^4 + \dots \Rightarrow S(1-x) = \frac{1}{1-x} \quad \Rightarrow \quad S = \frac{1}{(1-x)^2}$$

ekani kelib chiqadi. Bu yerda $|x| < 1$.

17. $x_n - x_{n-1} = f(x_{n-1}) - f(x_{n-2})$ ga Lagranj teoremasini qo‘llaymiz. $\exists c_n \in (a_n, b_n)$ topiladiki, bu yerda $a_n = \min(x_n, x_{n-1})$, $b_n = \max(x_n, x_{n-1})$ $|x_n - x_{n-1}| = |f'(c_n)(x_{n-1} - x_{n-2})| \le q|x_{n-1} - x_{n-2}|$ tengsizlik o‘rinli bo‘ladi. Lagranj teoremasini ketma-ket qo‘llab, quyidagilarni topamiz: $|x_n - x_{n-1}| \le q|x_{n-1} - x_{n-2}| \le q^2|x_{n-2} - x_{n-3}| \le ... \le q^{n-2}|x_2 - x_1|$ demak, $\forall n \in \mathbb{N}$ uchun $|x_n - x_{n-1}| \le \frac{q^n|x_2 - x_1|}{q^2}$. Endi $x_1 + \sum_{n=1}^{\infty}(x_{n+1} - x_n)$ funksional qatorni tekshiramiz. Bu qator

$$|x_n - x_{n-1}| \le \frac{q^n|x_2 - x_1|}{q^2} \Rightarrow |x_{n+1} - x_n| \le \frac{q^{n+1}|x_2 - x_1|}{q^2} \Rightarrow \sum_{n=1}^{\infty}\frac{q^{n+1}|x_2 - x_1|}{q^2}$$

Veyershtrass alomatiga ko‘ra yaqinlashuvchi. U holda uning yig‘indisi $S_n = x_n$ yaqinlashuvchi.

18. $[1, n]$ segmaentni teng n ta bo‘lakka bo‘lib, quyidagilarga ega bo‘lamiz:

$$\int_1^n f(x)dx = \int_1^2 f(x)dx + \int_2^3 f(x)dx + ... + \int_{n-1}^n f(x)dx <$$
$$< f(2)(2-1) + f(3)(3-2) + ... + f(n)(n-(n-1)) = f(2) + f(3) + ... + f(n)$$
$$\int_1^n f(x)dx = \int_1^2 f(x)dx + \int_2^3 f(x)dx + ... + \int_{n-1}^n f(x)dx > f(1) + f(2) + ... + f(n-1)$$

Demak,

$$f(1) + f(2) + ... + f(n-1) < \int_1^n f(x)dx = \int_1^2 f(x)dx + \int_2^3 f(x)dx + ... + \int_{n-1}^n f(x)dx <$$
$$< f(2) + f(3) + ... + f(n)$$

Bularni aniqlashda funksiyaning monoton o‘suvchi ekanidan foydalanildi.

19. Teskarisini faraz qilamiz. $\det A = 0$ bo‘lsin. U holda $AX = 0$ tenglama noldan farqli yechimga ega.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = 0 \end{cases}$$

tenglamalar sistemasining noldan farqli yechimlari biror $\alpha_1, \alpha_2, ..., \alpha_n$ son-lari bo'lsin. Ular tenglamalar sistemasidagi har bir tenglikni ayniyatga aylantiradi. Ya'ni

$$\begin{cases} a_{11}\alpha_1 + a_{12}\alpha_2 + ... + a_{1n}\alpha_n = 0 \\ a_{21}\alpha_1 + a_{22}\alpha_2 + ... + a_{2n}\alpha_n = 0 \\ .... \\ a_{n1}\alpha_1 + a_{n2}\alpha_2 + ... + a_{nn}\alpha_n = 0 \end{cases}$$

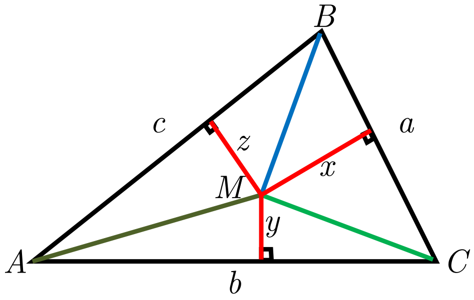
$\max(|\alpha_1|, |\alpha_2|, ..., |\alpha_n|) = |\alpha_k|$ desak, $a_{k1}\alpha_1 + a_{k2}\alpha_2 + ... + a_{kn}\alpha_n = 0$, $\alpha_k \neq 0$ ga ko'ra $a_{kk}\alpha_{ki} = \sum_{i \neq k}^{n} a_{ki}\alpha_i \Rightarrow |a_{kk}| = \dfrac{\sum_{i \neq k}^{n} |a_{ki}||\alpha_i|}{|\alpha_{ki}|} \leq \left| \sum_{i \neq k}^{n} |a_{ki}| \right| < |a_{kk}|$ o'rinli. Bu esa ziddiyat. Demak, $\det A \neq 0$, ya'ni A matritsa teskarilanuvchi.

20. *Ko'rsatma*: a) $x'_n = \dfrac{1}{n}$ va $x''_n = \dfrac{1}{n+1}$ ketma-ketliklarga tekis uzluksizlikning Geyne ta'rifidan foydalaning;

b) *ko'rsatma*: $x'_n = \pi n$ va $x''_n = \pi n + \dfrac{1}{n}$ ketma-ketliklarga tekis uzluksizlikning Geyne ta'rifidan foydalaning;

c) *ko'rsatma*: $x'_n = \dfrac{1}{\pi / 2 + 2\pi n}$ va $x''_n = \dfrac{1}{\pi n}$ ketma-ketliklarga tekis uzluksizlikning Geyne ta'rifidan foydalaning.

21.



$$S_{AMB}=\frac{1}{2}cz,\quad S_{AMC}=\frac{1}{2}by,\quad S_{CMB}=\frac{1}{2}ax,\quad S_{ABC}=\frac{1}{2}ah_a=\frac{1}{2}bh_b=\frac{1}{2}ch_c$$

$$\Rightarrow \frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}=\frac{S_{AMB}+S_{AMC}+S_{CMB}}{S_{ABC}}=1$$

ekanidan foydalanamiz. Endi $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\left(\frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}\right)$ ko‘paytmaga Koshi-Bunyakovskiy tengsizligini qo‘llaymiz u holda

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\left(\frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}\right)\geq\left(\sqrt{\frac{a}{h_a}}+\sqrt{\frac{b}{h_b}}+\sqrt{\frac{c}{h_c}}\right)^2=$$

$$=\left(\frac{a}{\sqrt{2S}}+\frac{b}{\sqrt{2S}}+\frac{c}{\sqrt{2S}}\right)^2=\frac{(a+b+c)^2}{2S}=\frac{(a+b+c)^2}{2\cdot\frac{1}{2}(a+b+c)r}=\frac{a+b+c}{r}$$

Koshi-Bunyakovskiy tengsizligida tenglik belgisi $\frac{\frac{a}{x}}{\frac{x}{h_a}}=\frac{\frac{b}{y}}{\frac{y}{h_b}}=\frac{\frac{c}{z}}{\frac{z}{h_c}}\Rightarrow\frac{ah_a}{x^2}=\frac{bh_b}{y^2}=\frac{ch_c}{z^2}\Rightarrow x=y=z=r$ da bajariladi. U holda M nuqta uchburchakning bissektrisalari kesishgan nuqtada bo‘lar ekan.

22. 1 sonining n-darajali ildizi quyidagi ko‘rinishda $z^{n-1}+z^{n-2}+...+z+1=(z-w)(z-w^2)...(z-w^{n-1})$ bu yerda, $w=\exp(2\pi i/n)$ ekani bizga ma’lum. U holda $z=1$ bo‘lsa, $n=(1-w)(1-w^2)...(1-w^{n-1})$ tenglik o‘rinli bo‘ladi.

$1-w^k=1-e^{(2\pi ik/n)}=-e^{(\pi ik/n)}$, $\left(k=\overline{1,n-1}\right)$ va

$e^{(\pi ik/n)}-e^{(-\pi ik/n)}=-2ie^{(\pi ik/n)}\sin\frac{\pi k}{n}$ ga ko‘ra:

$$\prod_{k=1}^{n-1}(1-w^k)=2^{n-1}(-1)^{n-1}i^{n-1}e^{\left(\frac{\pi i}{n}(1+2+...+n-1)\right)}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=$$

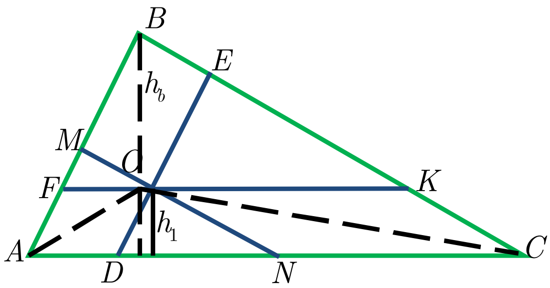
$$=2^{n-1}(-1)^{n-1}i^{n-1}e^{\left(\frac{\pi i}{2}(n-1)\right)}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=2^{n-1}(-1)^{n-1}i^{n-1}i^{n-1}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=$$

$$=2^{n-1}(-1)^{n-1}(i^2)^{n-1}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=2^{n-1}\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}$$

u holda $\prod_{k=1}^{n-1}\sin\frac{\pi k}{n}=\frac{n}{2^{n-1}}$ tenglik oʻrinli.

23. Biz $g(x)=f(x)+x-1$ funksiyani $[0,1]$ segmentda qaraymiz. $g(0)=-1,\ g(1)=1$ u holda Bolsano-Koshining 1-teoremasiga koʻra, $\exists c\in(0,1)$ topiladiki, $g(c)=0$ tenglik oʻrinli boʻladi. Bundan esa $f(c)=1-c$ ekani ke-lib chiqadi. Endi $[0,1]$ segmentni $[0,c]$ va $[c,1]$ boʻlgan ikkita segmentga ajratamiz. Masala shartiga koʻra, $f(x)\in C[0,c]\cap C'(0,c)$ Lagranj teoremasiga koʻra, $\exists a\in(0,c)$ mavjudki $f'(a)=\frac{f(c)-f(0)}{c-0}$ tenglik oʻrinli. Xuddi shun-day $f(x)\in C[c,1]\cap C'(c,1)$ yana Lagranj teoremasiga koʻra $\exists b\in(c,1)$ topilib $f'(b)=\frac{f(1)-f(c)}{1-c}$ oʻrinli boʻladi. Demak, $a\neq b$ uchun $f'(a)f'(b)=\frac{1-c-0}{c}\cdot\frac{1-(1-c)}{1-c}=1$. Isbot tugadi.

24.



$\frac{S_{AOC}}{S_{ABC}}=\frac{\frac{AC}{2}\cdot h_1}{\frac{AC}{2}\cdot h_b}=\frac{h_1}{h_b}=\frac{AF}{AB}$ chunki, $\sin A=\frac{h_b}{AB}=\frac{h_1}{OD}=\frac{h_1}{AF}$ xuddi shunga oʻxshash $\frac{S_{BOC}}{S_{ABC}}=\frac{CN}{AC}$ va $\frac{S_{AOB}}{S_{ABC}}=\frac{BE}{BC}$ $\Rightarrow$ $\frac{AF}{AB}+\frac{BE}{BC}+\frac{CN}{AC}=\frac{S_{AOC}+S_{AOB}+S_{BOC}}{S_{ABC}}=1$

25. Quyidagicha belgilash kiritamiz:

$$A_n=a_1+a_2+\ldots+a_n-n\sqrt[n]{a_1a_2\ldots a_n}\quad\Rightarrow A_{n+1}-A_n=$$
$$=a_{n+1}+n\sqrt[n]{a_1a_2\ldots a_n}-(n+1)\sqrt[n+1]{a_1a_2\ldots a_na_{n+1}}=$$
$$=a_{n+1}+n\sqrt[n]{a_1a_2\ldots a_n}-(n+1)\left(\sqrt[n]{a_1a_2\ldots a_na_n}\right)^{\frac{n}{n+1}}\cdot\left(a_{n+1}\right)^{\frac{1}{n+1}}\quad\Rightarrow$$
$$x=\left(a_{n+1}\right)^{\frac{1}{n+1}},\ y=\left(\sqrt[n]{a_1a_2\ldots a_na_n}\right)^{\frac{1}{n+1}}\quad\Rightarrow A_{n+1}-A_n=x^{n+1}+ny^{n+1}-\left(n+1\right)xy^n=$$
$$=y^{n+1}\left(\left(\frac{x}{y}\right)^{n+1}-\left(n+1\right)\frac{x}{y}+n\right)\quad\Rightarrow\frac{x}{y}=1+z\quad\Rightarrow$$
$$A_{n+1}-A_n=y^{n+1}\left(\left(1+z\right)^{n+1}-1-\left(n+1\right)z\right)$$

Bernulli tengsizligiga ko'ra, $(1+z)^{\alpha} > 1+\alpha z$ ekanidan

$$A_{n+1} - A_n = y^{n+1}\left((1+z)^{n+1} - 1 - (n+1)z\right) >$$
$$> y^{n+1}\left(1+(n+1)z - 1 - (n+1)z\right) = 0 \Rightarrow A_{n+1} > A_n.$$

Xuddi shunga o'xshash, $A_{n+1} > A_n > A_{n-1} > ... > A_2 > A_1$ $A_2 = \left(\sqrt{a_1} - \sqrt{a_2}\right)^2$ ekanini e'tiborga olsak, $A_n \geq \left(\sqrt{a_1} - \sqrt{a_2}\right)^2$ ya'ni berilgan tengsizlikning isboti kelib chiqadi. Tenglik ishorasi faqat $a_1 = a_2 = ... = a_n \geq 0$ da bajariladi.

26. $x + \frac{1}{x} = 2\cos\alpha \Rightarrow x^2 - 2x\cos\alpha + 1 = 0 \Rightarrow x = \cos\alpha \pm i\sin\alpha$ bu yerda $i^2 = -1$ u holda Muavr formulasidan foydalansak,

$$x^n + \frac{1}{x^n} = (\cos\alpha \pm i\sin\alpha)^n + (\cos\alpha \pm i\sin\alpha)^{-n} =$$
$$= \cos n\alpha \pm i\sin n\alpha + \cos(-n)\alpha \pm i\sin(-n)\alpha = 2\cos n\alpha$$

27. Buni isbotlash uchun quyidagi yordamchi tengsizlikdan foydalanamiz. $a \geq 0, b \geq 0, c \geq 0$ sonlari uchun $(a+b-c)(a-b+c)(b+c-a) \leq abc$ buni isbotlash oson. Biz bu tengsizlikning chap tomonidagi qavslarni ochib, uni soddalashtirgandan so'ng u quyidagi ko'rinishga keladi:

$a^3 + b^3 + c^3 \geq ab^2 + a^2b + ac^2 + a^2c + bc^2 + b^2c + 3abc$ bu tengsizlikni ikkala tomonini 3 ga ko'paytirib, o'ng tomoniga $27abc$ ni qo'shamiz, natijada $3(a^3 + b^3 + c^3) + 18abc \geq 3(ab^2 + a^2b + ac^2 + a^2c + bc^2 + b^2c)$ hosil bo'ladi. Endi buning ikkala tomoniga $a^3 + b^3 + c^3 + 6abc$ ni qo'shamiz:

$$4(a^3 + b^3 + c^3) + 24abc \geq a^3 + b^3 + c^3 + 3(ab^2 + a^2b + ac^2 + a^2c + bc^2 + b^2c) + 6abc \Rightarrow$$
$$\Rightarrow 4((a^3 + b^3 + c^3) + 6abc) \geq (a+b+c)^3 \Rightarrow a^3 + b^3 + c^3 + 6abc \geq \frac{1}{4}(a+b+c)^3.$$

28. Javob: (2,2,5). Agar $x = 2$ bo'lsa, u holda $y = 2, z = 5$ bo'lishi ko'rinib turibdi. Endi x 2 dan boshqa tub son bo'lsin. Demak, u toq ham-dir. U holda toq sonni har qanday natural darajaga ko'targanda yana toq son hosil bo'ladi. Ya'ni tenglikning chap tomoni juft son. z bo'lsa 2 dan boshqa juft-tub qiymat qabul qila olmaydi. Bu holda tub sonlarda yechim mavjud emas. Tenglama tub sonlarda yagona yechimga ega ekan.

29. $f(x) = \frac{1}{1-x^2} = \frac{1}{2}\left(\frac{1}{1+x} + \frac{1}{1-x}\right) = \frac{1}{2}\left((1+x)^{-1} + (1-x)^{-1}\right)$

$n = 1$ da $f'(x) = \frac{1}{2}\left(-(1+x)^{-2} + (1-x)^{-2}\right)$

$n=2$ da $f''(x)=\frac{1}{2}\left(2\left(1+x\right)^{-3}+2\left(1-x\right)^{-3}\right)$

$n=3$ da $f'''(x)=\frac{1}{2}\left(-6\left(1+x\right)^{-4}+6\left(1-x\right)^{-4}\right)$

$n=k$ da $f^{(k)}(x)=\frac{1}{2}\left(\left(-1\right)^{k}k!\left(1+x\right)^{-(k+1)}+k!\left(1-x\right)^{-(k+1)}\right)$ deb faraz qilamiz va bu tasdiqning $n=k+1$ uchun o‘rinli ekanini ko‘rsatamiz.

$n=k+1$ da

$f^{(k+1)}(x)=\left(f^{k}\left(x\right)\right)'=\frac{1}{2}\left(\left(-1\right)^{k+1}\left(k+1\right)!\left(1+x\right)^{-(k+2)}+\left(k+1\right)!\left(1-x\right)^{-(k+2)}\right)$. Demak, ixtiyoriy natural n larda

$$f^{(n)}(x)=\frac{1}{2}\left(\left(-1\right)^{n}n!\left(1+x\right)^{-(n+1)}+n!\left(1-x\right)^{-(n+1)}\right)$$

tenglik o‘rinli ekan. U holda

$$f^{(n)}(0)=\frac{1}{2}\left(\left(-1\right)^{n}n!\left(1+0\right)^{-(n+1)}+n!\left(1-0\right)^{-(n+1)}\right)=$$
$$=\frac{n!}{2}\left(1+\left(-1\right)^{n}\right)=\begin{cases} n!, & agar\ \ n-juft\ bo'lsa,\\ 0, & agar\ \ n-toq\ bo'lsa.\end{cases}$$

30. $f\left(x\right)=\sqrt{1+x^2}$ funksiyaga Iensen tengsizligini qo‘llaymiz:

$f'\left(x\right)=\frac{x}{\sqrt{1+x^2}}$, $f''\left(x\right)=\dfrac{\sqrt{1+x^2}-\dfrac{x^2}{\sqrt{1+x^2}}}{1+x^2}=\dfrac{1}{\left(1+x^2\right)\sqrt{1+x^2}}>0$. Demak, Iensen tengsizligiga ko‘ra $f''\left(x\right)\geq 0,\ x\in\left(a,b\right)$ bo‘lsa, ixtiyoriy $x_1,x_2,...,x_n\in\left(a,b\right)$ va $p_1+p_2+...+p_n=1$ tenglikni qanoatlantiruvchi ixtiyoriy $p_1\geq 0,p_2\geq 0,...,p_n\geq 0$ sonlari uchun ushbu

$$f\left(p_1x_1+p_2x_2+...+p_nx_n\right)\leq p_1f\left(x_1\right)+p_2f\left(x_2\right)+...+p_nf\left(x_n\right) \qquad (1)$$

tengsizlik o‘rinli. Biz yuqoridagi tengsizlikda $p_i,\ x_i\ \left(i=\overline{1,n}\right)$ larning ixtiyoriyligidan ularni quyidagicha tanlab olamiz:

$p_i=\frac{a_i}{a_1+a_2+...+a_n}$, $x_i=\frac{b_i}{a_i}$ $\left(i=\overline{1,n}\right)$ bularni (1) ga qo‘yamiz:

$$\sqrt{1+\left(\frac{b_1+b_2+...+b_n}{a_1+a_2+...+a_n}\right)^2}\leq\frac{a_1\sqrt{1+\left(\frac{b_1}{a_1}\right)^2}}{a_1+a_2+...+a_n}+\frac{a_2\sqrt{1+\left(\frac{b_2}{a_2}\right)^2}}{a_1+a_2+...+a_n}+...+\frac{a_n\sqrt{1+\left(\frac{b_n}{a_n}\right)^2}}{a_1+a_2+...+a_n}\quad\Rightarrow$$

$$\Rightarrow\sqrt{(a_1+a_2+...+a_n)^2+\left(b_1+b_2+...+b_n\right)^2}\leq\sqrt{a_1^2+b_1^2}+\sqrt{a_2^2+b_2^2}+...+\sqrt{a_n^2+b_n^2}.$$

Isbot tugadi.

31. Biz quyidagi ikkita yordamchi funksiyani $[x_1,x_2]$ segmentda qaraymiz: $\phi(x)=\frac{f(x)}{x}$ va $\psi(x)=\frac{1}{x}$ ko'rinib turibdiki, bu funksiyalar $[x_1,x_2]$ segmentda aniqlangan, uzluksiz va differensiallanuvchi hamda $\psi(x)=\frac{1}{x}$ nolga teng emas. U holda biz qarayotgan ikkala funksiya $[x_1,x_2]$ $\left(0<x_1<x_2\right)$ da Koshhi teoremasining barcha shartlarini qanoatlantiradi, ya'ni, $\exists\xi\in(x_1,x_2)$ to-piladiki, $\frac{\phi'(\xi)}{\psi'(\xi)}=\frac{\phi(x_2)-\phi(x_1)}{\psi(x_2)-\psi(x_1)}$ tenglik o'rinli bo'ladi. $\phi(x)=\frac{f(x)}{x}$, $\psi(x)=\frac{1}{x}$ larni hisobga olgan holda $\frac{1}{x_2-x_1}\begin{vmatrix} x_1 & x_2 \\ f(x_1) & f(x_2)\end{vmatrix}=f(\xi)-\xi f'(\xi)$ ga ega bo'lamiz. Shuni isbotlash talab qilingan edi.

32. Teylor formulasiga ko'ra $\left(x\neq x_0\right)$

$$f(x)-f(x_0)=(x-x_0)f'(x_0)+\frac{(x-x_0)^2 f''(\overline{x_0})}{2} \qquad \left(x_0-r<\overline{x_0}<x_0+r\right)$$

bu tenglikni ikkala tarafini ham $\left(x_0-r,x_0+r\right)$ oraliqda integrallab, quyidagiga ega bo'lamiz:

$$\int_{x_0-r}^{x_0+r}[f(x)-f(x_0)]=\int_{x_0-r}^{x_0+r}(x-x_0)f'(x_0)dx+\int_{x_0-r}^{x_0+r}\frac{(x-x_0)^2 f''(\overline{x_0})}{2}dx=\int_{x_0-r}^{x_0+r}\frac{(x-x_0)^2 f''(\overline{x_0})}{2}dx.$$

O'rta qiymat haqidagi teoremaga ko'ra $\exists\xi\in(x_0-r,x_0+r)$ topiladiki,

$$\int_{x_0-r}^{x_0+r}[f(x)-f(x_0)]=\int_{x_0-r}^{x_0+r}\frac{(x-x_0)^2 f''(\overline{x_0})}{2}dx=f''(\xi)\int_{x_0-r}^{x_0+r}\frac{(x-x_0)^2}{2}dx=\frac{r^3}{3}f''(\xi)$$

bo'ladi. Bundan $f''(\xi)=\frac{3}{r^3}\int_{x_0-r}^{x_0+r}\left(f(x)-f(x_0)\right)dx$ tenglik kelib chiqadi.

33. Bizga ma'lumki, uch xonali sonni $\overline{xyz}=100x+10y+z$ ko'rinishda yoyish mumkin. Tengsizlikni quyidagi ko'rinishga keltiramiz:

$$(100a+10b+c)(100b+10c+a)(100c+10a+b)\geq 111^3abc\Rightarrow$$

$$\Rightarrow\ \left(\frac{100a+10b+c}{a}\right)\left(\frac{100b+10c+a}{b}\right)\left(\frac{100c+10a+b}{c}\right)\geq 111^3\ \Rightarrow$$

$$\Rightarrow\ \left(100+10\frac{b}{a}+\frac{c}{a}\right)\left(100+10\frac{c}{b}+\frac{a}{b}\right)\left(100+10\frac{a}{c}+\frac{b}{c}\right)\geq 111^3$$

Oxirgi tengsizlikni isbotlash kifoya. Buning uchun qavslarni ochib chiqamiz:

$$(100+10\frac{b}{a}+\frac{c}{a})(100+10\frac{c}{b}+\frac{a}{b})(100+10\frac{a}{c}+\frac{b}{c})=10^6+10^5(\frac{a}{c}+\frac{c}{b}+\frac{b}{a})+2\cdot 10^4(\frac{a}{b}+\frac{b}{c}+\frac{c}{a})+$$
$$+.10^3(\frac{a^2}{bc}+\frac{c^2}{ab}+\frac{b^2}{ac})+2\cdot 10^2(\frac{a}{c}+\frac{c}{b}+\frac{b}{a})+10(\frac{a}{b}+\frac{b}{c}+\frac{c}{a})+3\cdot 10^3+1\geq 10^6+3\cdot 10^5+6\cdot 10^4+$$
$$+6\cdot 10^3+6\cdot 10^2+3\cdot 10+1=(100+10+1)^3=111^3.$$

Bu yerda Koshi tengsizligining $n=3$ holidan foydalanildi. Isbot tugadi.

34. $n=1$ da $x_2=\frac{1}{4-6036}=-\frac{1}{6032}<\frac{1}{3}$, $n=2$ da $x_3=\frac{1}{4+\frac{1}{6032}}<\frac{1}{3}$

$n=3$ da $x_4=\frac{1}{4-3x_3}<\frac{1}{4-3\cdot\frac{1}{3}}=\frac{1}{3}$, $n=k$ da $x_{k+1}<\frac{1}{3}$ deb faraz qilib, bu tasdiqning $n=k+1$ uchun ham o‘rinli ekanini ko‘rsatamiz.

$n=k+1$ da $x_{k+2}=\frac{1}{4-3x_{k+1}}<\frac{1}{4-3\cdot\frac{1}{3}}=\frac{1}{3}$. Demak, $\forall n\in\mathbb{N}$ uchun $x_n<\frac{1}{3}$. Endi ketma-ketlikning monoton o‘suvchiligini ko‘rsatamiz.

$$x_{n+1}-x_n=\frac{1}{4-3x_n}-x_n=\frac{1-4x_n+3x_n^2}{4-3x_n}=\frac{3(x_n-1)(x_n-\frac{1}{3})}{4-3x_n}>0$$ chunki, $\forall n\in\mathbb{N}$ uchun $x_n<\frac{1}{3}$. U holda $x_{n+1}-x_n>0\Rightarrow x_{n+1}>x_n$ ya’ni ketma-ketlik monoton o‘suvchi. Endi monoton ketma-ketliklar haqidagi teormega ko‘ra $\{x_n\}$ ketma-ketlik yaqinlashuvchi. Uning limitini biror c deb olamiz. $x_{n+1}=\frac{1}{4-3x_n}$ ning ikkala tomonini $n\to+\infty$ da limitga o‘tib, $3c^2-4c+1=0$ dan $c=1$ va $c=\frac{1}{3}$ larni topamiz. Yuqoridagilarga ko‘ra $\lim\limits_{n\to+\infty}x_n=\frac{1}{3}$ ekanini topamiz.

35. $f(x)$ funksiya $[a,b]$ segmentning ixtiyoriy nuqtasida uzluksiz differensiallanuvchi bo‘lgani uchun $\forall x\in[a,b]$ da $|f'(x)|$ chegaralangan bo‘ladi va uning maksimal qiymatini M bilan belgilasak, $\exists\varepsilon\in[a,b]$ topiladiki,

$$M=\max_{a\leq x\leq b}|f'(x)|=f'(\varepsilon)$$

bo‘ladi. $[a,b]$ segmentni teng ikki qismga bo‘lib, $\left[a,\frac{a+b}{2}\right]$ va $\left[\frac{a+b}{2},b\right]$ segmentlarning har birida $f(x)$ differensiallanuvchi bo‘lgani uchun quyidagini yoza olamiz:

$$\exists \xi \in \left(a, x\right), \ a \le x \le \frac{a+b}{2} \qquad f\left(x\right) = f'\left(\xi\right)\left(x-a\right) \le M\left(x-a\right)$$

$$\exists \eta \in \left(x, b\right), \ \frac{a+b}{2} \le x \le b \qquad f\left(x\right) = f'\left(\eta\right)\left(b-x\right) \le M\left(b-x\right)$$

Bularni e'tiborga olgan holda quyidagi mulohazani yoza olamiz:

$$\int_a^b f\left(x\right)dx = \int_a^{\frac{a+b}{2}} f\left(x\right)dx + \int_{\frac{a+b}{2}}^b f\left(x\right)dx = \int_a^{\frac{a+b}{2}} f'\left(\xi\right)\left(x-a\right)dx + \int_{\frac{a+b}{2}}^b f'\left(\eta\right)\left(b-x\right)dx \le$$

$$\le M\int_a^{\frac{a+b}{2}} \left(x-a\right)dx + M\int_{\frac{a+b}{2}}^b \left(b-x\right)dx = M\frac{\left(b-a\right)^2}{4} = \left|f'\left(\varepsilon\right)\right| \cdot \frac{\left(b-a\right)^2}{4} \Rightarrow$$

bundan

$$\left|f'(\varepsilon)\right| \ge \frac{4}{(b-a)^2}\int_0^1 \left|f(x)\right|dx$$

tengsizlik kelib chiqadi.

36. $f(x) = \log_x(x+1) \quad (x \ge 2)$ funksiyani qaraymiz.

$$f'(x) = \left(\frac{\ln(x+1)}{\ln x}\right)' = \frac{\frac{1}{x+1}\cdot \ln x - \frac{1}{x}\ln(x+1)}{\ln^2 x} = \frac{\ln x^{\frac{1}{x+1}} - \ln(x+1)^{\frac{1}{x}}}{\ln^2 x} \qquad x > 1$$

ekanidan $(x+1)^{\frac{1}{x}} > x^{\frac{1}{x}} > x^{\frac{1}{x+1}}$ tengsizlik o'rinli. Demak, $f'(x) < 0$. U holda $f(x)$ funksiya $x \ge 2$ da kamayuvchi.

$$n+1 > n \Rightarrow f(n+1) < f(n) \Rightarrow \log_{n+1}(n+2) < \log_n(n+1)$$

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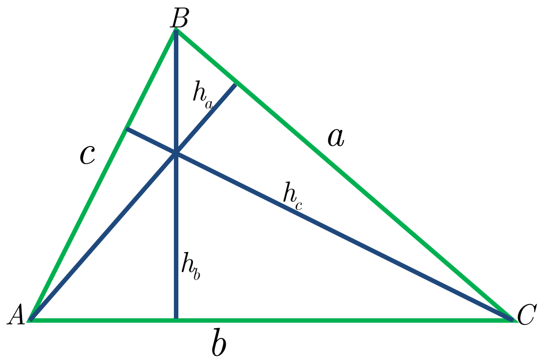


37. Chizmadan ko'rinadiki $a\sin\beta = h_c, \ \ b\sin\gamma = h_a, \ \ c\sin\alpha = h_b$ tengliklar o'rinli. Bundan tashqari, $a = 2R\sin\alpha, b = 2R\sin\beta, \ c = 2R\sin\lambda$ ekanini hisobga olsak, quyidagiga ega bo'lamiz:

$$9R(a\cos\alpha + b\cos\beta + c\cos\gamma) = P(a\sin\beta + b\sin\gamma + c\sin\alpha) \quad \Rightarrow$$

$$P(h_a + h_b + h_c) = 9R(2R\sin\alpha\cos\alpha + 2R\sin\beta\cos\beta + 2R\sin\gamma\cos\gamma) =$$

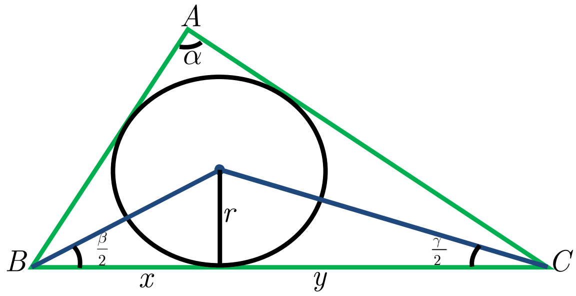
$$= 9R^2(\sin 2\alpha + \sin 2\beta + \sin 2\gamma) = 18S$$

Bundan tashqari,

$$P(h_a + h_b + h_c) = (a+b+c)(h_a + h_b + h_c) \ge 9\sqrt[3]{abc}\cdot\sqrt[3]{h_a h_b h_c} = 18S$$

bu yerda tenglik belgisi faqat $a=b=c,\ h_a=h_b=h_c$ da bajariladi. Demak, biz izlayotgan uchburchak muntazam uchburchak ekan.

38. Bizga ma'lumki, $a=2R\sin\alpha$ $\Rightarrow\ 2R=\dfrac{a}{\sin\alpha}$. Endi r ni ham uchburchakning a tomoni va burchaklari orqali ifodalaymiz. Chizmadan koʻrinib turibdiki,



$$a=x+y=rctg\frac{\beta}{2}+rctg\frac{\gamma}{2}=\frac{r\sin\dfrac{\beta+\gamma}{2}}{\sin\dfrac{\beta}{2}\sin\dfrac{\gamma}{2}}=\frac{r\cos\dfrac{\alpha}{2}}{\sin\dfrac{\beta}{2}\sin\dfrac{\gamma}{2}}\ \Rightarrow\ r=\frac{a\sin\dfrac{\beta}{2}\sin\dfrac{\gamma}{2}}{\cos\dfrac{\alpha}{2}}$$

$$\frac{2R}{r}=\frac{\dfrac{a}{\sin\alpha}}{\dfrac{a\sin\dfrac{\beta}{2}\sin\dfrac{\gamma}{2}}{\cos\dfrac{\alpha}{2}}}=\frac{1}{2\sin\dfrac{\alpha}{2}\sin\dfrac{\beta}{2}\sin\dfrac{\gamma}{2}}=\frac{1}{\sin\dfrac{\alpha}{2}}\cdot\frac{1}{\cos\left(\dfrac{\beta-\gamma}{2}\right)-\cos\left(\dfrac{\beta+\gamma}{2}\right)}\geq$$

$$\geq\frac{1}{\sin\dfrac{\alpha}{2}}\cdot\frac{1}{1-\sin\dfrac{\alpha}{2}}=\frac{1}{\sin\dfrac{\alpha}{2}(1-\sin\dfrac{\alpha}{2})}.$$

39. 2-ustundan boshlab barcha ustun elementlarini 1-ustun elementlariga qoʻshib chiqamiz:

$$\begin{vmatrix} x+c_1+c_2+\dots c_n & c_1 & c_2 & \dots & c_n \\ x+c_1+c_2+\dots c_n & x & c_2 & \dots & c_n \\ \dots\dots\dots\dots & & & & \\ x+c_1+c_2+\dots c_n & c_2 & c_3 & \dots & x \end{vmatrix}=0\quad\Rightarrow(x+c_1+c_2+\dots c_n)\begin{vmatrix} 1 & c_1 & c_2 & \dots & c_n \\ 1 & x & c_2 & \dots & c_n \\ \dots\dots\dots & & & & \\ 1 & c_2 & c_3 & \dots & x \end{vmatrix}=0$$

endi 1-satr elementlarini -1 ga koʻpaytirib, 2-satr elementlariga, 2-satr elementlarini -1 ga koʻpaytirib 3-satr elementlariga qoʻshamiz va hokazo. $(n-1)$-satr elementlarini -1 ga koʻpaytirib, n-satr elementlariga qoʻshgandan keyin quyidagiga ega boʻlamiz:

$$(x + c_1 + c_2 + ...c_n)\begin{vmatrix} 1 & c_1 & c_2 ... & c_n \\ 0 & x - c_1 & 0 ... & 0 \\ 0 & 0 & x - c_2 ... & 0 \\ .... \\ 0 & 0 & 0 ... & x - c_n \end{vmatrix} = 0$$

$(x + c_1 + c_2 + ... + c_n)(x - c_1)(x - c_2)...(x - c_n) = 0$ bu tenglamadan esa $x = -(c_1 + c_2 + ... + c_n), x = c_1, x = c_2, ..., x = c_n$ yechimlarni olamiz.

40. Agar $x \neq y$ va $x, y \in N$ bo‘lsa, $x > y$ yoki $x < y$ bo‘lishi aniq. Faraz qilaylik, $x > y$ bo‘lsin. U holda tenglik bajarilishi uchun x ga y bo‘linishi kerak. $x = yn, (n \in \mathbb{N})$ deb olaylik. Tenglikka qo‘ysak, $(yn)^y = y^{yn} \Rightarrow yn = y^n \Rightarrow y^{n-1} = n \Rightarrow y = n^{\frac{1}{n-1}}$ va $x = n^{\frac{n}{n-1}}$ ekani kelib chiqadi. Oxirgi tengliklar esa $n = 2$ ya’ni, $x = 2, y = 4$ bo‘lganda bajariladi. Xuddi shunga o‘xshash $x < y$ hol uchun $y = 2, x = 4$ javoblarga ega bo‘lamiz. Demak, yechim $(2,4),(4,2)$ ga teng.

41. Buning uchun determinantni biror D_n deb belgilab olib, uni 1-va 2-satrlari bo‘yicha yoyamiz:

$$D_n = \begin{vmatrix} 1 & 1 & 0 & 0 & ... & 0 & 0 \\ -1 & 1 & 1 & 0 & ... & 0 & 0 \\ 0 & -1 & 1 & 1 & ... & 0 & 0 \\ .... \\ 0 & 0 & 0 & 0 & ... & -1 & 1 \end{vmatrix} = D_{n-1} + \begin{vmatrix} 1 & 0 & 0 & ... & 0 & 0 \\ -1 & 1 & 1 & ... & 0 & 0 \\ .... \\ 0 & 0 & 0 & 0 & ... -1 & 1 \end{vmatrix}$$

hosil bo‘lgan $(n-1)$-tartibli determinantni ham 1-satr bo‘yicha yoysak, $D_n = D_{n-1} + D_{n-2}$ tenglikka ega bo‘lamiz. Bu Fibonachchi sonlari ketma-ketligining rekkurent formulasini ifodalaydi. Endi buni tekshirib ko‘rish qoldi.

$$D_1 = 1, D_2 = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2, D_3 = \begin{vmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{vmatrix} = 3.$$

bu esa rekkurent formulani qanoatlantiradi. Demak, determinantning n-hadi Fibonachchi sonlari ket-ma-ketligining n-hadiga teng ekan.

42. $A=\left(\dfrac{f\left(a+\frac{1}{n}\right)}{f(a)}\right)^n$ deb belgilash kiritamiz va ikkala tomonini natural logarifm ostiga olamiz: $\ln A=\ln\left(\dfrac{f\left(a+\frac{1}{n}\right)}{f(a)}\right)^n=\dfrac{\ln\left(\dfrac{f\left(a+\frac{1}{n}\right)}{f(a)}\right)}{\frac{1}{n}}$ bu esa $n\to\infty$ da $\frac{\infty}{\infty}$ ko‘rinishidagi aniqmaslikni ifodalaydi. $n=x$ deb almashtirib, bunga Lopital qoidasini qo‘llaymiz:

$$\lim_{x\to\infty}\frac{\ln\left(\dfrac{f\left(a+\frac{1}{n}\right)}{f(a)}\right)}{\frac{1}{n}}=\lim_{x\to\infty}\frac{\ln\left(\dfrac{f\left(a+\frac{1}{x}\right)}{f(a)}\right)}{\frac{1}{x}}=\lim_{x\to\infty}\frac{-\dfrac{1}{x^2}\cdot\dfrac{f'\left(a+\frac{1}{x}\right)}{f(a)}}{-\dfrac{1}{x^2}\cdot\dfrac{f\left(a+\frac{1}{x}\right)}{f(a)}}=\frac{f'(a)}{f(a)}\Rightarrow A=e^{\frac{f'(a)}{f(a)}}$$

43. Limit ichidagi integral $tg\phi=\frac{a}{b},\left(0<\phi<\frac{\pi}{2}\right)$ desak, ko‘rinishga keladi. Integral ostidagi funksiyaning davri $T=\frac{\pi}{n}$ ga teng bo‘lgani uchun

$$I_n=2n\sqrt{a^2+b^2}\int_0^{\pi n}\left|\sin(\phi-nx)\right|dx=\begin{vmatrix}nx-\phi=t\\ ndx=dt\end{vmatrix}=2\sqrt{a^2+b^2}\int_{-\phi}^{\pi-\phi}\left|\sin t\right|dt=$$

$$=2\sqrt{a^2+b^2}\int_0^{\pi}\left|\sin t\right|dt=4\sqrt{a^2+b^2}$$

tenglik o‘rinli.

44. $f(x)$ funksiya $[a,b]$ segmentda uzluksizligdan hamda qavariqligidan shunday $\xi\in[0,b-a]$ topiladiki,

$$f\left(\frac{a+b}{2}\right)=f\left(\frac{a+\xi}{2}+\frac{b-\xi}{2}\right)\geq\frac{1}{2}\left(f(a+\xi)+f(b-\xi)\right)$$

Bu oxirgi munosabatni ξ bo‘yicha $[0,b-a]$ da integrallasak hamda $a+\xi=t\ \ va\ \ b-\xi=z$ o‘zgartirish kiritsak, u holda

$$(b-a)f\left(\frac{a+b}{2}\right)\geq\frac{1}{2}\left(\int_0^{b-a}f(a+\xi)d\xi+\int_0^{b-a}f(b-\xi)d\xi\right)=\int_a^b f(x)dx$$

kelib chiqadi, ya'ni talab qilinayotgan munosabatni o'ng tarafi kelib chiqadi:

Endi $[a,b]$ segmentda quyidagicha bo'laklash olamiz.

$P=\left\{x_i+i\frac{b-a}{n}; i=\overline{1,n}\right\}$ va $\xi_i=x_i$ desak $\Delta x_i=\frac{b-a}{n}$ ekanini e'tiborga olsak u holda $S_P(f)=\frac{b-a}{n}\sum_{i=0}^{n-1}f\left(a+i\frac{b-a}{n}\right)=\frac{b-a}{n}\left(\sum_{i=0}^{n-1}f\left(1-\frac{i}{n}\right)a+\frac{i}{n}b\right)$ ekani kelib chiqadi. $f(x)$ funksiya $[a,b]$ segmentda qavariqligidan

$$S_P(f)\geq\frac{b-a}{n}\sum_{i=0}^{n-1}\left(\left(1-\frac{i}{n}\right)f(a)+\frac{i}{n}f(b)\right)=\frac{b-a}{n}\left(\frac{n+1}{2}f(a)+\frac{n-1}{2}f(b)\right).$$

Agar oxirgi munosabatdan $n\to\infty$ limitga o'tsak, u holda $\int_a^b f(x)dx\geq\frac{(b-a)}{2}(f(a)+f(b))$ kelib chiqadi. Bular talab qilinayotgan munosabatni to'liq isbotlaydi.

45. $ab\leq\frac{a^2+b^2}{2}=\frac{c^2}{2}$ va $a+b=\sqrt{a^2+b^2+2ab}\leq\sqrt{c^2+2\cdot\frac{c^2}{2}}=c\sqrt{2}$ endi $ab(a+b+c)\leq\frac{c^2}{2}\cdot\left(c\sqrt{2}+c\right)=\frac{\sqrt{2}+1}{2}\cdot c^3=\frac{2\sqrt{2}+2}{4}\cdot c^3<\frac{3+2}{4}\cdot c^3=\frac{5}{4}\cdot c^3$. Isbot tugadi.

46. $\begin{pmatrix}x'\\y'\end{pmatrix}=\begin{pmatrix}3&-1\\2&0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$ differensial tenglamani qaraymiz. Uning xarakteristik tenglamasini yechamiz:

$$\det(A-\lambda I)=\begin{vmatrix}3-\lambda&-1\\2&-\lambda\end{vmatrix}=\lambda^2-3\lambda+2=0\Rightarrow\lambda_1=1,\ \lambda_2=2.$$

Endi har bir xos qiymatga mos bo'lgan xos vektorni aniqlaymiz:

$$(A-1\cdot I)v_1=\begin{pmatrix}2&-1\\2&-1\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=0\Rightarrow 2a-b=0\Rightarrow a=1,\ b=2\Rightarrow v_1=\begin{pmatrix}1\\2\end{pmatrix}$$

$$(A-2\cdot I)v_2=\begin{pmatrix}1&-1\\2&-2\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=0\Rightarrow a-b=0\Rightarrow a=1,\ b=1\Rightarrow v_2=\begin{pmatrix}1\\1\end{pmatrix}$$

Sistemaning umumiy yechimi quyidagi ko'rinishda bo'ladi:

$u(t)=\begin{pmatrix}x\\y\end{pmatrix}=C_1\begin{pmatrix}1\\2\end{pmatrix}e^t+C_2\begin{pmatrix}1\\1\end{pmatrix}e^{2t}=\begin{pmatrix}C_1e^t+C_2e^{2t}\\2C_1e^t+C_2e^{2t}\end{pmatrix}$ bunga $t=0$ ni qo‘yib, $u(0)=\begin{pmatrix}x(0)\\y(0)\end{pmatrix}=\begin{pmatrix}C_1+C_2\\2C_1+C_2\end{pmatrix}$ ni hosil qilamiz. Natijada quyidagilarga ega bo‘lamiz:

$$u(0)=e_1 \Leftrightarrow \begin{pmatrix}C_1+C_2\\2C_1+C_2\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}\Rightarrow\begin{cases}C_1=-1\\C_2=2\end{cases}\Rightarrow u_1(t)=\begin{pmatrix}2e^{2t}-e^t\\2e^{2t}-2e^t\end{pmatrix}$$

$$u(0)=e_2 \Leftrightarrow \begin{pmatrix}C_1+C_2\\2C_1+C_2\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}\Rightarrow\begin{cases}C_1=1\\C_2=-1\end{cases}\Rightarrow u_2(t)=\begin{pmatrix}e^t-e^{2t}\\2e^t-e^{2t}\end{pmatrix}$$

$u_1(t)$ va $u_2(t)$ vektorlardan matritsa tuzamiz. $e^{tA}=\begin{pmatrix}2e^{2t}-e^t & e^t-2e^{2t}\\2e^{2t}-2e^t & 2e^t-e^{2t}\end{pmatrix}$

bunga $t=1$ ni qo‘ysak, $e^A=\begin{pmatrix}2e^2-e & e-2e^2\\2e^2-2e & 2e-e^2\end{pmatrix}$ biz izlayotgan matritsa hosil bo‘ladi.

47. Sistemaning determinanti

$$D=\begin{vmatrix}a_{11}-\frac{1}{2} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22}-\frac{1}{2} & & a_{2n}\\ \dots & \dots & \dots & \dots\\ \dots & \dots & \dots & \dots\\ a_{n1} & a_{n2} & & a_{nn}-\frac{1}{2}\end{vmatrix}$$

Agar biz

$$P(\lambda)=\begin{vmatrix}a_{11}-\lambda & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22}-\lambda & & a_{2n}\\ \dots & \dots & \dots & \dots\\ \dots & \dots & \dots & \dots\\ a_{n1} & a_{n2} & & a_{nn}-\lambda\end{vmatrix}$$

belgilash kiritsak, u holda $D=P\left(\frac{1}{2}\right)$ bo‘ladi. Ikkinchi tomondan, $P(\lambda)=(-1)^n\lambda^n+b_1\lambda^{n-1}+\dots+b_n$ ko‘rinishda bo‘ladi, bu yerda b_i – butun son va $i=\overline{1,n}$. Agarda $P\left(\frac{1}{2}\right)=0$ bo‘lsa, u holda $(-1)^n\frac{1}{2^n}+b_1\frac{1}{2^{n-1}}+\dots+b_n=0$ va bu oxirgi tenglikni ikkala tarafini ham 2^n ga ko‘paytirsak, $(-1)^n+2b_1+2^2b_2\dots+2^nb_n=0$ bo‘ladi. Ushbu $2b_1+2^2b_2\dots+2^nb_n=N$ belgilash kiritsak, u holda $(-1)^n + 2N = 0$ ega bo'lamiz. Bunday bo'lishi mumkin emas, chunki N butun. Demak, $D = P\left(\frac{1}{2}\right) \neq 0$ ekan, u holda sistema yagona yechimga ega va bu yechim $x_1 = x_2 = ... = x_n = 0$ bo'ladi.

48. Biz $f(x) = (1+x)\ln\left(1+\frac{1}{x}\right), (x > 0)$ funksiyani qaraymiz. Bu funksiya uchun $f'(x) = \ln\left(1+\frac{1}{x}\right) - \frac{1}{x^2}\cdot\frac{1+x}{1+\frac{1}{x}} = \ln\left(1+\frac{1}{x}\right) - \frac{1}{x} \leq \frac{1}{x} - \frac{1}{x} = 0$ ekanini ko'rish qiyin emas. Bu esa funksiyaning berilgan oraliqda monoton kamayuvchi eka-nini bildiradi. U holda $n < n+1$ ga funksiyani ta'sir ettirsak:

$$f(n) > f(n+1) \Rightarrow (1+n)\ln\left(1+\frac{1}{n}\right) > (1+n+1)\ln\left(1+\frac{1}{n+1}\right) \Rightarrow$$
$$\Rightarrow \left(1+\frac{1}{n}\right)^{n+1} > \left(1+\frac{1}{n+1}\right)^{n+2}$$

tengsizlik hosil bo'ladi. Shuni isbotlash talab etilgan edi.

49. Bu integralni hisoblash uchun uni I bilan belgilaymiz. $I = \int \sin(\ln x)dx$ endi buni bo'laklab integrallash qoidasiga asosan integrallaymiz.

$u = \sin(\ln x) \quad du = \frac{1}{x}\cos(\ln x)$ natijada quyidagi tenglik kelib chiqadi.
$dv = dx \quad v = x$

$$I = x\sin(\ln x) - \int x\frac{1}{x}\cos(\ln x)dx = x\sin(\ln x) - \int \cos(\ln x)dx$$

$I = x\sin(\ln x) - \int \cos(\ln x)dx$ buni ham bo'laklab integrallaymiz.

$u = \cos(\ln x) \quad du = -\frac{1}{x}\sin(\ln x)dx$
$dv = dx \quad v = x$

$$I = x\sin(\ln x) - \left(x\cos(\ln x) + \int \sin(\ln x)dx\right)$$
$$I = x\sin(\ln x) - (x\cos(\ln x) + I)$$

$I = \frac{x\sin(\ln x) - x\cos(\ln x)}{2}$ bu esa integralni javobi.

50. Bu integralni oldin I deb belgilab olib, so'ngra ikki marta bo'-laklab integrallash kifoya.

$$I=\int e^x\sin x dx=\begin{vmatrix} u=\sin x, & du=\cos x dx \\ dv=e^x, & v=e^x\end{vmatrix}=e^x\sin x-\int e^x\cos x dx=$$
$$=\begin{vmatrix} u=\cos x, & du=-\sin x dx \\ dv=e^x, & v=e^x\end{vmatrix}=e^x\sin x-e^x\cos x-I \Rightarrow I=\frac{e^x\sin x-e^x\cos x}{2}+C$$

51. $x_1=\frac{1}{2}\left(x_0+\frac{1}{x_0}\right)=\frac{1}{2}\left(2+\frac{1}{2}\right)=\frac{5}{4}$. Ketma-ketlikning har bir hadi musbat bo‘lgani uchun $x_{n+1}=\frac{1}{2}\left(x_n+\frac{1}{x_n}\right)\geq\frac{1}{2}\cdot 2\sqrt{x_n\cdot\frac{1}{x_n}}\geq 1$ tengsizlik o‘rinli. Bu esa ketma-ketlikning quyidan chegaralanganligini bildiradi. Endi quyidagi ayirmani qaraymiz:

$$x_{n+1}-x_n=\frac{1}{2}\left(x_n-x_{n-1}+\frac{1}{x_n}-\frac{1}{x_{n-1}}\right)=\frac{1}{2}\left(x_n-x_{n-1}-\frac{x_n-x_{n-1}}{x_nx_{n-1}}\right)=$$
$$=\frac{1}{2}\left(x_n-x_{n-1}\right)\left(1-\frac{1}{x_nx_{n-1}}\right)=\frac{1}{2^2}\left(x_{n-1}-x_{n-2}\right)\left(1-\frac{1}{x_nx_{n-1}}\right)\left(1-\frac{1}{x_{n-1}x_{n-2}}\right)=..$$
$$...=\frac{1}{2^2}\left(1-\frac{1}{x_nx_{n-1}}\right)\left(1-\frac{1}{x_{n-1}x_{n-2}}\right)\cdot...\cdot\left(1-\frac{1}{x_1x_0}\right)\left(x_1-x_0\right)<0$$

Chunki ko‘paytmadagi har bir qavs ichidagi ifoda musbat faqat oxirgi qavs ichidagi ayirma $x_1-x_0=\frac{5}{4}-2<0$ manfiy qiymat qabul qiladi. Bundan $x_{n+1}-x_n<0 \Rightarrow x_{n+1}<x_n$, ya’ni ketma-ketlik monoton kamayuvchi. Demak, ketma-ketlik monoton kamayuvchi va quyidan chegaralangan. U holda monoton ketma-ketliklar haqidagi teoremaga ko‘ra, u yaqinlashuvchi.

52. 1-qadam. $n=2$ da tensizlikning chap qismi $2!=2$ va tengsizlikning o‘ng qismi $\left(\frac{2+1}{2}\right)^2=\left(\frac{3}{2}\right)^2=\frac{9}{4}=2{,}25$. $2<2{,}25$ bo‘lganligi sababli, 1-qadam isbotlandi.

2-qadam. $n=k$ da tengsizlikning bajarilishi berilgan: $k!<\left(\frac{k+1}{2}\right)^k$, $k\geq 2$. $n=k+1$ da quyidagi tengsizlikning bajarilishini isbotlash lozim: $(k+1)!<\left(\frac{k+2}{2}\right)^{k+1}$, $k\geq 2$.

Isboti. $(k+1)!=k!\cdot(k+1)<\left(\frac{k+1}{2}\right)^k\cdot(k+1)=\left(\frac{k+2}{2}\right)^{k+1}$ songa ko‘paytiramiz va bo‘lamiz:

$$=\left(\frac{k+2}{2}\right)^{k+1}\cdot\frac{(k+1)^k\cdot(k+1)\cdot 2^{k+1}}{2^k\cdot(k+2)^{k+1}}=\left(\frac{k+2}{2}\right)^{k+1}\cdot\frac{2\cdot(k+1)^{k+1}}{(k+2)^{k+1}}<\frac{2\cdot(k+1)^{k+1}}{(k+2)^{k+1}}<1$$

tengsizlikning bajarilishini isbotlaymiz.

$$\frac{2\cdot(k+1)^{k+1}}{(k+2)^{k+1}}=\frac{2}{(\frac{k+2}{k+1})^{k+1}}=2\cdot\frac{1}{(1+\frac{1}{k+1})^{k+1}}$$

$$\left(1+\frac{1}{k+1}\right)^{k+1}=1+\frac{k+1}{k+1}+\underbrace{\frac{(k+1)\cdot k}{2!}\cdot\frac{1}{(k+1)^2}+....+\left(\frac{1}{k+1}\right)^{k+1}}_{>0}>2\cdot$$

$$\frac{1}{\left(1+\frac{1}{k+1}\right)^{k+1}}<\frac{1}{2}\ \Rightarrow\ 2\cdot\left(\frac{k+1}{k+2}\right)^{k+1}<2\cdot\frac{1}{2}=1\quad \text{va}\ \ k!<\left(\frac{k+2}{2}\right)^{k+1}\cdot 1=\left(\frac{k+2}{2}\right)^{k+1}.$$

2-qadam isbotlandi.

Matematik induksiya prinsipiga ko‘ra, tengsizlik ixtiyoriy $n\geq 2$ natural son uchun bajariladi.

53. 1-usul. 1-qadam. $n=1$ da tenglikning chap qismi $(a+b)^1$ ga teng. Ushbu tenglikning o‘ng qismi $\sum_{m=0}^{1}C_1^m a^{1-m}b^m=\frac{1!}{0!1!}a+\frac{1!}{1!0!}b=a+b.$

2-qadam. $n=k$ da tenglikning o‘rinli ekanligi berilgan $(a+b)^k=\sum_{m=0}^{k}C_k^m\,a^{k-m}b^m$, $n=k+1$ da tenglikning o‘rinli ekanligini isbotlaymiz: $(a+b)^{k+1}=\sum_{m=0}^{k+1}C_{k+1}^m\,a^{k+1-m}b^m$. Haqiqatdan,

$$(a+b)^{k+1}=(a+b)\cdot(a+b)^k=(a+b)\cdot\sum_{m=0}^{k}C_k^m\,a^{k-m}b^m=$$

$$=\sum_{m=0}^{k}C_k^m\,a^{k+1-m}b^m+\sum_{m=0}^{k}C_k^m\,a^{k-m}b^{m+1}=$$

Ikkinchi qo‘shiluvchida yig‘indini $m=1$ da boshlaymiz. Ikkinchi qo‘shiluvchida m ning o‘rniga $m-1$ olinadi.

$$=\sum_{m=0}^{k}C_k^m\,a^{k+1-m}b^m+\sum_{m=1}^{k+1}C_k^{m-1}\,a^{k-(m-1)}b^{m-1+1}=$$

$$=\sum_{m=0}^{k}C_k^m\,a^{k+1-m}b^m+\sum_{m=1}^{k+1}C_k^{m-1}\,a^{k+1-m}b^m=$$

Birinchi yig‘indida birinchi qo‘shiluvchini, ikkinchi yig‘indida – oxirgi qo‘shiluvchini alohida yozamiz.

Ikkala yig‘indi $m=1$ dan $m=k$ gacha yig‘iladi. a,b sonlarning darajalari yig‘indisi ushbu belgilar bilan mos tushadi:

$$= C_k^0 a^{k+1} + \sum_{m=1}^{k} (C_k^m + C_k^{m-1})\, a^{k+1-m} b^m + C_k^k b^{k+1} = C_k^0 = C_{k+1}^0 = C_k^k = C_{k+1}^{k+1} = 1.$$

$$C_k^m + C_k^{m-1} = \frac{k!}{m!(k-m)!} + \frac{k!}{(m-1)!(k-(m-1))!} =$$

$$= \frac{k!}{m(m-1)!(k-m)!} + \frac{k!}{(m-1)!(k+1-m)(k-m)!} =$$

$$= \frac{k!(k+1-m+m)}{m!(k+1-m)!} = \frac{(k+1)!}{m!((k+1)-m)!} = C_{k+1}^m.$$

$$= \sum_{m=0}^{k+1} C_{k+1}^m\, a^{k+1-m} b^m.$$

2-qadam isbotlandi. 1- va 2- qadamlardan berilgan tenglikning ixtiyoriy n uchun o‘rinli ekanligi kelib chiqadi.

2-usul. $f(x) = (x+b)^n$ funksiyani qaraymiz. Uni yoyib chiqsak, biror $f(x) = A_n x^n + A_{n-1}x^{n-1} + ... + A_k x^k + ... + A_1 x + A_0$ ko‘rinishga keladi. Bu yerda $A_i, \left(i=\overline{0,n}\right)$ lar koeffitsiyentlar. U holda $A_0 = f(0) = b^n$. Endi uning hosilasini qaraymiz:

$$f'(x) = nA_n x^{n-1} + (n-1)A_{n-1}x^{n-2} + ... + A_1 \Rightarrow A_1 = f'(0) = nb^{n-1}.$$

Xuddi shunga o‘xshash $A_k = \dfrac{f^{(k)}(0)}{k!} = \dfrac{n(n-1)...(n-k+1)b^{n-k}}{k!} = C_n^k b^{n-k}$.

Demak, $(x+b)^n = \sum_{k=0}^{n} C_n^k x^k b^{n-k}$. Bu esa $x=a$ da $(a+b)^n = \sum_{k=0}^{n} C_n^k a^k b^{n-k}$ ga teng.

54. Quyidagi funksiyani qaraymiz: $g(x) = f(x) - \dfrac{x^2}{2}$.

Ma’lumki, $g'(x) = f'(x) - x$ va $g'(0) = f'(0) - 0 = 1$, $g'(1) = f'(1) - 1 = -1$

Demak, $g'(x)$ funksiya segmentning chetki nuqtalarida turli ishorali qiymatlar qabul qilyapti. Bolsono-Koshining 1-teoremasiga ko‘ra,

$g'(x)$ funksiya uchun $\exists c \in (0,1)$ topiladiki, $g'(c) = 0$ bundan $f'(c) - c = 0$ yoki $f'(c) = c$.

55. Quyidagi funksiyani qaraymiz: $g(x) = f(x)e^{-x}$. Ma'lumki, $g'(x) = e^{-x}(f'(x) - f(x))$, $g(0) = g(1) = 0$. Endi $g'(x)$ funksiya Roll teoremasining barcha shartlarini qanoatlantiradi. Roll teoremasiga koʻra, $\exists x \in (0,1)$ uchun $g'(x) = 0$ boʻladi. Bundan esa $f'(x) = f(x)$.

56. $P(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ... + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$ boʻlsin u holda

$P'(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ... + \frac{x^{n-2}}{(n-2)!} + \frac{x^{n-1}}{(n-1)!} = P(x) - \frac{x^n}{n!}$ $P(x) = P'(x) + \frac{x^n}{n!}$

$P(x)$ karrali ildizga ega boʻlsin. U holda

$$\exists \alpha \in R \quad P'(\alpha) = P(\alpha) = 0, \quad 0 = 0 + \frac{\alpha^n}{n!} \quad \alpha = 0$$

Demak, $\alpha = 0$, $P(x)$ koʻpxadning ildizi. Lekin, $P(0) = 1 \neq 0$ ziddiyat $P(x)$ karrali ildizga ega emas.

57. $a_1 = \frac{a_0 + 3}{4} = \frac{3}{4}$, $a_2 = \frac{a_1 + 3}{4} = a_1 = \frac{a_1}{4} + \frac{3}{4}$ $a_3 = \frac{a_2}{4} + \frac{3}{4} = \frac{3}{4^3} + \frac{3}{4^2} + \frac{3}{4}$

demak, $a_n = \frac{3}{4^n} + \frac{3}{4^{n-1}} + ... + \frac{3}{4} = \frac{\frac{3}{4}\left(1 - \frac{1}{4^n}\right)}{1 - \frac{1}{4}} = 1 - \frac{1}{4^n}$ u holda

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1 - \frac{1}{4^n}\right) = 1$$

58. $[x] + [2x] + [3x] = 6$ $x = k + \alpha$ boʻlsin $k = [x]$, $\alpha = \{x\}$ $k - x$ sonning butun qismi, $\alpha - x$ sonning kasr qismi

$[k + \alpha] + [2k + 2\alpha] + [3k + 3\alpha] = 6$
$k + 2k + [2\alpha] + 3k + [3\alpha] = 6$
$6k + [2\alpha] + [3\alpha] = 6$

Quyidagi hollarni qaraymiz:

1-hol. $0 \le \alpha < \frac{1}{3} \to 0 \le 3\alpha < 1, 0 \le 2\alpha < \frac{2}{3} < 1$

Demak, $[2\alpha] = [3\alpha] = 0,\ 6k = 6\ k = 1$ $x = k + \alpha$ $x = 1 + \alpha.\ x \in [0 : \frac{1}{3})$

2-hol. $\frac{1}{3} \le \alpha < \frac{1}{2} \Rightarrow 1 \le 3\alpha < \frac{3}{2},\ \frac{2}{3} < 2\alpha < 1$ $[3\alpha] = 1, [2\alpha] = 0$, $6k + 1 = 6$,

$k = \frac{5}{6}$ k-butun son boʻlishi kerak. Demak, bu holda yechim yoʻq.

3-hol. $\frac{1}{2} \le \alpha < \frac{2}{3} \Rightarrow \frac{3}{2} \le 3\alpha < 2, 1 \le 2\alpha < \frac{4}{3} \Rightarrow [3\alpha] = 1$ $[2\alpha] = 1$

$6k+1+1=6,\quad 6k=4\ k=\frac{4}{6}$ k-butun son bo‘lishi kerak. Demak, bu holda yechim yo‘q.

4-hol. $\frac{2}{3}\le\alpha<1$ bundan $2\le 3\alpha<3,\ \frac{4}{3}\le 2\alpha<2$ $[3\alpha]=2,\quad [2\alpha]=1$ va $6k+2+1=6\Rightarrow k=\frac{3}{6}$ lekin k-butun son bo‘lishi kerak. Demak bu holda yechim yo‘q.

Demak , umumiy yechim $x=1+\alpha$ bu yerda $0\le\alpha<\frac{1}{3}$.

59. $I=\int\frac{\sin x}{\sin x+\cos x+\sqrt{2}}dx\quad G=\int\frac{\cos x}{\sin x+\cos x+\sqrt{2}}dx$ bo‘lsin.

$$I-G=\int\frac{\sin x-\cos x}{\sin x+\cos x+\sqrt{2}}dx=-\int\frac{d(\sin x+\cos x)}{\sin x+\cos x+\sqrt{2}}=-\ln\left(\sin x+\cos x+\sqrt{2}\right)$$

$$I+G=\int\frac{\sin x+\cos x}{\sin x+\cos x+\sqrt{2}}dx=\int\left(1-\frac{\sqrt{2}}{\sin x+\cos x+\sqrt{2}}\right)dx=$$

$$=x-\int\frac{\sqrt{2}dx}{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)+\sqrt{2}}=x-\int\frac{dx}{\sqrt{2}\cos^2\left(\frac{x}{2}-\frac{\pi}{4}\right)}=\left|\begin{array}{l}\frac{x}{2}-\frac{\pi}{4}=t\\ x=2t+\frac{\pi}{2}\\ dx=2dt\end{array}\right|=$$

$$=x-\int\frac{2dt}{\cos^2 t}=x-2tgt=x-2tg\left(\frac{x}{2}-\frac{\pi}{4}\right)$$

$$\begin{cases}I-G=-\ln\left(\sin x+\cos x+\sqrt{2}\right)\\ I+G=x-2tg\left(\frac{x}{2}-\frac{\pi}{4}\right)\end{cases}$$

bu tengliklarni hadma-had qo‘shib I ni topamiz.

$$I=\frac{x}{2}-tg\left(\frac{x}{2}-\frac{\pi}{4}\right)-\frac{1}{2}\ln\left|\left(\sin x+\cos x+\sqrt{2}\right)\right|+C$$

60. Quyidagi integrallarni hisoblang.

a) $\int\sqrt{tgx}dx$

$$\int\sqrt{tgx}dx=\left|\begin{array}{l}\sqrt{tgx}=t\\ tgx=t^2,dx=\frac{2t}{1+t^4}dt\\ x=arctgt^2\end{array}\right|=\int\frac{2t^2}{t^4+1}dt+\int\frac{\left(t^2+1\right)+\left(t^2-1\right)}{t^4+1}dt=$$

$$=\int\frac{t^2+1}{t^4+1}dt+\int\frac{t^2-1}{t^4+1}dt=\int\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt+\int\frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt=$$

$$= \int \frac{d\left(1-\frac{1}{t}\right)}{\left(1-\frac{1}{t}\right)^2+2} + \int \frac{d\left(1-\frac{1}{t}\right)}{\left(1-\frac{1}{t}\right)^2-2} = \frac{1}{\sqrt{2}} artg\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) +$$

$$+\ln\left|\frac{t^2-t\sqrt{2}+1}{t^2+t\sqrt{2}+1}\right| + C = \frac{1}{\sqrt{2}} arctg\left(\frac{tg^2x-1}{\sqrt{2tgx}}\right) + \frac{1}{2\sqrt{2}}\ln\left|\frac{tgx-\sqrt{2tgx}+1}{tgx+\sqrt{2tgx}+1}\right| + C$$

b) $$\int \frac{dx}{\sin x} = \int \frac{\sin x dx}{\sin^2 x} = -\int \frac{d(\cos x)}{1-\cos^2 x} = \left|\cos x = t\right| =$$
$$= \int \frac{dt}{t^2-1} = \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C = \frac{1}{2}\ln\left|\frac{1-\cos x}{1+\cos x}\right| + C$$

c) $\int\limits_0^1 xf\left(x^2\right)dx$,bu yerda $\int\limits_0^1 f\left(x\right)dx = a$.

$$\int\limits_0^1 xf\left(x^2\right)dx = \frac{1}{2}\int\limits_0^1 f\left(x^2\right)d\left(x^2\right) = \left|x^2 = t\right| = \frac{1}{2}\int\limits_0^1 f\left(t\right)d\left(t\right) = \frac{1}{2}a .$$

61. $f(x) = x^3 - 3x^2 + 1$ bo‘lsin.

$$f(0.6) = 0,6^3 - 0,6^2 + 1 = 0,216 - 1,08 + 1 = 0,136 > 0 .$$

$$f(0,7) = 0,7^3 - 3\cdot 0,7^2 + 1 = 0,343 - 1,47 + 1 = -0,127 < 0.$$

Balsano-Koshining birinchi teoremasiga ko‘ra, shunday $x_0 \in (0.6, 0.7)$ nuqta topiladiki , $f(x_0) = 0$ bo‘ladi. Demak, x_0 bu tenglamaning ildizidir.

62. $f(0) = 1 > 0$
$f(1) = -6 < 0$

Bolsano-Koshining birinchi teoremasiga ko‘ra, shunday $x_0 \in (0,1)$ topiladiki, $f(x_0) = 0$. [0,1] segmentni o‘nta teng bo‘lakka bo‘lamiz:

$$[0,1] = \bigcup_{k=1}^{10}\left[\frac{k-1}{10};\frac{k}{10}\right]$$

Endi har bir segmentning chegaralarida funksiyaning qiymatlari ko‘paytmasi manfiy bo‘ladiganini topamiz. $[0;0,1]$ da tekshiramiz. $f(0) = 0 - 8\cdot 0 + 1 > 0,$ $f(0,1) = 0,01 - 0,8 + 1 = 0,21 > 0$ $[0;0,1]$ da yechim yo‘q. Chunki $f(0)\cdot f(0,1) > 0$ $[0,1;0,2]$ da tekshiramiz.

$$f(0,1) = 0,21 > 0, \ f(0,2) = 0,04 - 1,6 + 1 = -0,56 < 0.$$

Demak, bu segmentda $f(x)$ Bolsano-Koshining birinchi teoremasining shartlarini qanotlantiradi. Bunga ko‘ra, $\exists x_0 \in (0,1;0,2)$ topilib, $f(x_0) = 0$ $0,1 < x_0 < 0,2$.

Demak, x_0 bu tenglamaning ildizi va verguldan keyingi birinchi raqami 1, ya'ni $x_0 \approx 0{,}1$.

63. Quyidagi hollarni qaraymiz:

1-hol. $x_1 + x_2 + ...x_n = 0 \quad 0 \le |x_1| + ...|x_n|$. Ravshanki bu tengsizlik hamisha o'rinli chunki , $\forall k \in \{1,2...n\}$ da $|x_k| \ge 0$.

2-hol. $x_1 + x_2 + ... + x_n \ne 0$ bu holda

$$1 = \frac{x_1 + x_2 + ... + x_n}{x_1 + x_2 + ... + x_n} = \frac{x_1}{x_1 + x_2 + ... + x_n} + \frac{x_2}{x_1 + x_2 + ... + x_n} + ...$$
$$.. + \frac{x}{x_1 + x_2 + ... + x_n} \le \left|\frac{x_1}{x_1 + x_2 + ... + x_n}\right| + \left|\frac{x_2}{x_1 + x_2 + ... + x_n}\right| + ...$$
$$... + \left|\frac{x_n}{x_1 + x_2 + ... + x_n}\right| = \frac{|x_1| + |x_2| + ...|x_n|}{x_1 + x_2 + ... + x_n}$$

Bundan quyidagi tengsizlikka ega bo'lamiz.

$$|x_1 + x_2 + ... + x_n| \le |x_1| + |x_2| + ... + |x_n|$$

64. 2^{nx} sonning raqamlari soni p, 5^{nx} niki esa q bo'lsin. U holda 2^{nx} va 5^{nx} sonlari ketma-ket yozilsa, $p+q$ xonali son hosil bo'ladi. Demak, $p+q$ ni topishimiz kerak. Ravshanki,

$$10^{p-k} < 2^{nx} < 10^p$$
$$10^{q-1} < 5^{nx} < 10^q$$

Bu tengsizliklarni hadma-had ko'paytirib topamiz:

$$10^{p+q-2} < 10^{nx} < 10^{p+q} \Rightarrow p + q - 2 < nx < p + q$$

nx butun son ekanidan $nx = p + q - 1$ kelib chiqadi. Demak, $p + q = nx + 1$.

65. $f(x) \in c[a,b]$ bo'lsin, Veyershtrasning 2-teoremasiga ko'ra f(x) funksiyaning [a,b] da eng katta va eng kichik qiymatlari mavjud. $m = \min\limits_{x \in [a,b]} f(x)$, $M = \max\limits_{x \in [a,b]} f(x)$ va $\forall x \in [a.b]$ uchun $m \le f(x) \le M$ bo'ladi. $x_1 x_2 x_3 \in [a.b]$ bo'lsin. U holda

$$m = \frac{m + m + m}{3} \le \frac{f(x_1) + f(x_1) + f(x_3)}{3} \le \frac{M + M + M}{3} \le M.$$

Demak, $\frac{f(x_1)+f(x_1)+f(x_3)}{3} \in (m,M)$. Bolsano-Koshining birinchi teoremasiga ko‘ra, $\exists\xi \in (a,b)$ topilib, $f(\xi) = \frac{f(x_1)+f(x_1)+f(x_3)}{3}$ tenglik o‘rinli bo‘ladi.

66. $f(x) \in C^1[0,\infty)$, $|f'(x)| \le M$ bo‘lsin u xolda $\forall x', x'' \in [0,\infty)$ nuqta olaylik. Lagranj teoremasiga ko‘ra, $\exists C \in (x', x'')$ topilib, $f(x') - f(x'') = f'(c)(x' - x'')$ u holda quyidagiga ega bo‘lamiz:

$$|f(x') - f(x'')| = |f'(c)||x' - x''| \le M|x' - x''| \quad |x' - x''| < \delta$$

bo‘lsa, $|f(x') - f(x'')| < M\delta$ bo‘ladi $M\delta = \varepsilon$ deb olinib $\delta = \frac{\varepsilon}{M}$ ekanini topamiz. Demak, $f(x)$ funksiya uchun $\forall\varepsilon > 0$ uchun $\exists\delta(\varepsilon) = \frac{\varepsilon}{M}$ topilib, $|x' - x''| < \delta$ tengsizlikni qanoatlantiruvchi $\forall x', x'' \in [0,\infty)$ uchun $|f(x') - f(x'')| < \varepsilon$ o‘rinli.

Demak, $f(x)$ $[0,\infty)$ da tekis uzliksiz.

67.

$$i = \cos\left(\frac{\pi}{2} + 2\pi k\right) + i\sin\left(\frac{\pi}{2} + 2\pi k\right) = e^{i\left(\frac{\pi}{2}+2\pi k\right)}, \quad i^i = e^{i^2\left(\frac{\pi}{2}+2\pi k\right)} = e^{-\frac{\pi}{2}+2\pi k}, \quad k \in \mathbb{Z}$$

Demak , i^i ning qiymati cheksiz ko‘p qiymatli.

68. Quyidagi hollarni qaraymiz:

1-hol. $z_1 + z_2 = 0$ $0 \le |z_1| + |z_2|$ ravshanki bu tenglik doimo o‘rinli.

2-hol. $z_1 + z_2 \ne 0$ $\forall z \in C$ uchun $\operatorname{Re} z \le |z|$

$$1 = \operatorname{Re}1 = \operatorname{Re}\left(\frac{z_1}{z_1 + z_2}\right) + \operatorname{Re}\left(\frac{z_2}{z_1 + z_2}\right) \le \frac{|z_1|}{|z_1 + z_2|} + \frac{|z_1|}{|z_1 + z_2|}.$$

Bu yerdan $|z_1 + z_2| \le |z_1| + |z_2|$ kelib chiqadi.

69. 68- misolga qarang.

70. Ikkinchi tur Eyler integrali $\Gamma(a) = \int_0^{+\infty} x^{a-1}e^{-x}dx$ Gamma funksiya va $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ ekanidan

$$\int_0^{+\infty} e^{-x^2}dx = \begin{vmatrix} x^2 = y \\ x = \sqrt{y} \\ dx = \frac{1}{2}y^{-\frac{1}{2}}dy \end{vmatrix} = \frac{1}{2}\int_0^{+\infty} y^{-\frac{1}{2}}e^{-y}dy = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$$

ekani kelib chiqadi.

71. Buning uchun Dirixle funksiyasidan foydalanamiz. Dirixle funksiyasi quyidagicha edi. $D(x)=\begin{cases}1, \ agar \ \ x-ratsional \ son \ \ bo'lsa,\\ 0, \ agar \ \ x-irratsional \ \ son \ bo'lsa.\end{cases}$ u holda quyidagi funksiya $f(x)=(x-1)(x-2)...(x-2013)D(x)$ masala shartini qanoatlantiradi. Bunday funksiyalardan ko‘plab topish mumkin.

72. Buning uchun quyidagi ikkita funksiya haqida gapirish yetarli: $f(x)=x-\frac{1}{x}$ va $g(x)=\sin x+\frac{1}{x}$ funksiyalarning $[0,1]$ segmentdagi aniq integrali mavjud emas. $f(x)+g(x)=x+\sin x$ yig‘indining esa $[0,1]$ segmentda aniq integrali mavjud. O‘zlari integrallanuvchi emas, lekin yig‘indisi integrallanuvchi bo‘lgan funksiyalarga juda ko‘plab misol topish mumkin.

73. $f'(\sin^2 x)=1+\cos^2 x=1+1-\sin^2 x=2-\sin^2 x$ endi $\sin^2 x=t$ deb belgilab olamiz.

$$f'(t)=2-t \quad \Rightarrow \quad f(t)=2t-\frac{t^2}{2}+C \quad \Rightarrow \quad f(x)=2x-\frac{x^2}{2}+C$$

74. Lemma. Simmetrik oraliqda toq funksiyani integrali nolga teng.

Isboti. Bizga $[-a,a]$ oraliqda aniqlangan, uzluksiz va toq bo‘lgan $f(x)$ funksiya berilgan bo‘lsin. U holda

$$\int_{-a}^{a} f(x)dx=\int_{-a}^{0} f(x)dx+\int_{0}^{a} f(x)dx=F(0)-F(-a)+F(a)-F(0)=-F(a)+F(a)=0$$

bu yerda $F(x)$-juft funksiya.

U holda $f(t)=\frac{t}{t^2+x^2}\cos x$ funksiya t ga nisbatan toq funksiya ekanidan, yuqoridagi lemmaga ko‘ra limit ichidagi integral nolga teng. Bu esa limitning ham nol ekanini ko‘rsatadi.

75. Buning uchun $f_1(x)=x^2$ funksiyaga tegishli bo‘lgan ixtiyoriy nuqtani olib, bu nuqtadan $f_2(x)=x-1$ to‘g‘ri chiziqqacha bo‘lgan masofani topamiz. Masalan, $(1,1)$ nuqta birinchi funksiyani qanoatlantiradi. U holda nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofani formulasiga ko‘ra,

$$d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}=\frac{|1-1-1|}{\sqrt{2}}=\frac{1}{\sqrt{2}}.$$

76. $\frac{1}{x\sqrt{2}}=a,\ \frac{1}{y\sqrt{3}}=b,\ \frac{1}{2z}=c$ deb belgilash kiritsak, u holda $Q(a,b,c)=2a^2+6b^2+12c^2$ ifodaning eng katta qiymatini topsak masala yechiladi. a,b,c musbat sonlar quyidagi shartlarni qanoatlantiradi:

$$\max\{a,b,\}<c\le\frac{1}{2} \qquad (1)$$

$$c\sqrt{2}+a\sqrt{3}\ge 2\sqrt{6}ac \qquad (2)$$

$$c\sqrt{2}+b\sqrt{5}\ge 2\sqrt{10}bc \qquad (3)$$

(2) dan $\frac{\sqrt{2}}{a}+\frac{\sqrt{3}}{c}\ge 2\sqrt{6}\Rightarrow\frac{2}{a^2}+\frac{3}{c^2}\ge 12\Rightarrow\frac{1}{6}a^2\left(\frac{2}{a^2}+\frac{3}{c^2}\right)\ge 2a^2$,

Bundan,

$$a^2+c^2=2a^2+c^2-a^2\le\frac{1}{6}a^2\left(\frac{2}{a^2}+\frac{3}{c^2}\right)+c^2\left(1-\frac{a^2}{c^2}\right)\le\frac{1}{6}a^2\left(\frac{2}{a^2}+\frac{3}{c^2}\right)+\frac{1}{2}\left(1-\frac{a^2}{c^2}\right)=\frac{5}{6}$$

Xuddi shunday (1) va (3) dan $b^2+c^2\le\frac{7}{10}$ ekanligini topamiz.

Shunday qilib, $Q(a,b,c,)=2(a^2+c^2)+6(b^2+c^2)+4c^2\le\frac{118}{15}$

tenglik $Q(a,b,c)=Q\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{5}},\frac{1}{\sqrt{2}}\right)=\frac{118}{15}$ bajariladi va $a=\frac{1}{\sqrt{3}},\ b=\frac{1}{\sqrt{5}},\ c=\frac{1}{\sqrt{2}}$ qiymatlar (1)-(2)-(3) shartlarni qanoatlantiradi. Bundan

$\max P(x,y,z)=\max Q(a,b,c)=\frac{118}{15}$.

77. O'rta arifmetik va o'rta geometrik miqdorlar haqidagi Koshi tengsizligini qo'llaymiz:

$$1+\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{3}\right)+...+\left(1-\frac{1}{n}\right)=1+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+...+\frac{n-1}{n}\ge$$

$$\ge n\sqrt[n]{1\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}...\frac{n-1}{n}}=n\sqrt[n]{\frac{1}{n}}=n^{\frac{n-1}{n}}.$$

78. $\frac{1}{1+x_i^2}=y_i\quad(\mathrm{i}=1,2,\dots 2011)$ deb belgilash kiritsak, u holda $\mathrm{y}_1+\mathrm{y}_2+...+\mathrm{y}_{2011}=1$ bo'ladi. O'rta arifmetik va o'rta geometrik miqdorlar haqidagi Koshi tengsizligini quyidagi usulda qo'llasak,

$$1-y_i=y_1+y_2+...+y_{2002}-y_i\ge 2001\sqrt[2001]{\frac{y_1y_2...y_{2002}}{y_i}}\quad(\mathrm{i}=1,2,\dots 2011)$$

ekanligini topamiz va bu tengsizliklarni hadma-had ko'paytirib,

$$\prod_{i=1}^{2002}(1-y_i)\ge\prod_{i=1}^{2002}2001\sqrt[2001]{\frac{y_1y_2...y_{2002}}{y_i}}=2001^{2002}y_1y_2...y_{2002},\quad\prod_{i=1}^{2002}\frac{1-y_i}{y_i}\ge 2001^{2002}$$

yoki $\prod_{i=1}^{2002} x_i \geq 2001^{1001}$ tengsizlikni hosil qilamiz.

79. $y_i = \dfrac{1998}{x_i + 1998}$ almashtirish kiritamiz. Ravshanki, $y_i \geq 0, \quad i = 1,2,...,n$ va $y_1 + y_2 + ... + y_n = 1$. Demak, $1 - y_i = \sum_{j \neq i} y_j$. Koshi tengsizligiga koʻra, $1 - y_i \geq (n-1)\sqrt[n-1]{\prod_{j \neq i} y_j}$. Bu tengsizliklarni barchasini koʻpaytirsak, $\prod_{i=1}^{n}(1 - y_i) \geq (n-1)^n \prod_{i=1}^{n} y_i$ yoki $\prod_{i=1}^{n} \dfrac{1 - y_i}{y_i} \geq (n-1)^n$ tengsizlikni hosil qilamiz. $\dfrac{1 - y_i}{y_i} = \dfrac{x_i}{1998}$ boʻlgani uchun bundan $x_1 x_2 ... x_n \geq 1998^n (n-1)^n$ tengsizlikni hosil qilamiz.

80. Oʻrta arifmetik va oʻrta geometrik miqdorlar haqidagi Koshi tengsizligi va $x \in \left(0; \dfrac{\pi}{2}\right)$ uchun $\sin x \left(\sin 2x - 1\right) \leq a$ yoki $\sin x \leq \sqrt{\dfrac{tgx}{2}}$ tengsizliklarni oʻrinli ekanligini etiborga olib,

$$\sin x_1 \cdot \sin x_2 \cdot ... \cdot \sin x_n \leq \left(\frac{\sin x_1 + \sin x_2 + ... + \sin x_n}{n}\right)^n \leq$$
$$\leq \left(\frac{\sqrt{tgx_1} + \sqrt{tgx_2} + ... + \sqrt{tgx_n}}{n}\right)^n \cdot 2^{-\frac{n}{2}} \leq$$
$$\leq \left(\sqrt{\frac{tgx_1 + tgx_2 + ... + tgx_n}{n}}\right)^n \cdot 2^{-\frac{n}{2}} \leq 2^{-\frac{n}{2}}$$

munosabatni hosil qilamiz.

81. Istalgan natural n uchun $\dfrac{1}{2n-1} > \dfrac{1}{2n}$ ekanligini etiborga olsak, $\dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{5} + ... + \dfrac{1}{2n-1} > \dfrac{1}{2} + \dfrac{1}{4} + ... + \dfrac{1}{2n}$ munosabat oʻrinlidir. Endi $\underbrace{\dfrac{1}{2} + \dfrac{1}{2} + ... + \dfrac{1}{2}}_{n} > \dfrac{1}{2} + \dfrac{1}{4} + ... + \dfrac{1}{2n}$ yoki $\dfrac{1}{2} > \left(\dfrac{1}{2} + \dfrac{1}{4} + ... + \dfrac{1}{2n}\right)\dfrac{1}{n}$ ekanligidan foydalansak, U holda

$$1 + \frac{1}{3} + ... + \frac{1}{2n-1} = \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{2n-1} > \left(\frac{1}{2} + \frac{1}{4} + ... + \frac{1}{2n}\right)\frac{1}{n} + \left(\frac{1}{2} + \frac{1}{4} + ... + \frac{1}{2n}\right) =$$
$$= \frac{n+1}{n}\left(\frac{1}{2} + \frac{1}{4} + ... + \frac{1}{2n}\right)$$

boʻladi.

82. a) umumiylikni chegaralamasdan $a \ge b \ge c$ deb olib, uchburchak tengsizligini qo'llasak, $1 = a + b + c > 2a \Rightarrow b \le a < \frac{1}{2}$ va bundan

$$a^n + b^n < \frac{1}{2^n} + \frac{1}{2^n} = \frac{2}{2^n} \Rightarrow (a^n + b^n)^{\frac{1}{n}} < \frac{2^{\frac{1}{n}}}{2} \qquad (*)$$

$$\left(b + \frac{c}{2}\right)^n = b^n + \frac{n}{2} c\, b^{n-1} + ... + \frac{c^n}{2^n} > b^n + c^n \text{ (chunki } \frac{n}{2} c b^{n-1} > c^n \text{)}.$$

Xuddi shunday, $\left(a + \frac{c}{2}\right)^n > a^n + c^n$. Demak,

$$(b^n + c^n)^{\frac{1}{n}} + (a^n + b^n)^{\frac{1}{n}} < b + \frac{c}{2} + a + \frac{c}{2} = 1 \cdot \qquad (**)$$

(*) va (**) larni hadma-had qo'shib, isboti talab etilgan tengsizlikni hosil qilamiz;

b) α, β, γ uchburchak burchaklari uchun $1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma$ tenglikdan foydalanib,

$$\frac{\cos\alpha}{\cos\beta\cos\gamma} \cdot \frac{\cos\beta}{\cos\gamma\cos\alpha} \cdot \frac{\cos\gamma}{\cos\alpha\cos\beta} = \frac{\cos\alpha}{\cos\beta\cos\gamma} + \frac{\cos\beta}{\cos\gamma\cos\alpha} + \frac{\cos\gamma}{\cos\alpha\cos\beta} + 2$$

ifodani hosil qilamiz. $x = \frac{\cos\alpha}{\cos\beta\cos\gamma}$, $y = \frac{\cos\beta}{\cos\gamma\cos\alpha}$, $z = \frac{\cos\gamma}{\cos\alpha\cos\beta}$ deb belgilash kiritib, $\cos\alpha + \cos\beta + \cos\gamma \le \frac{3}{2}$ tengsizlikdan foydalansak,

$$\frac{1}{\sqrt{xy}} + \frac{1}{\sqrt{yz} + \sqrt{zx}} \frac{1}{} \le \frac{3}{2} \Leftrightarrow \ 2(\sqrt{x} + \sqrt{y} + \sqrt{z}) \le 3\sqrt{xyz} \Leftrightarrow$$

$$4(x + y + z + 2(\sqrt{xy} + \sqrt{yz} + \sqrt{zx})) \le 9xyz \Leftrightarrow$$

$$8(\sqrt{xy} + \sqrt{yz} + \sqrt{zx}) \le 9(x + y + z + 2) - 4(x + y + z) = 5(x + y + z) + 18$$

83. Buni isbotlash uchun tenglikning o'ng tomonidan uning chap tomonini keltirib chiqaramiz:

$$(a - b)(a^{n-1} + a^{n-2}b + ... + b^{n-1}) = a^n + a^{n-1}b + a^{n-2}b^2 + ..$$
$$. + ab^{n-1} - a^{n-1}b - a^{n-2}b^2 - ... - b^n = a^n - b^n$$

bu yerda o'rtadagi hadlarning barchasi ixchamlashib ketadi.

84. Bizga ma'lumki, $|\sin x| \le 1$ tengsizlik ixtiyoriy x larda o'rinli. U holda biz $\sin x$ ning darajasini qanchalik oshirsak, u shunchalik nolga yaqinlashadi. Demak, o'zining absolyut qiymatidan kichik.\ $|\sin^n x| \ge 0, |\sin x| \ge 0$ ekani yuqoridagi tasdiqni isbotlaydi. Tenglik belgisi esa, $|\sin x| = 1$ va $|\sin x| = 0$ bo'lganda bajarilishi ko'rinib turibdi.

85. Matematik induksiya metodini qo‘llaymiz.

$n=1$ da tenglik bajariladi.

$n=2$ da $\left|\sin 2x\right| = 2\left|\sin x\right|\left|\cos x\right| \le 2\left|\sin x\right|$ chunki $\left|\cos x\right| \le 1$.

Bu tasdiqni $n=k$ da $\left|\sin kx\right| \le k\left|\sin x\right|$ ni to‘g‘ri deb olib, $n=k+1$ da to‘g‘riligini isbotlaymiz:

$n=k+1$ da

$$\left|\sin\left(k+1\right)x\right| = \left|\sin(kx+x)\right| = \left|\sin kx\cos x + \cos kx \sin x\right| \le \left|\sin kx \cos x\right| + \left|\cos kx \sin x\right| \le$$
$$\le \left|\sin kx\right| + \left|\sin x\right| \le k\left|\sin x\right| + \left|\sin x\right| = \left(k+1\right)\left|\sin x\right|$$

Demak, ixtiyoriy natural n lar uchun $\left|\sin nx\right| \le n\left|\sin x\right|$ tengsizlik o‘rinli ekan.

86. $S_k = \dfrac{1}{k+1} + \dfrac{1}{k+2} + ... + \dfrac{1}{3k+1}$ orqali belgilaymiz.

1-qadam. $n=1$ da $S_1 = \dfrac{1}{1+1} + \dfrac{1}{1+2} + \dfrac{1}{3\cdot 1+1} = \dfrac{13}{12} > 1$ ga ega bo‘lamiz. 1-qadam isbotlandi.

2-qadam. $n=k$ da quyidagi tengsizlikning bajarilishi berilgan:

$$S_k = \frac{1}{k+1} + \frac{1}{k+2} + ... + \frac{1}{3k+1} > 1.$$

Quyidagi tengsizlikning bajarilishini isbotlaymiz:

$$S_{k+1} = \frac{1}{k+2} + \frac{1}{k+3} + ... + \frac{1}{3k+1} + \frac{1}{3k+2} + \frac{1}{3k+3} + \frac{1}{3k+4} > 1.$$

Isboti.

$$S_{k+1} = \frac{1}{k+2} + \frac{1}{k+3} + ... + \frac{1}{3k+1} + \frac{1}{3k+2} + \frac{1}{3k+3} + \frac{1}{3k+4} + \left(\frac{1}{k+1} - \frac{1}{k+1}\right) =$$
$$= \underbrace{\frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + ... + \frac{1}{3k+1}}_{=S_k>1} + \underbrace{\frac{1}{3k+2} + \frac{1}{3k+3} + \frac{1}{3k+4} - \frac{1}{k+1}}_{>0} > 1.$$

“$k>0$” tengsizlik quyidagicha kelib chiqadi:

$$\frac{1}{3k+2} + \frac{1}{3k+3} + \frac{1}{3k+4} - \frac{1}{k+1} = \frac{1}{3k+2} + \frac{1}{3k+4} - \frac{2}{3k+3} =$$
$$= \frac{(3k+4)(3k+3) + (3k+2)(3k+3) - (6k+4)(3k+4)}{(3k+2)(3k+3)(3k+4)} =$$
$$= \frac{2}{(3k+2)(3k+3)(3k+4)} > 0.$$ 2-qadam isbotlandi.

87. 1-qadam. $n=6$ da: $\frac{5^6}{6!}=\frac{5^5}{5!}\left(\frac{5}{6}\right)^{6-5}$ ega bo‘lamiz. 1-qadam isbotlandi.

2-qadam. $\frac{5^k}{k!}\leq\frac{5^5}{5!}\left(\frac{5}{6}\right)^{k-5}$, $k\geq 6$ tengsizlikning bajarilishi berilgan. Quyidagi tengsizlikning bajarilishini isbotlash lozim: $\frac{5^{k+1}}{(k+1)!}\leq\frac{5^5}{5!}\left(\frac{5}{6}\right)^{k+1-5}$.

Isboti. $n=k\geq 6$ da quyidagiga ega bo‘lamiz:

$$\frac{5^{k+1}}{(k+1)!}=\underbrace{\frac{5^k}{k!}}_{\leq\frac{5^5}{5!}\left(\frac{5}{6}\right)^{k-5}}\cdot\underbrace{\frac{5}{k+1}}_{<\frac{5}{6}}\leq\frac{5^5}{5!}\left(\frac{5}{6}\right)^{k-5}\frac{5}{6}=\frac{5^5}{5!}\left(\frac{5}{6}\right)^{k+1-5}.$$

2-qadam isbotlandi. Matematik induksiya prinsipiga ko‘ra, berilgan tengsizlik ixtiyoriy $n\geq 6$ natural son uchun bajariladi.

88. $n=1,2,3$ da yig‘indini topamiz:

$$S_1=arctg\frac{1}{2}=arctg\frac{1}{1+1};$$

$$S_2=arctg\frac{1}{2}+arctg\frac{1}{8}=arctg\frac{\frac{1}{2}+\frac{1}{8}}{1-\frac{1}{2}\cdot\frac{1}{8}}=arctg\frac{2}{3}=arctg\frac{2}{2+1};$$

$$S_3=arctg\frac{2}{3}+arctg\frac{1}{18}=arctg\frac{\frac{2}{3}+\frac{1}{18}}{1-\frac{2}{3}\cdot\frac{1}{18}}=arctg\frac{3}{4}=arctg\frac{3}{3+1}.$$

Quyidagini isbotlaymiz:

$$S_n=arctg\frac{1}{2}+arctg\frac{1}{8}+...+arctg\frac{1}{2n^2}=arctg\frac{n}{n+1},\quad \forall n\in\mathbb{N}. \qquad (*)$$

Matematik induksiya metodi bilan formulani isbotlaymiz:

1-qadam. $n=1$ qiymatda hisoblanganda S_1 isbotlandi.

2-qadam. $n=k$ da (*) tenglikning bajarilishi berilgan. $n=k+1$ da $S_{k+1}=arctg\frac{k+1}{k+1+1}$ tenglikning bajarilishini isbotlaymiz:

$$S_{k+1}=\underbrace{arctg\frac{1}{2}+arctg\frac{1}{8}+..+arctg\frac{1}{2k^2}}_{=S_k}+arctg\frac{1}{2(k+1)^2}=$$

$$= S_k + arctg\frac{1}{2(k+1)^2} = arctg\frac{k}{k+1} + arctg\frac{1}{2(k+1)^2} =$$

$$= arctg\frac{\frac{k}{k+1}+\frac{1}{2(k+1)^2}}{1-\frac{k}{k+1}\cdot\frac{1}{2(k+1)^2}} = arctg\frac{k+1}{k+2}.$$

Bu yerda

$$\frac{\frac{k}{k+1}+\frac{1}{2(k+1)^2}}{1-\frac{k}{k+1}\cdot\frac{1}{2(k+1)^2}} = \frac{(2k^2+2k+1)\cdot 2\cdot(k+1)^3}{2\cdot(k+1)^2\cdot(2k^3+6k^2+5k+2)} = \frac{(2k^2+2k+1)\cdot(k+1)}{(2k^3+6k^2+5k+2)} =$$

$$= \frac{k+1}{k+2}.$$

Oxirgi tenglik quyidagidan kelib chiqadi:

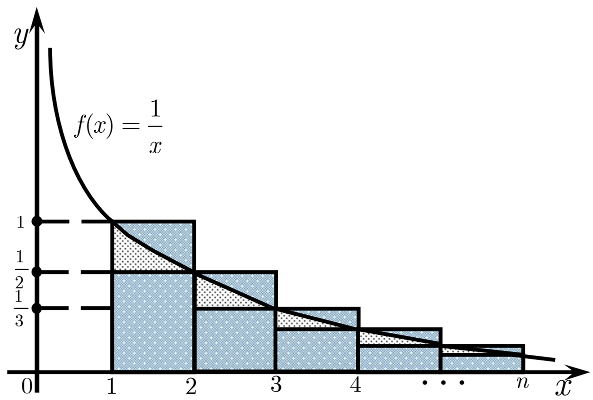
$$(2k^3+6k^2+5k+2)(2k^2+2k+1)(k+2)$$

2-qadam isbotlandi. Matematik induksiya prinsipiga ko‘ra, (*) tenglikning ixtiyoriy n natural sonlarda bajarilishi kelib chiqadi.

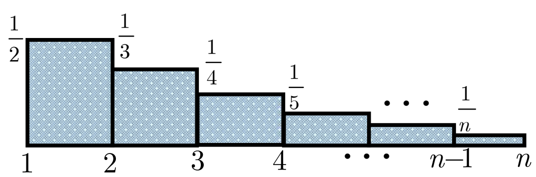
89. Dastlab quyidagi tengsizlikni isbotlaymiz:

$$\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n} < \int_1^n \frac{dx}{x} = \ln n < 1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n-1}, \ \forall\, n\in N, \ n\geq 2.$$

$\int_1^n \frac{dx}{x} = \ln n$ integral $f(x)=\frac{1}{x}$ egri chiziq, $x=1$, $x=n$ va $y=0$ chiziqlar bilan chegaralangan shakl yuzasiga teng.

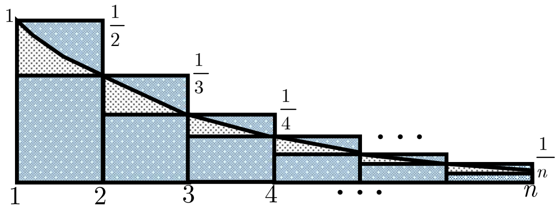


Bu yuza to‘g‘ri to‘rtburchaklar yuzalarining yig‘indisidan kattadir.



$$\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}<\ln n \quad \forall\, n\in\mathbb{N},\ \ n\geq 2.$$

Bu yuza esa to'rtburchaklar yuzalarining yig'indisidan kichikdir.



$$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n-1}>\ln n,\ \ \forall\, n\in\mathbb{N},\ \ n\geq 2\cdot$$

Matematik induksiya metodi bilan tengsizliklarni isbotlaymiz:

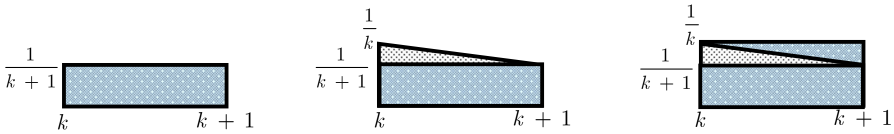
1-qadam. Uchta rasm va $[0,1]$ oraliqda aniqlangan integral xossasidan quyidagi tengsizlik kelib chiqadi $\frac{1}{2}\cdot 1<\int\limits_1^2\frac{dx}{x}=\ln 2<1\cdot 1\cdot$

Bundan $n=2$ da tengsizliklarning o'rinli ekanligi tasdiqlanadi. 1-qadam isbotlandi.

2-qadam. $n=k$ quyidagi tengsizlikning bajarilishi berilgan $\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{k}<\ln k<1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{k-1},\quad k\geq 2\cdot$ $n=k+1$ da ushbu tengsizlikning bajarilishini isbotlash lozim:

$$\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{k}+\frac{1}{k+1}<\ln(k+1)<1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{k-1}+\frac{1}{k},\quad k\geq 2\cdot \qquad (2)$$

Isboti. Quyidagi shakllarning S_1, S_2, S_3 yuzalarini taqqoslaymiz:



$$S_1=\frac{1}{k+1}\cdot 1\quad<\quad S_2=\int\limits_k^{k+1}\frac{dx}{x}=\ln\frac{k+1}{k}\quad<\quad S_3=\frac{1}{k}\cdot 1$$

(2) tengsizlikning chap qismini taqqoslaymiz:

$$\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{k}+\frac{1}{k+1}<\ln k+\frac{1}{k+1}\cdot 1<\ln k+\int\limits_k^{k+1}\frac{dx}{x}=$$
$$=\ln k+\ln(k+1)-\ln k=\ln(k+1),\quad k\geq 2$$

(2) tengsizlikning o'ng qismini taqqoslaymiz:

$$\ln(k+1)=\ln k+\ln(k+1)-\ln k=$$

$$= \ln k + \underbrace{\int_{k}^{k+1} \frac{dx}{x}}_{<\frac{1}{k}} < 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{k-1} + \frac{1}{k}, \quad k \geq 2\,.$$

2-qadam isbotlandi. 1 va 2-qadamlarning isbotidan ixtiyoriy natural $n \geq 2$ uchun tasdiq o'rinli ekanligi kelib chiqadi.

90. 1-qadamni isbotiga o'xshash to'rt marotaba x ni t ga almashtirish va t bo'yicha 0 dan $x \in \left(0, \frac{\pi}{2}\right)$ gacha mos tengsizlikni integrallash protsedurasini bajarish kerak. (1) tengsizlikning o'ng qismida x ni t ga almashtirish va t bo'yicha 0 dan $x \in \left(0, \frac{\pi}{2}\right)$ gacha ushbu tengsizlikni integrallab, quyidagini hosil qilamiz:

$$1 - \cos x = \int_0^x \sin t dt \leq \int_0^x \left(t - \frac{t^3}{3!} + ... + \frac{t^{2k+1}}{(2k+1)!}\right)dt = \frac{x^2}{2} - \frac{x^4}{4!} + ... + \frac{x^{2k+2}}{(2k+2)}.$$

$\Rightarrow \cos x \geq 1 - \frac{x^2}{2} + \frac{x^4}{4!} - ... - \frac{x^{2k+2}}{(2k+2)}$. Ikkinchi marotaba x ni t ga almashtirish va t bo'yicha 0 dan $x \in \left(0, \frac{\pi}{2}\right)$ gacha integtallab, quyidagiga ega bo'lamiz:

$$\sin x = \int_0^x \cos t dt \geq \int_0^x \left(1 - \frac{t^2}{2} + ... - \frac{t^{2k+2}}{(2k+2)!}\right)dt = x - \frac{x^3}{3!} + ... - \frac{t^{2k+3}}{(2k+3)!}.$$

Bu quyidagiga teng kuchli

$$\sin x \geq x - \frac{x^3}{3!} + ... - \frac{t^{2k+3}}{(2k+3)!}, \quad x \in \left(0, \frac{\pi}{2}\right). \tag{2}$$

(1) tengsizlikning chap qismi isbotlandi. (2) tengsizlikni uchinchi marotaba x ni t ga almashtirish va t bo'yicha $x \in \left(0, \frac{\pi}{2}\right)$ gacha integrallab, quyidagiga ega bo'lamiz:

$$1 - \cos x = \int_0^x \sin t dt \geq \int_0^x \left(t - \frac{t^3}{3!} + ... - \frac{t^{2k+3}}{(2k+3)!}\right)dt = \frac{x^2}{2} - \frac{x^4}{4!} + ... - \frac{x^{2k+4}}{(2k+4)!}.$$

Bundan $\cos x \le 1 - \frac{x^2}{2} + \frac{x^4}{4!} - ... + \frac{x^{2k+4}}{(2k+4)!}$. To‘rtinchi marotaba ushbu tengsizlikni x ni t ga almashtirish va t bo‘yicha $x \in \left(0, \frac{\pi}{2}\right)$ gacha integrallab, quyidagiga ega bo‘lamiz:

$$\sin x = \int_0^x \cos t dt \le \int_0^t \left(1 - \frac{t^2}{2} + ... + \frac{t^{2k+4}}{(2k+4)!}\right) dt = x - \frac{x^3}{3!} + ... + \frac{x^{2k+5}}{(2k+5)!}.$$

Ya’ni quyidagi tengsizlik bajariladi: $\sin x \le x - \frac{x^3}{3!} + ... + \frac{x^{2k+5}}{(2k+5)!}$.

(1) tengsizlikning o‘ng qismi, ushbu tengsizlik va (2) tengsizlikdan (1) tengsizlik hosil qilinadi. 2-qadam isbotlandi.

91. Ushbu masalani yechishda matematik induksiya metodidan foydalanmasdan isbotlash mumkin. $10^n + 10^{n-1} + ... + 1$ yig‘indi $q = 10$ maxrajli $n + 1$ ta haddan iborat geometrik progressiyani anglatadi. U holda

$$\left(10^n + 10^{n-1} + ... + 1\right) \cdot \left(10^{n+1} + 5\right) + 1 = \left(\frac{10^{n+1} - 1}{10 - 1}\right) \cdot \left(10^{n+1} + 5\right) + 1 =$$

$$= \frac{\left(10^{n+1}\right)^2 - 10^{n+1} + 5 \cdot 10^{n+1} - 5 + 9}{9} = \frac{\left(10^{n+1}\right)^2 + 2 \cdot 10^{n+1} \cdot 2 + 2^2}{9} = \left(\frac{10^{n+1} + 2}{3}\right)^2$$

ixtiyoriy n natural son uchun o‘rinli bo‘ladi.

92. Quyidagi belgilash kiritamiz: $f(x) = x^{3m} + x^{3n+1} + x^{3p+2}$ va $g(x) = x^2 + x + 1$. Ma’lumki, $f(x)$ ko‘phad $g(x)$ ko‘phadga bo‘linishi uchun $g(x)$ ning ildizlari $f(x)$ ning ham ildizlari bo‘lishi kerak. Demak, biz $g(x) = 0$ tenglamaning yechimlari $f(x)$ ni ham ildizlari bo‘lishini ko‘rsatsak bo‘ldi.

$g(x) = 0 \Rightarrow x^2 + x + 1 = 0 \Rightarrow x_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i = \cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}$ hamda $x_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i = \cos\frac{4\pi}{3} + i \sin\frac{4\pi}{3}$. Endi $\begin{cases} f(x_1) = 0 \\ f(x_2) = 0 \end{cases}$ munosabatning o‘rinli ekanini ko‘rsatamiz. Buning uchun Muavr formulasiga ko‘ra $x_1^3 = 1$ va $x_2^3 = 1$ ekanligini e’tiborga olsak:

$$f(x_1) = x_1^{3m} + x_1^{3n+1} + x_1^{3p+2} = \left(x_1^3\right)^m + \left(x_1^3\right)^n \cdot x_1 + \left(x_1^3\right)^p \cdot x_1^2 =$$
$$= 1 + x_1 + x_1^2 = 0 \Rightarrow f(x_1) = 0$$

Xuddi shunday,

$$f(x_2)=x_2^{3m}+x_2^{3n+1}+x_2^{3p+2}=\left(x_2^3\right)^m+\left(x_2^3\right)^n\cdot x_2+\left(x_2^3\right)^p\cdot x_2^2=$$
$$=1+x_2+x_2^2=0\;\Rightarrow f(x_2)=0$$

Demak, $.\,g(x)=x^2+x+1.$ ko‘phadni ildizlari $f(x)=x^{3m}+x^{3n+1}+x^{3p+2}$ ko‘phadning ham ildizlari ekan, ya’ni $\forall m,n,p\in N$ larda $x^{3m}+x^{3n+1}+x^{3p+2}$ ko‘phad x^2+x+1 ko‘phadga qoldiqsiz bo‘linar ekan.

93. Quyidagi belgilash kiritamiz: $f(x)=x^{3m}-x^{3n+1}+x^{3p+2}$ va $g(x)=x^2-x+1.$ Biz $g(x)$ ning ildizlari $f(x)$ ni ham ildizi ekanligini ko‘rsatsak yetarli.

$$g(x)=0\Rightarrow x^2-x+1=0\Rightarrow\begin{cases}x_1=\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}=\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\\ x_2=\dfrac{1}{2}-i\dfrac{\sqrt{3}}{2}=\cos\dfrac{5\pi}{3}+i\sin\dfrac{5\pi}{3}\end{cases}$$

Bulardan ko‘rinadiki, Muavr fo‘rmulasiga ko‘ra, $x_1^3=x_1^3=-1$ bo‘ladi hamda $x_1^2=x_1-1$ va $x_2^2=x_2-1$ ekanligini e’tiborga olib, bularni $f(x)$ ga qo‘yamiz:

$$f(x_1)=x_1^{3m}-x_1^{3n+1}+x_1^{3p+2}=\left(x_1^3\right)^m-\left(x_1^3\right)^n\cdot x_1+\left(x_1^3\right)^p\cdot x_1^2=$$
$$=(-1)^m-(-1)^n x_1+(-1)^p x_1^2=(-1)^m-(-1)^n x_1+(-1)^p(x_1-1)=$$
$$=\left[(-1)^m-(-1)^p\right]+x_1\left[(-1)^p-(-1)^n\right]$$

Bu oxirgi ifoda nol bo‘lishi uchun qo‘shilivchi qavslarining har biri nol bo‘lishi kerak, ya’ni $(-1)^m=(-1)^p=(-1)^n$ bo‘lishi kerak. Bu munosabat esa m,n,p lar bir vaqtda juft yoki bir vaqtda toq bo‘lganda bajariladi. $f(x_2)=0$ bo‘lishi uchun xuddu shu shart kelib chiqadi. Demak, biz quyidagi xulosaga keldik: $x^{3m}-x^{3n+1}+x^{3p+2}$ ko‘phad x^2-x+1 ko‘phadga qoldiqsiz bo‘linishi uchun m,n,p lar bir vaqtda juft yoki bir vaqtda toq bo‘lishi kerak.

94. $g(x)=x^4+x^2+1$ ko‘phadni o‘zaro tub ko‘paytuvchilarga ajratamiz:

$$g(x)=x^4+x^2+1=\left(x^2-x+1\right)\left(x^2+x+1\right)$$

92-misolda ixtiyoriy natural $\forall m,n,p\in N$ larda $x^{3m}+x^{3n+1}+x^{3p+2}$ ko‘phad x^2+x+1 ko‘phadga qoldiqsiz bo‘linadi. Demak, biz x^2-x+1 ni ildizlari

$f(x)=x^{3m}+x^{3n+1}+x^{3p+2}$ ni ham ilzdizlari bo‘ladigan m,n,p larni topishimiz kerak.

$$g(x)=0 \Rightarrow x^2-x+1=0 \Rightarrow \begin{cases} x_1=\frac{1}{2}+i\frac{\sqrt{3}}{2}=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3} \\ x_2=\frac{1}{2}-i\frac{\sqrt{3}}{2}=\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3} \end{cases}$$

Bulardan ko‘rinadiki, Muavr fo‘rmulasiga ko‘ra $x_1^3=x_1^3=-1$ bo‘ladi hamda $x_1^2=x_1-1$ va $x_2^2=x_2-1$ ekanligini e’tiborga olib, bularni $f(x)$ ga qo‘yamiz:

$$f(x_1)=x_1^{3m}+x_1^{3n+1}+x_1^{3p+2}=\left(x_1^3\right)^m+\left(x_1^3\right)^n\cdot x_1+\left(x_1^3\right)^p\cdot x_1^2=$$
$$=(-1)^m+(-1)^n x_1+(-1)^p x_1^2=(-1)^m+(-1)^n x_1+(-1)^p(x_1-1)=$$
$$=\left[(-1)^m-(-1)^p\right]+x_1\left[(-1)^p-(-1)^{n+1}\right]$$

Demak, $f(x_1)=0$ bo‘lishi uchun $(-1)^m=(-1)^p=(-1)^{n+1}$ bo‘lishi, ya’ni $m,p,n+1$ sonlari bir vaqtda toq yoki bir vaqtda juft bo‘lishi kerak.

$f(x_2)=0$ bo‘lishi uchun xuddu shu shart kelib chiqadi.

Bulardan quyidagi xulosaga kelamiz: $x^{3m}+x^{3n+1}+x^{3p+2}$ ko‘phad x^4+x^2+1 ko‘phadga qoldiqsiz bo‘linishi uchun $m,p,n+1$ sonlari bir vaqtda toq yoki bir vaqtda juft bo‘lishi kerak.

95. Biz $g(x)=x^2+x+1$ ko‘phad ildizlari $f(x)=x^{2m}+x^m+1$ ko‘phadning ham ildizi bo‘ladigan m larni topishimiz kerak.

$g(x)=0 \Rightarrow x^2+x+1=0 \Rightarrow x_1=-\frac{1}{2}+\frac{\sqrt{3}}{2}i=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$ hamda $x_2=-\frac{1}{2}-\frac{\sqrt{3}}{2}i=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}$. Bularni $f(x)$ ga qo‘yib, quyidagiga ega bo‘lamiz:

$$f(x_1)=x_1^{2m}+x_1^m+1=\cos\frac{4\pi m}{3}+i\sin\frac{4\pi m}{3}+\cos\frac{2\pi m}{3}+i\sin\frac{2\pi m}{3}+1=$$
$$=2\cos\pi m\cos\frac{\pi m}{3}+i\sin\pi m\cos\frac{\pi m}{3}+1=2(-1)^m\cos\frac{\pi m}{3}+1.$$

Bu oxirgi ifoda, ya’ni $f(x_1)=0$ bo‘lishi uchun m soni 3 ga bo‘linmasligi kerak, ya’ni $m=3k+1$ yoki $m=3k+2$ ko‘rinishda bo‘lishi kerak. $f(x_2)=0$ bo‘lishi uchun ham shu shart kelib chiqadi. Bulardan quyidagi xulosaga kelamiz: $x^{2m}+x^m+1$ ko‘phad x^2+x+1 ko‘phadga bo‘linishi uchun m soni $m = 3k+1$ yoki $m = 3k+2$ ko‘rinishda bo‘lishi kerak.

96. Buning uchun $g(x) = x^{k-1} + x^{k-2} + ... + 1$ ko‘phadning ildizi $f(x) = x^{ka_1} + x^{ka_2+1} + ... + x^{ka_k+k-1}$ ko‘phadning ham ildizi bo‘lishini ko‘rsatamiz.

$$x^{k-1} + x^{k-2} + ... + 1 = 0 \qquad \Big| \cdot (x-1)$$

$$x^k - 1 = 0 \qquad\qquad x^k = 1$$

Demak, $g(x)$ ko‘phadning ildizlari 1 soning k – darajali ildizlaridir. Bu ildizlarni x_i $i = \overline{1,k}$ bilan belgilasak. Berilishiga ko‘ra har x_i uchun

$$x_i^k = 1$$

bo‘ladi. Bulardan,

$$f(x_i) = x_i^{ka_1} + x_i^{ka_2+1} + ... + x_i^{ka_k+k-1} = \left(x_i^k\right)^{a_1} + \left(x_i^k\right)^{a_2} \cdot x_i + ... + \left(x_i^k\right)^{a_k} \cdot x^{k-1} =$$
$$= x_i^{k-1} + x_i^{k-2} + ... + 1 = 0.$$

Isbot tugadi.

97. $\left(x^2 + x + 1\right)^2 = \left(x - x_1\right)^2 \left(x - x_2\right)^2$ yuqoridagi misollarda ko‘rsatilgandi

$$x_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$
$$x_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

x_1 *va* x_2 sonlari $\left(x^2 + x + 1\right)^2$ ko‘phadning karrali ildizlari. Bu ildizlari $f(x)$-ni ham ildizi bo‘lishi uchun m natural soni

$m = 6n + 2$ *va* $m = 6n + 4$ $(n \in N)$ ko‘rinishda bo‘lishi kerak edi. Lekin bu yerda x_1 ildiz karrali ildiz bo‘lishi uchun $f'(x_1) = m\left(\left(1 + x_1\right)^{m-1} + x_1^{m-1}\right) = 0$ ham nol bo‘ladigan m larni topamiz. Bunda faqat ikkita hol bo‘lishi mumkin:

$$m = 6n + 2 \Rightarrow f'(x_1) = m\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} + \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = im\sqrt{3} \neq 0$$
$$m = 6n + 5 \Rightarrow f'(x_1) = m\left(\cos\pi + i\sin\pi + \cos 2\pi + i\sin 2\pi\right) = 0$$

$f(x_2) = 0$ bo‘lishi uchun ham xuddi shunday shart kelib chiqadi.

Demak, bulardan quyidagi xulosaga kelamiz:

$\left(x+1\right)^m + x^m + 1$ ko‘phad $\left(x^2+x+1\right)^2$ ko‘phadga bo‘linishi uchun m natural soni $m = 6n+4$ $\left(n \in \mathbb{N}\right)$ ko‘rinishda bo‘lishi kerak.

98. Biz $g\left(x\right) = x^2 + x + 1$ ko‘phad ildizi $f\left(x\right) = \left(x+1\right)^m + x^m + 1$ ko‘phadni ham ildizi bo‘ladigan m larni topishimiz kerak.

$g\left(x\right) = 0 \Rightarrow x^2 + x + 1 = 0 \Rightarrow x_1 = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i = \cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}$

$x_2 = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i = \cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}$. Bularni hamda $1 + x_1 = \cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}$, shu bilan birga, Muavr formulasini e’tiborga olsak, quyidagilarga ega bo‘lamiz:

$$f\left(x_1\right) = \left(x_1 + 1\right)^m + x_1^m + 1 = \cos\frac{\pi m}{3} + i\sin\frac{\pi m}{3} + \cos\frac{2\pi m}{3} + i\sin\frac{2\pi m}{3} + 1 \cdot$$

Kosinus va sinus funksiyalarning davri $2\pi k$ $k \in Z$ bo‘lgani uchun $\dfrac{\pi m}{3} = 2\pi k \Rightarrow m = 6k$ larni e’tiborga olib, quyidagi hollarni qarab chiqamiz.

$m = 6n \Rightarrow f\left(x_1\right) = \cos 2\pi n + i\sin 2\pi n + \cos 4\pi n + i\sin 4\pi n + 1 = 3 \neq 0$

$m = 6n+1 \Rightarrow f\left(x_1\right) = \cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3} + \cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3} + 1 = i\sqrt{3} + 1 \neq 0$

$m = 6n+2 \Rightarrow f\left(x_1\right) = \cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3} + \cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3} + 1 = 0$

$m = 6n+3 \Rightarrow f\left(x_1\right) = \cos\pi + i\sin\pi + \cos 2\pi + i\sin 2\pi + 1 = 1 \neq 0$

$m = 6n+4 \Rightarrow f\left(x_1\right) = \cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3} + \cos\dfrac{8\pi}{3} + i\sin\dfrac{8\pi}{3} + 1 = 0$

$m = 6n+5 \Rightarrow f\left(x_1\right) = \cos\dfrac{5\pi}{3} + i\sin\dfrac{5\pi}{3} + \cos\dfrac{10\pi}{3} + i\sin\dfrac{10\pi}{3} + 1 = -i\sqrt{3} + 1 \neq 0$

Demak, $f\left(x_1\right) = 0$ bo‘lishi uchun m natural soni $m = 6n+2$ *va* $m = 6n+4$ $\left(n \in \mathbb{N}\right)$ ko‘rinishda bo‘lishi kerak. $f\left(x_2\right) = 0$ bo‘lishi uchun ham xuddi shu shart kelib chiqadi. Bulardan quyidagi xulosaga kelamiz: $\left(x+1\right)^m + x^m + 1$ ko‘phad $x^2 + x + 1$ ko‘phadga bo‘linishi uchun m natural soni $m = 6n+2$ *va* $m = 6n+4$ $\left(n \in \mathbb{N}\right)$ ko‘rinishda bo‘lishi kerak.

99. $\left(x^2 + x + 1\right)^2 = \left(x - x_1\right)^2\left(x - x_2\right)^2$ yuqoridagi misollarda ko‘rsatilgandi

$$x_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$

$$x_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

x_1 *va* x_2 sonlari $(x^2+x+1)^2$ koʻphadning karrali ildizlari. Bu ildizlari $f(x)$ning ham ildizi boʻlishi uchun m natural soni $m = 6n+1$ *va* $m = 6n+5$ $(n \in \mathbb{N})$ koʻrinishda boʻlishi kerak edi. Lekin bu yerda x_1 ildiz karrali ildiz boʻlishi uchun $f'(x_1) = m\left((1+x_1)^{m-1} - x^{m-1}\right) = 0$ ham nol boʻladigan m larni topamiz. Bunda faqat ikkita hol boʻlishi mumkin:

$$m = 6n+1 \Rightarrow f'(x_1) = m(\cos 2\pi n + i\sin 2\pi n - \cos 4\pi n - i\sin 4\pi n) = 0$$
$$m = 6n+5 \Rightarrow f'(x_1) =$$
$$= m(\cos(2\pi n + \pi) + i\sin(2\pi n + \pi) - \cos(4\pi n + 2\pi) - i\sin(4\pi n + 2\pi)) = -2m \neq 0$$

$f(x_2) = 0$ boʻlishi uchun ham xuddi shunday shart kelib chiqadi.

Demak, bulardan quyidagi xulosaga kelamiz:

$(x+1)^m - x^m - 1$ koʻphad $(x^2+x+1)^2$ koʻphadga boʻlinishi uchun m natural soni $m = 6n+1$ $(n \in \mathbb{N})$ koʻrinishda boʻlishi kerak.

100. $(a+b-c)^2 > 0 \Rightarrow a^2+b^2+c^2+2(ab-bc-ac) > 0$ $\Rightarrow \frac{5}{3} + 2(ab-bc-ac) > 0 \Rightarrow bc+ac-ab < \frac{5}{6} < 1$ oxirgi tengsizlikning ikkala tarafini $abc > 0$ ga boʻlsak, $\frac{1}{a}+\frac{1}{b}-\frac{1}{c} < \frac{1}{abc}$ tengsizlik hosil boʻladi.

101. Bu tengsizlikni isbotlash uchun avval uni quyidagi koʻrinishga keltirib olamiz: $\left(1+\frac{1}{n}\right)^{n+1} < e\left(1+\frac{1}{2n}\right)$

Quyidagi yordamchi funksiyani qaraymiz:

$$f(x) = x + x\ln\left(1+\frac{x}{2}\right) - (1+x)\ln(1+x) \qquad \left(0 < x \leq \frac{1}{n}\right)$$

Bu funksiyani oʻsuvchi yoki kamayuvchi ekanini uning hosilasi yordamida tekshiramiz. Agar biz $x > 0$ larda $\ln x \leq x - 1$ tengsizlikni ham e’tiborga olsak,

$$f'(x) = \frac{x}{x+2} - \ln\frac{1+x}{1+\frac{x}{2}} > \frac{x}{x+2} - \frac{1+x}{1+\frac{x}{2}} + 1 = 0 \Rightarrow x \in \left(0, \frac{1}{n}\right) \quad f'(x) > 0$$

ekanligi, ya'ni funksiya $\left(0,\frac{1}{n}\right)$ oraliqda o'suvchi ekanligi kelib chiqadi.

Bundan $\forall x \in \left(0,\frac{1}{n}\right)$ da $f\left(\frac{1}{n}\right) > f(0) = 0 \Rightarrow$ $\left(1+\frac{1}{n}\right)^{n+1} < e\left(1+\frac{1}{2n}\right)$

Demak, $\forall n \in \mathbb{N}$ lar uchun $\frac{e}{2n+1} < e - \left(1+\frac{1}{n}\right)^n$ tengsizlik o'rinli ekan.

102. Isboti. 4-shartga ko'ra, $\forall \varepsilon > 0$ $\frac{\varepsilon}{4n_0 M}$ ga ko'ra,

$\exists n_0(\varepsilon) \in \mathbb{N}$ $\forall n \in \mathbb{N}$ topiladiki, $\forall n > n_0$ da $|x_n - a| < \frac{\varepsilon}{2}$ o'rinli bo'ladi. $\lim\limits_{x\to\infty} x_n = a$

dan $|x_n| < M$ $(M > 0)$, $\forall n \in N$ $|x_n - a| < |x_n| + |a| < M + M < 2M$

3- shartga ko'ra, $\forall \varepsilon > 0, \frac{\varepsilon}{4n'M}$ ga ko'ra $\exists n' \in \mathbb{N}$ $\forall n > n'$ lar uchun

$|P_{n_k}| < \frac{\varepsilon}{4n'M}$ quyidagini qaraymiz. $\forall \varepsilon > 0$ uchun

$\max(n', n_0) = \overline{n}$, $\exists \overline{n} \in \mathbb{N}, \forall n > \overline{n}$

$$\left|\sum_{k=1}^{n} P_{n_k} x_k - a\right| = \left|\sum_{k=1}^{n} P_{n_k} x_k - \sum_{k=1}^{n} P_{n_k}(x_k - k)\right| = \left|\sum_{k=1}^{n} P_{n_k}(x_k - a)\right| =$$

$$= \left|P_{n_1}(x_1 - a) + P_{n_2}(x_2 - a) + \ldots + P_{n_{n_0}}(x_{n_0} - a) + P_{n_{n_0+1}}(x_{n_0+1} - a) + \ldots. + P_{n_n}(x_n - a)\right| <$$

$$< \frac{\varepsilon}{4n_0 M} \cdot 2M \cdot n_0 + \frac{\varepsilon}{2} \cdot 1 < \varepsilon.$$

Bundan $\lim\limits_{n\to\infty} \sum_{k=1}^{n} P_{n_k} x_k = a$ ekani kelib chiqadi.

103. Masala shartiga ko'ra $a \le x_i \le b, (i = 1,2,3,\ldots,n)$. U holda $0 \ge (x_i - a)(x_i - b)$ tengsizlik o'rinli. Bu tengsizlikni har bir x_i lar uchun yozib chiqamiz:

$$\begin{cases} x_1(a+b) \ge x_1^2 + ab \\ x_2(a+b) \ge x_2^2 + ab \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ x_n(a+b) \ge x_n^2 + ab \end{cases} \Rightarrow \begin{cases} a+b \ge x_1 + \frac{ab}{x_1} \\ a+b \ge x_2 + \frac{ab}{x_2} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a+b \ge x_n + \frac{ab}{x_n} \end{cases} \Rightarrow n(a+b) \ge (x_1 + x_2 + x_n) + ab\left(\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}\right) \ge$$

$$\ge 2\sqrt{ab(x_1 + x_2 + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}\right)} \Rightarrow (x_1 + x_2 + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}\right) \le \frac{n^2(a+b)^2}{4ab}.$$

104.

$$\lim_{x\to 0}\frac{1-\cos x\cos 2x\cos 3x.....\cos nx}{x^2}=\lim_{x\to 0}\left[\frac{1-\cos x}{x^2}+\frac{(1-\cos 2x)\cos x}{x^2}+\right.$$
$$+\frac{(1-\cos 3x)\cos x\cos 2x}{x^2}+\frac{(1-\cos 4x)\cos x\cos 2x\cos 3x}{x^2}+...$$
$$\left....+\frac{(1-\cos nx)\cos x\cos 2x\cos 3x...\cos nx}{x^2}\right]=\lim_{x\to 0}\frac{1-\cos kx}{x^2}=$$
$$=\lim_{x\to 0}\frac{2\sin^2\frac{kx}{2}}{\left(\frac{kx}{2}\right)^2}\cdot\left(\frac{k}{2}\right)^2=\frac{2k^2}{4}=\frac{k^2}{2}\Rightarrow \lim_{x\to 0}\frac{1-\cos x\cos 2x\cos 3x.....\cos nx}{x^2}=$$
$$=\frac{1^2}{2}+\frac{2^2}{2}+....+\frac{n^2}{2}=\frac{1^2+2^2+..+n^2}{4}=\frac{n(n+1)(2n+1)}{12}.$$

105. 3-misoldagi kabi $x\geq e$ da $f(x)=\frac{\ln x}{x}$ funksiya kamayuvchi va $\pi>e$ ekanidan $f(\pi)<f(e)\ \Rightarrow \frac{\ln\pi}{\pi}<\frac{\ln e}{e}\ \Rightarrow\ \pi^e<e^\pi$ ekani kelib chiqadi.

106. Biz yana $f(x)=\frac{\ln x}{x}$ funksiyani $x>1$ oraliqda qaraymiz. 3-misoldan ma'lumki, bu funksiya monoton kamayuvchi. U holda $a>b>1$ ga funksiyani ta'sir ettirsak,

$$f(a)<f(b)\ \Rightarrow\ \frac{\ln a}{a}<\frac{\ln b}{b}\ \Rightarrow\ a^b<b^a \qquad (1)$$

Endi (1) ga funksiyani yana bir marta qo‘llasak, $a^{b^a}>b^{a^b}$ hosil bo‘ladi.

107. Quyidagi tenglamani qaraymiz: $z^{2n+1}=1$. Ravshanki, bu tenglamaning ildizlari quyidagi formuladan topiladi:

$$z_k=\cos\frac{2\pi k}{2n+1}+i\sin\frac{2\pi k}{2n+1},\quad k=\overline{0,2n}\,.$$

Muavr formulasiga ko‘ra, ixtiyoriy $\alpha\in R$ uchun

$$(\cos\alpha+i\sin\alpha)^{2n+1}=\cos(2n+1)\alpha+i\sin(2n+1)\alpha$$

Bu tenglikning chap tomonini Nyuton binomi formulasiga ko‘ra yoyamiz:

$$(\cos\alpha+i\sin\alpha)^{2n+1}=\cos^{2n+1}\alpha+iC_{2n+1}^1\cos^{2n}\alpha\sin\alpha-C_{2n+1}^2\cos^{2n-1}\alpha\sin^2\alpha-$$
$$-iC_{2n+1}^3\cos^{2n-2}\alpha\sin^3\alpha+...+(-1)^nC_{2n+1}^{2n}\cos\alpha\sin^{2n}\alpha+i(-1)^n\sin^{2n+1}\alpha=$$
$$=\cos(2n+1)\alpha+i\sin(2n+1)\alpha$$

bu yerda, $C_n^k=\frac{n!}{(n-k)!k!}$

Tenglikning mos mavhum qisimlarini tenglashtirib, quyidagiga ega bo‘lamiz:

$$\sin(2n+1)\alpha = C_{2n+1}^{1}\cos^{2n}\alpha\sin\alpha - C_{2n+1}^{3}\cos^{2n-2}\alpha\sin^{3}\alpha + ... + (-1)^{n}\sin^{2n+1}\alpha$$

Agar $\sin\alpha \neq 0$ bo'lsa, u holda

$$\frac{\sin(2n+1)\alpha}{\sin^{2n+1}\alpha} = C_{2n+1}^{1}ctg^{2n}\alpha - C_{2n+1}^{3}ctg^{2n-2}\alpha + ... + (-1)^{n}$$

ya'ni ushbu tenglamani hosil qilamiz:

$$\sin(2n+1)\alpha = 0 \Leftrightarrow C_{2n+1}^{1}ctg^{2n}\alpha - C_{2n+1}^{3}ctg^{2n-2}\alpha + ... + (-1)^{n} = 0$$

bunda $x = ctg^{2}\alpha$ belgilash orqali quyidagi tenglamani hosil qilamiz:

$$C_{2n+1}^{1}x^{n} - C_{2n+1}^{3}x^{n-1} + ... + (-1)^{n} = 0$$

Ma'lumki, bu tenglamaning ildizlari $\frac{\sin(2n+1)\alpha}{\sin^{2n+1}\alpha} = 0$ tenglikdan topiladi.

$$(2n+1)\alpha = \pi k,\ \alpha = \frac{\pi k}{2n+1},\ \left(k = \overline{1,2n}\right)$$

Ravshanki,

$$ctg^{2}\frac{\pi(n+1)}{2n+1} = ctg^{2}\frac{\pi n}{2n+1},\ ctg^{2}\frac{\pi(n+2)}{2n+1} = ctg^{2}\frac{\pi(n-1)}{2n+1}, ..., ctg^{2}\frac{\pi(2n-2)}{2n+1} =$$
$$= ctg^{2}\frac{3\pi}{2n+1},$$
$$ctg^{2}\frac{\pi(2n-1)}{2n+1} = ctg^{2}\frac{2\pi n}{2n+1}, ctg^{2}\frac{2\pi n}{2n+1} ctg^{2}\frac{\pi}{2n+1}$$

Demak, $C_{2n+1}^{1}x^{n} - C_{2n+1}^{3}x^{n-1} + ... + (-1)^{n} = 0$

tenglamaning ildizlari $ctg^{2}\frac{\pi}{2n+1}, ctg^{2}\frac{2\pi}{2n+1}, ..., ctg^{2}\frac{\pi n}{2n+1}$ sonlar bo'ladi.

Bu tenglamani quyidagicha yozib olamiz:

$x^{n} - \frac{2n(n-1)}{3}x^{n-1} + ... + \frac{(-1)^{n}}{2n+1} = 0$. Viet teoremasiga ko'ra,

$$ctg^{2}\frac{\pi}{2n+1} + ctg^{2}\frac{2\pi}{2n+1} + ... + ctg^{2}\frac{\pi n}{2n+1} = \frac{n(2n-1)}{3}$$

$ctg^{2}\frac{\pi k}{2n+1} = \frac{1}{\sin^{2}\frac{\pi k}{2n+1}} - 1\ \left(k = \overline{1,n}\right)$ ayniyatdan foydalanib, quyidagi tenglikga ega bo'lamiz:

$$\frac{1}{\sin^{2}\frac{\pi}{2n+1}} + \frac{1}{\sin^{2}\frac{2\pi}{2n+1}} + ... + \frac{1}{\sin^{2}\frac{\pi n}{2n+1}} = \frac{n(2n+1)}{3}$$

Ma'lumki, ixtiyoriy $x \in \left(0, \frac{\pi}{2}\right)$ sonlar uchun quyidagi tenglik o'rinli.

$$\sin x < x$$

Demak, $\dfrac{2n^2+2n}{3}=\dfrac{1}{\left(\dfrac{\pi}{2n+1}\right)^2}+\dfrac{1}{\left(\dfrac{2\pi}{2n+1}\right)^2}+...+\dfrac{1}{\left(\dfrac{n\pi}{2n+1}\right)^2}$

Bundan quyidagi tengsizlik kelib chiqadi:

$$1+\frac{1}{2^2}+...+\frac{1}{n^2}<\frac{\pi^2}{6}\left(1-\frac{1}{(2n+1)^2}\right)$$

Shuningdek, ixtiyoriy $x\in\left(0,\dfrac{\pi}{2}\right)$ uchun $tg\,x>x$ bo‘ladi. Buni ushbu $ctg^2\dfrac{\pi}{2n+1}+ctg^2\dfrac{2\pi}{2n+1}+...+ctg^2\dfrac{\pi n}{2n+1}=\dfrac{n(2n-1)}{3}$ tenglikka qo‘llab, quyidagilarga ega bo‘lamiz:

$$\frac{n(2n-1)}{3}<\frac{(2n+1)^2}{\pi^2}\left(1+\frac{1}{2^2}+...+\frac{1}{n^2}\right),$$

$$1+\frac{1}{2^2}+...+\frac{1}{n^2}>\frac{\pi^2}{6}\left(1-\frac{6n+1}{(2n+1)^2}\right)$$

Demak, ixtiyoriy $n\in\mathbb{N}$ uchun

$$\frac{\pi^2}{6}\left(1-\frac{6n+1}{(2n+1)^2}\right)<1+\frac{1}{2^2}+...+\frac{1}{n^2}<\frac{\pi^2}{6}\left(1-\frac{1}{(2n+1)^2}\right)$$

Natija: $\sum\limits_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}=\dfrac{\pi^2}{8}$, $\sum\limits_{k=1}^{n-1}\sin\dfrac{\pi k}{n}=\dfrac{n}{2^{n-1}}$, $n\in\mathbb{N}$ tengliklar o‘rinli bo‘ladi.

108. 107-misoldagi tengsizlikda $n\to\infty$ da limitga o‘tsak, $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^2}=\dfrac{\pi^2}{6}$ ekani kelib chiqadi.

109. Shartga ko‘ra, $\vec{a}+\vec{b}=\alpha\vec{c}$ va $\vec{b}+\vec{c}=\beta\vec{a}$ bu tengliklarni ayirsak, $\vec{a}-\vec{c}=\alpha\vec{c}-\beta\vec{a}\ \Rightarrow(1+\beta)\vec{a}=(1+\alpha)\vec{c}$ tenglikka ega bo‘lamiz. $\vec{a}$ va $\vec{c}$ lar collinear bo‘lmagani uchun bu tenglik faqat $\alpha=\beta=-1$ bo‘lganda bajariladi. Bundan $\vec{a}+\vec{b}=-\vec{c}\ \Rightarrow\vec{a}+\vec{b}+\vec{c}=0$ $\left|\vec{a}+\vec{b}+\vec{c}\right|=0$ ekani kelib chiqadi.

110. Tenglikning o‘ng tomonini $f(x)=xe^{-x}+e^{-x}+\dfrac{x^2}{2}-1=0$ deb belgilaymiz. Uning hosilasi barcha haqiqiy x lar uchun $f'(x)=x\left(1-e^{-x}\right)\geq0$ bo‘lishini ko‘rish oson. U holda $f(x)=0$ tenglama yagona yechimga ega. Bu yechim esa $x=0$.

111. $x_0 = 1$ nuqtada o'tkazilgan urinma tenglamasi $y = f(1) + f'(1)(x-1)$ ko'rinishida bo'ladi. Bizdan $f(1)$ va $f'(1)$ larni topish talab etiladi. $x = 1$ da $f(1) - 4f(-1) = -9$ bo'ladi. $y = f(x)$ juft funksiya bo'lgani uchun $f(-1) = f(1) \Rightarrow f(1) = 3$ ekanini topamiz. $f'(2x^3 - x)(6x^2 - 1) - 4x^2(2x-1)f'(x^2 - x - 1) = 40x^4 - 24x^2 - 22x$. $x = 1$ da $5f'(1) - 8f(-1) - 4f'(-1) = -6$. $f(-1) = 3$ ekanidan $5f'(1) - 4f'(-1) = 18$. $f(x) = f(-x) \Rightarrow f'(x) = -f'(-x)$ u holda $f'(-1) = -f'(1) \Rightarrow f'(1) = 2$. Biz izlayotgan urinma tenglamasi $y = 2x + 1$ ko'rinishda ekan.

112. Bu limit ∞^0 ko'rinishidagi aniqmaslikni ifodalayapti. Biz uni A orqali belgilaylik. U holda $\ln A = \lim\limits_{x\to\infty} \dfrac{\ln tg\dfrac{\pi x}{2x+1}}{x}$ bu esa $\dfrac{\infty}{\infty}$ ko'rinishiga keladi. Lopital qoidasini qo'llasak,

$$\ln A = \lim_{x\to\infty} \frac{\dfrac{\pi}{(2x+1)^2}}{tg\dfrac{\pi x}{2x+1}\cos^2\dfrac{\pi x}{2x+1}} = \lim_{x\to\infty} \frac{\dfrac{\pi}{(2x+1)^2}}{\sin\dfrac{\pi x}{2x+1}\cos\dfrac{\pi x}{2x+1}} = \lim_{x\to\infty} \frac{\dfrac{2\pi}{(2x+1)^2}}{\sin\dfrac{2\pi x}{2x+1}} \text{ bu yerda}$$

$$\sin\frac{2\pi x}{2x+1} = \sin\frac{2\pi x + \pi - \pi}{2x+1} = \sin\frac{\pi(2x+1) - \pi}{2x+1} = \sin\left(\pi - \frac{\pi}{2x+1}\right) = \sin\frac{\pi}{2x+1} \approx \frac{\pi}{2x+1}\ (x \to \infty)$$

ekanini hisobga olsak, $\ln A = \lim\limits_{x\to\infty} \dfrac{\dfrac{2\pi}{(2x+1)^2}}{\dfrac{\pi}{2x+1}} = \lim\limits_{x\to\infty} \dfrac{\pi}{2x+1} = 0 \Rightarrow A = e^0 = 1$.

113. $f(x) = \cos^2 x \cdot \sin x$ funksiyaning $x \in [-\pi, \pi]$ sohada eng kichik qiymatini

topamiz. $f(x) = (1 - \sin^2 x) \cdot \sin x = \sin x - \sin^3 x \Rightarrow f'(x) = \cos x - 3\sin^2 x \cos x = 0 \Rightarrow$

$$\cos x = 0, \sin x = \pm\frac{1}{\sqrt{3}} \Rightarrow x = \pm\frac{\pi}{2};\ x = \pm\arcsin\frac{1}{\sqrt{3}};\ x = \pi - \arcsin\frac{1}{\sqrt{3}};\ x = \arcsin\frac{1}{\sqrt{3}} - \pi.$$

Yuqoridagilarni hisobga olsak, $\min\limits_{x\in[-\pi,\pi]} f(x) = -\dfrac{2}{3\sqrt{3}} > -\dfrac{2}{3}$ tengsizlikka ega bo'lamiz.

114.

$$\lim_{x\to+\infty}[(x+2)\ln(x+2)-2(x+1)\ln(x+1)+x\ln x]=\lim_{x\to+\infty}[(x+1)\ln(x+2)-(x+1)\ln(x+1)-$$
$$-(x\ln(x+1)-x\ln x)+\ln(x+2)-\ln(x+1)]=\lim_{x\to+\infty}\left(\ln\left(\frac{x+2}{x+!}\right)^{x+1}-\ln\left(\frac{x+1}{x}\right)^{x}+\ln\frac{x+2}{x+1}\right)=$$
$$=\ln\lim_{x\to+\infty}\left(1+\frac{1}{x+1}\right)^{x+1}-\ln\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x}+\ln\lim_{x\to+\infty}\left(1+\frac{1}{x+1}\right)=\ln e-\ln e+\ln 1=0.$$

115. $\overrightarrow{AB}=\vec{a},\overrightarrow{BC}=\vec{b},\overrightarrow{CA}=\vec{c}$ deb belgilab olamiz. Ma'lumki, $\vec{a}+\vec{b}+\vec{c}=0$ tenglik o‘rinli. Buning ikkala tomonini kvadratga oshirib,

$$\vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a},\vec{b})+2(\vec{b},\vec{c})+2(\vec{c},\vec{a})=0 \quad\Rightarrow (\vec{a},\vec{b})+(\vec{b},\vec{c})+(\vec{c},\vec{a})=$$
$$=-\frac{1}{2}\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2\right)<0$$

ga ega bo‘lamiz. Shuni isbotlash talab qilingan edi.

116. Limitni ikki hol uchun hisoblaymiz.

1-hol. $a>1$ bo‘lsin. U holda bu limitni A orqali va n ni x orqali belgilab olamiz: $\ln A=\lim\limits_{x\to\infty}\frac{1+a^x}{x}=\lim\limits_{x\to\infty}\frac{a^x\ln a}{1}=+\infty \;\Rightarrow A=+\infty$.

2-hol. $0<a\le 1$ bo‘lsin. Bunda ham yuqoridagiga o‘xshash $\ln A=\lim\limits_{x\to\infty}\frac{1+a^x}{x}=0 \;\Rightarrow A=1$. Javob: $\lim\limits_{n\to\infty}(1+a^n)^{\frac{1}{n}}=\begin{cases}+\infty, & a>1\\ 1, & 0<a\le 1\end{cases}$.

117.

$$\lim_{n\to\infty}\frac{2\sin\frac{\pi}{2^{n+1}}\cos\frac{\pi}{2^{n+1}}\ldots\cos\frac{\pi}{4}\cos\frac{\pi}{8}}{2\sin\frac{\pi}{2^{n+1}}}=\lim_{n\to\infty}\frac{\sin\frac{\pi}{2^n}\cos\frac{\pi}{2^n}\ldots\cos\frac{\pi}{4}\cos\frac{\pi}{8}}{2\sin\frac{\pi}{2^{n+1}}}=\ldots=\lim_{n\to\infty}\frac{\sin\frac{\pi}{2}}{2^n\cdot\sin\frac{\pi}{2^{n+1}}}$$

Bu yerda $\sin\frac{\pi}{2^{n+1}}\approx\frac{\pi}{2^{n+1}},\ (n\to\infty)$ ekanini hisobga olsak, bu limit

$$\lim_{n\to\infty}\frac{\sin\frac{\pi}{2}}{2^n\cdot\sin\frac{\pi}{2^{n+1}}}=\lim_{n\to\infty}\frac{\sin\frac{\pi}{2}}{2^n\cdot\frac{\pi}{2^{n+1}}}=\frac{2}{\pi}$$ ga tengligini topamiz. Javob: $\frac{2}{\pi}$.

118. $\det A\ne 0$ deb faraz qilaylik. U holda $AB=O$ ni chap tomondan A^{-1} ga ko‘paytiramiz. $A^{-1}AB=A^{-1}O \;\Rightarrow EB=O\Rightarrow B=O$ bu esa masala shartiga zid. Demak, $\det A=0$. Xuddi shunga o‘xshash $\det B=0$ ekanini topamiz.

119. Sistemadagi hamma tenglamalarni qo‘shsak,

$\left(1+2+...+10\right)\left(x_1+x_2+...+x_{10}\right)=10\cdot 55 \Rightarrow x_1+x_2+...+x_{10}=10$ ga ega bo‘lamiz. Birinchi tenglamadan ikkinchi tenglamani ayirsak, $9x_1-x_2-x_3-...-x_{10}=0 \Rightarrow 10x_1=10 \Rightarrow x_1=10$. Xuddi shunga o‘xshash $x_1=x_2=...=x_{10}=1$ ekanini topamiz.

120. Quyidagi hollarni qarab chiqamiz:

1-hol. $n \in \mathbb{N}$ bo‘lsin. $\lim\limits_{x\to\infty}\dfrac{\left(ax+1\right)^n}{x^n+b}=\left(\dfrac{\infty}{\infty}\right)=\lim\limits_{x\to\infty}\dfrac{x^n\left(a+\dfrac{1}{x^n}\right)^n}{x^n\left(1+\dfrac{b}{x^n}\right)}=a^n$.

2-hol. $n=0$ bo‘lsin. $\lim\limits_{x\to\infty}\dfrac{\left(ax+1\right)^n}{x^n+b}=\dfrac{1}{1+b}$.

3-hol. $n=-k,\ k\in\mathbb{N}$ bo‘lsin.

$$\lim_{x\to\infty}\frac{\left(ax+1\right)^n}{x^n+b}=\lim_{x\to\infty}\frac{\left(ax+1\right)^{-k}}{x^{-k}+b}=\lim_{n\to\infty}\frac{x^{-k}}{\left(1+bx^k\right)\left(ax+1\right)^k}=\begin{cases}0, & ab=0\\ \dfrac{1}{a^k}=a^n, & a\neq 0, b=0\\ \dfrac{1}{b}, & a=0, b\neq 0\\ \infty, & a=b=0\end{cases}.$$

121.

$$\lim_{n\to\infty}\frac{n^x-\left(n-1\right)^x}{\left(n+1\right)^{x-1}+\left(n+2\right)^{x-1}}=\lim_{n\to\infty}\frac{\left(n-1\right)^x\left(\left(\dfrac{n}{n-1}\right)^x-1\right)}{\left(n+1\right)^{x-1}+\left(n+2\right)^{x-1}}=$$

$$=\lim_{n\to\infty}\frac{\left(n-1\right)^x\left(\left(1+\dfrac{1}{n-1}\right)^x-1\right)}{\left(n+1\right)^{x-1}+\left(n+2\right)^{x-1}}=\left|\left(1+\frac{1}{n-1}\right)^x-1\approx\frac{x}{n-1},\ n\to\infty\right|=$$

$$=\lim_{n\to\infty}\frac{x\left(n-1\right)^{x-1}}{\left(n+1\right)^{x-1}+\left(n+2\right)^{x-1}}=\lim_{n\to\infty}\frac{x\left(1-\dfrac{1}{n}\right)^{x-1}}{\left(1+\dfrac{1}{n}\right)^{x-1}+\left(1+\dfrac{2}{n}\right)^{x-1}}=\frac{x}{2}=2013$$

U holda $x=4026$.

122. Agar $x=-1$ bo‘lsa, tenglikning chap tomoni 0 ga teng. Agar $x\in\left(-\infty,-1\right)\cup\left[1,+\infty\right)$ bo‘lsa, tenglikning chap tomoni $+\infty$ ga teng. U holda $\left|x\right|<1$ ekan, ya’ni

$$\lim_{n\to\infty}\frac{(1-x)(1+x)(1+x^2)(1+x^4)...(1+x^{2n})}{1-x}=\lim_{n\to\infty}\frac{(1-x^{4n})}{1-x}=\frac{1}{1-x}=2013\Rightarrow x=\frac{2012}{2013}$$ J

avob: $x=\frac{2012}{2013}$.

123. $$\lim_{x\to\infty}\frac{(x+a+b)^{2x+a+b}}{(x+a)^{x+a}(x+b)^{x+b}}=\lim_{x\to\infty}\frac{(x+a+b)^{x+a+b}}{(x+a)^{x+a}}\cdot\lim_{x\to\infty}\frac{(x+a+b)^{x+a+b}}{(x+b)^{x+b}}=$$

$$=\lim_{x\to\infty}\left(1+\frac{b}{x+a}\right)^{x+a}\cdot\lim_{x\to\infty}\left(1+\frac{a}{x+b}\right)^{x+b}=e^b\cdot e^a=e^{a+b}.$$

124. Ma'lumki, funksiya $[-1,1]$ segmentda aniqlangan. Funksiyaning kritik nuqtalarini topamiz.

$$f'(x)=\frac{3(\arcsin^2 x-\arccos^2 x)}{\sqrt{1-x^2}}=0\Rightarrow \arcsin x=\pm\arccos x\Rightarrow x=\frac{\sqrt{2}}{2}.$$

U holda

$$f(-1)=\frac{7\pi^3}{8},\ f(1)=\frac{\pi^3}{8},\quad f\left(\frac{\sqrt{2}}{2}\right)=\frac{\pi^3}{32}\quad\Rightarrow\min_{x\in[-1,1]}f(x)=\frac{\pi^3}{32},\ \max_{x\in[-1,1]}f(x)=\frac{7\pi^3}{8}$$

125. Teoremani ikki hol uchun isbotlaymiz.

1-hol. l-chekli bo'lsin.

3-shartga ko'ra, $\forall\varepsilon>0$ soni uchun $\frac{\varepsilon}{2}$ ga ko'ra, ketma-ketlikning shunday N nomerli hadi topiladi barcha $n\geq N$ tengsizlikni qanoatlantiruvchi hadlar uchun $\left|\frac{x_{n+1}-x_n}{y_{n+1}-y_n}-l\right|<\frac{\varepsilon}{2}$ tengsizlik bajariladi. Buni boshqacha ko'rinishda quyidagicha yozish mumkin:

$$l-\frac{\varepsilon}{2}<\frac{x_{n+1}-x_n}{y_{n+1}-y_n}<l+\frac{\varepsilon}{2}\quad\Leftrightarrow\left(l-\frac{\varepsilon}{2}\right)(y_{n+1}-y_n)<(x_{n+1}-x_n)<\left(l+\frac{\varepsilon}{2}\right)(y_{n+1}-y_n)$$

Oxirgi tengsizlikni har bir $n\geq N$ lar uchun yozib chiqamiz.

$$\begin{cases}\left(l-\frac{\varepsilon}{2}\right)(y_{N+1}-y_N)<(x_{N+1}-x_N)<\left(l+\frac{\varepsilon}{2}\right)(y_{N+1}-y_N)\\ \left(l-\frac{\varepsilon}{2}\right)(y_{N+2}-y_{N+1})<(x_{N+2}-x_{N+1})<\left(l+\frac{\varepsilon}{2}\right)(y_{N+2}-y_{N+1})\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ \left(l-\frac{\varepsilon}{2}\right)(y_n-y_{n-1})<(x_n-x_{n-1})<\left(l+\frac{\varepsilon}{2}\right)(y_n-y_{n-1})\end{cases}$$

Bularni qoshish natijasida $\left|\frac{x_n-x_N}{y_n-y_N}-l\right|<\frac{\varepsilon}{2}$ (1) ga ega bo'lamiz.

Endi quyidagi yordamchi ifodalarni qaraymiz:

$$\frac{y_n - y_N}{y_n} = 1 - \frac{y_N}{y_n} < 1 \qquad (2)$$

chunki N-tayinlangan son va $y_n \to +\infty, \left(n \to +\infty\right)$. $\forall \varepsilon > 0$ soni uchun $\frac{\varepsilon}{2}$ ga ko'ra, shunday N_0 nomer topiladiki, barcha $n \geq N_0$ tengsizlikni qanoatlantiruvchi hadlar uchun

$$\left|\frac{x_N - ly_N}{y_n} - l\right| < \frac{\varepsilon}{2} \qquad (3)$$

tengsizlik bajariladi, chunki, x_N, l, y_N chekli sonlar va $y_n \to +\infty, \left(n \to +\infty\right)$.

U holda $\forall \varepsilon > 0$ soni uchun shunday $\overline{N} = \max\{N, N_0\}$ nomer topiladiki, barcha $n \geq \overline{N}$ lar uchun

$$\left|\frac{x_n}{y_n} - l\right| = \left|\frac{x_N - ly_N}{y_n} + \frac{y_n - y_N}{y_n}\left(\frac{x_n - x_N}{y_n - y_N} - l\right)\right| \leq \left|\frac{x_N - ly_N}{y_n}\right| + \left|\frac{y_n - y_N}{y_n}\right|\left|\frac{x_n - x_N}{y_n - y_N} - l\right| < \frac{\varepsilon}{2} + 1 \cdot \frac{\varepsilon}{2} < \varepsilon.$$

Bu esa $\lim\limits_{n\to\infty} \frac{x_n}{y_n} = l$ ekanini bildiradi.

2-hol. l-cheksiz bo'lsin.

U holda $\lim\limits_{n\to\infty} \frac{x_{n+1} - x_n}{y_{n+1} - y_n} = \infty \quad \Rightarrow \lim\limits_{n\to\infty} \frac{y_{n+1} - y_n}{x_{n+1} - x_n} = 0$. Oxirgi tenglikda quyidagi shartlar o'rinli:

1) $x_{n+1} > x_n \left(n \in N\right)$ chunki, limiti $+\infty$ ga teng;

2) $\lim\limits_{n\to\infty} x_n = +\infty, \qquad \lim\limits_{n\to\infty} y_n = +\infty,$;

3) $\lim\limits_{n\to\infty} \frac{y_{n+1} - y_n}{x_{n+1} - x_n} = 0$.

Bu teoremaning l-chekli holiga tushadi. Bundan $\lim\limits_{n\to\infty} \frac{y_n}{x_n} = 0 \;\Rightarrow \lim\limits_{n\to\infty} \frac{x_n}{y_n} = \infty$ tenglikning o'rinli ekanligi kelib chiqadi.

126. Tekislikda bitta nuqtadan chiquvchi $\overline{r_1}$, $\overline{r_2}$, $\overline{r_3}$ birlik vektorlarni olamiz. Ular oralaridagi burchaklar 2α , 2β va 2γ bo'lsin. U holda ushbu $(\overline{r_1} + \overline{r_2} + \overline{r_3})^2 \geq 0$ tengsizlikka ko'ra,

$$3 + 2\cos 2\alpha + 2\cos 2\beta + 2\cos 2\gamma \geq 0,$$

$$3 + 2[3 - 2\sin^2\alpha - 2\sin^2\beta - 2\sin^2\gamma] \geq 0.$$

Oxirgi tengsizlikdan $\sin^2\alpha + \sin^2\beta + \sin^2\gamma \leq \frac{9}{4}$ kelib chiqadi.

127. Agar $\sin^2\alpha+\sin^2\beta+\sin^2\gamma\le\frac{9}{4}$ tengsizlikka $\sin\alpha=\frac{a}{2R}$, $\sin\beta=\frac{b}{2R}$, $\sin\gamma=\frac{c}{2R}$ ifodalarni qo'ysak, u holda $a^2+b^2+c^2\le 9R^2$ tengsizlik hosil bo'ladi.

128. Ushbu $\overline{OH}=\overline{OC}+\overline{CH}$ tenglik o'rinli. Demak, $\overline{CH}=\overline{OA}+\overline{OB}$ tenglikni isbotlash kifoya. $\overline{CH}$ vektor ham, $\overline{OA}+\overline{OB}=2\overline{OK}$ vektor ham $\overline{AB}$ vektorga perpendikular bo'lgani uchun ular o'zaro parallel bo'ladi. Demak, bu vektorlar kollinear ekan $\overline{CH}=\lambda\cdot(\overline{OA}+\overline{OB})$. Bu vektorlar bir tomonga yo'nalganligi uchun $\lambda\ge 0$ bo'ladi. λ ning qiymatini topish uchun $\overline{CH}$ va $\overline{OA}+\overline{OB}$ vektorlar uzunliklarini topamiz. Quyidagi ifodalarni topish qiyin emas $\left|\overline{OA}+\overline{OB}\right|^2=4R^2\cos^2\gamma$, $\left|\overline{CH}\right|^2=4R^2\cos^2\gamma$. Bu yerda R orqali ABC uchburchakka tashqi chizilgan aylana radiusi va γ orqali AB tomon qarshisidagi burchak belgilangan. Demak, $\lambda=1$ ekan, ya'ni ushbu $\overline{CH}=\overline{OA}+\overline{OB}$ tenglik o'rinli ekan.

129. Determinantni W bilan belgilaymiz. U holda $W^2<1$ ekanini isbotlash yetarli.

$$W^2=(y-x)^2(z-x)^2(z-y)^2\le\left(\frac{(y-x)^2+(z-x)^2+(z-y)^2}{3}\right)^3\le\left(\frac{3(x^2+y^2+z^2)}{3}\right)^3=1.$$

Bundan isbotlanishi kerak bo'lgan tengsizlik kelib chiqadi.

130. $k=[n\sqrt{2}]<n\sqrt{2}$ orqali $n\sqrt{2}$ sonning butun qismini belgilaylik. U holda $1\le 2n^2-k^2=\left(\sqrt{2}n-k\right)\left(\sqrt{2}n+k\right)<\left\{n\sqrt{2}\right\}\cdot 2n\sqrt{2}\Rightarrow\{n\sqrt{2}\}>\frac{1}{2n\sqrt{2}}$ tengsizlik o'rinli bo'ladi. Shuni isbotlash talab qilingan edi.

131. $\begin{cases}2^n=(1+1)^n=C_n^0+C_n^1+C_n^2+C_n^3+C_n^4+...\\0^n=(1-1)^n=C_n^0-C_n^1+C_n^2-C_n^3+C_n^5-...\end{cases}$ bu tengliklarni qo'shsak, $2\left(C_n^0+C_n^2+C_n^4+...\right)=2^n\Rightarrow C_n^0+C_n^2+C_n^4+...=2^{n-1}$ ga ega bo'lamiz.

Yuqoridagi birinchi tenglikdan ikkinchisini ayirsak, $2\left(C_n^1+C_n^3+C_n^5+...\right)=2^n\Rightarrow C_n^1+C_n^3+C_n^5+...=2^{n-1}$ ga ega bo'lamiz.

132. $(1+i)^n = C_n^0 + C_n^1 i + C_n^2 i^2 + C_n^3 i^3 + C_n^4 i^4 + ... C_n^{n-1} i^{n-1} + C_n^n i^n$, lekin ikkinchi tomondan, $(1+i)^n = \left(\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^n = 2^{\frac{n}{2}}\left(\cos\frac{\pi n}{4} + i\sin\frac{\pi n}{4}\right)$ bu ikkita ifodani tenglashtirsak,

$$2^{\frac{n}{2}}\left(\cos\frac{\pi n}{4} + i\sin\frac{\pi n}{4}\right) = \left(C_n^0 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - C_n^{10} + ...\right) + i\left(C_n^1 - C_n^3 + C_n^5 - C_n^7 + ...\right)$$

ayniyatga ega bo‘lamiz, bu tenglikning haqiqiy va mavhum qismlarini tenglashtirib, quyidagilarni hosil qilamiz:

$$C_n^0 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - C_n^{10} + ... = 2^{\frac{n}{2}}\cos\frac{\pi n}{4} \text{ va}$$

$$C_n^1 - C_n^3 + C_n^5 - C_n^7 + C_n^9 ... = 2^{\frac{n}{2}}\sin\frac{\pi n}{4}$$

133. a) $z = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ deb belgilab olaylik. U holda quyidagilarga egamiz:

$$1 + z = 1 + \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = 1 - \frac{1}{2} + \frac{\sqrt{3}}{2}i = \frac{1}{2} + \frac{\sqrt{3}}{2}i = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$$

$$1 + z^2 = 1 + \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^2 = 1 + \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = 1 - \frac{1}{2} - \frac{\sqrt{3}}{2}i =$$
$$= \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$$

$$z^{3n} = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^{3n} = \cos\frac{6\pi n}{3} + i\sin\frac{6\pi n}{3} = 1$$

$$1 + z^{2n} + z^{4n} = \cos\frac{4\pi n}{3} + i\sin\frac{4\pi n}{3} + \cos\frac{8\pi n}{3} + i\sin\frac{8\pi n}{3} = 0, \quad (n \in \mathbb{N})$$

Bularni bilgan holda quyidagi ifodalarni qaraymiz:

a) $$(1+z)^n = C_n^0 + C_n^1 z + C_n^2 z^2 + C_n^3 z^3 + C_n^4 z^4 + C_n^5 z^5 + C_n^6 z^6 + ...$$
$$(1+z^2)^n = C_n^0 + C_n^1 z^2 + C_n^2 z^4 + C_n^3 z^6 + C_n^4 z^8 + C_n^5 z^{10} + C_n^6 z^{12} + ...$$
bundan tashqari, 131-misolga ko‘ra, $2^n = C_n^0 + C_n^1 + C_n^2 + C_n^3 + C_n^4 + ...$ ekanini hisobga olib, bu uchala tengliklarni qo‘shsak,

$$2^n + (1+z)^n + (1+z^2)^n = 2^n + \cos\frac{\pi n}{3} + i\sin\frac{\pi n}{3} + \cos\frac{5\pi n}{3} + i\sin\frac{5\pi n}{3} = 3C_n^0 +$$
$$+ C_n^1\left(1 + z + z^2\right) + C_n^2\left(1 + z^2 + z^4\right) + C_n^3\left(1 + z^3 + z^6\right) + C_n^4\left(1 + z^4 + z^8\right) +$$
$$+ C_n^5\left(1 + z^5 + z^{10}\right) + C_n^6\left(1 + z^6 + z^{12}\right) + ... \Rightarrow$$
$$\Rightarrow 2^n + \cos\frac{\pi n}{3} + \cos\left(2\pi n - \frac{\pi n}{3}\right) + 2i\sin\pi n \cdot \cos\frac{2\pi n}{3} = 3C_n^0 + 3C_n^3 + 3C_n^6 + ... \Rightarrow$$
$$\Rightarrow C_n^0 + C_n^3 + C_n^6 + ... = \frac{1}{3}\left(2^n + \cos\frac{\pi n}{3}\right)$$

tenglikka ega bo‘lamiz;

b) $$\left(1+z\right)^n = C_n^0 + C_n^1 z + C_n^2 z^2 + C_n^3 z^3 + C_n^4 z^4 + C_n^5 z^5 + C_n^6 z^6 + \dots$$
$$\left(1+z^2\right)^n = C_n^0 + C_n^1 z^2 + C_n^2 z^4 + C_n^3 z^6 + C_n^4 z^8 + C_n^5 z^{10} + C_n^6 z^{12} + \dots$$

tengliklarning birinchisini z ga va ikkinchisini z^2 ga ko‘paytiramiz. Natijada

$$z\left(1+z\right)^n = C_n^0 z + C_n^1 z^2 + C_n^2 z^3 + C_n^3 z^4 + C_n^4 z^5 + C_n^5 z^6 + C_n^6 z^7 + \dots$$
$$z^2\left(1+z^2\right)^n = C_n^0 z^2 + C_n^1 z^4 + C_n^2 z^6 + C_n^3 z^8 + C_n^4 z^{10} + C_n^5 z^{12} + C_n^6 z^{14} + \dots$$

ifodalarga ega bo‘lamiz. Yana $2^n = C_n^0 + C_n^1 + C_n^2 + C_n^3 + C_n^4 + \dots$ ekanini hisobga olib, bu uchala tengliklarni qo‘shsak,

$$2^n + z\left(1+z\right)^n + z^2\left(1+z^2\right)^n = 2^n + \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^2 \cdot \left(\cos\frac{n}{3} + i\sin\frac{\pi}{3}\right)^n +$$
$$+\left(\cos\frac{n}{3} + i\sin\frac{\pi}{3}\right)^4 \cdot \left(\cos\frac{n}{3} + i\sin\frac{\pi}{3}\right)^{5n} = 2^n + \cos\frac{\left(n+2\right)\pi}{3} + i\sin\frac{\left(n+2\right)\pi}{3} +$$
$$+\cos\frac{\left(5n+4\right)\pi}{3} + i\sin\frac{\left(5n+4\right)\pi}{3} = C_n^0\left(1+z+z^2\right) + C_n^1\left(1+z^2+z^4\right) +$$
$$+C_n^2\left(1+z^3+z^6\right) + C_n^3\left(1+z^4+z^8\right) + C_n^4\left(1+z^5+z^{10}\right) + C_n^5\left(1+z^6+z^{12}\right) +$$
$$+C_n^6\left(1+z^7+z^{14}\right) + \dots \Rightarrow 2^n + \cos\left(2\pi + \frac{\pi\left(n-4\right)}{3}\right) + \cos\left(2\pi n - \frac{\pi\left(n-4\right)}{3}\right) +$$
$$+2i\sin 2\left(n+1\right)\pi \cdot \cos\frac{\left(2n+1\right)\pi}{3} =$$
$$= 3C_n^2 + 3C_n^5 + 3C_n^8 + \dots \Rightarrow C_n^2 + C_n^5 + C_n^8 + \dots = \frac{1}{3}\left(2^n + \cos\frac{\pi\left(n-4\right)}{3}\right)$$

c) yuqoridagilar asosida ushbu
$$\begin{cases} 2^n = C_n^0 + C_n^1 + C_n^2 + C_n^3 + C_n^4 + \dots \\ C_n^0 + C_n^3 + C_n^6 + \dots = \frac{1}{3}\left(2^n + \cos\frac{\pi n}{3}\right) \\ C_n^2 + C_n^5 + C_n^8 + \dots = \frac{1}{3}\left(2^n + \cos\frac{\pi\left(n-4\right)}{3}\right) \end{cases}$$

tengliklarga egamiz. Ikkinchi tenglikka uchinchi tenglikni qo‘shib, birinchi tenglikdan ayirsak,

$$2^n - \frac{1}{3}\left(2^n + \cos\frac{\pi n}{3} + 2^n + \cos\frac{\pi\left(n-4\right)}{3}\right) = \left(C_n^0 + C_n^1 + C_n^2 + C_n^3 + C_n^4 + \dots\right) -$$
$$-\left(\left(C_n^0 + C_n^3 + C_n^6 + \dots\right) + \left(C_n^2 + C_n^5 + C_n^8 + \dots\right)\right) \Rightarrow C_n^1 + C_n^4 + C_n^7 + \dots =$$
$$= \frac{1}{3}\left(2^n - 2\cos\frac{\pi\left(n-2\right)}{3}\cos\frac{2\pi}{3}\right) \Rightarrow C_n^1 + C_n^4 + C_n^7 + \dots = \frac{1}{3}\left(2^n + \cos\frac{\pi\left(n-2\right)}{3}\right)$$

ekanligini topishimiz mumkin.

134. a) 131- va 132-misollarga ko‘ra,

$$\begin{cases} C_n^0+C_n^2+C_n^4+C_n^6+C_n^8+C_n^{10}+...=2^{n-1} \\ C_n^0-C_n^2+C_n^4-C_n^6+C_n^8-C_n^{10}+...=2^{\frac{n}{2}}\cos\frac{\pi n}{4} \end{cases}$$ tengliklar o'rinli.

Bu tengliklarni qo'shsak,

$$2\left(C_n^0+C_n^4+C_n^8+...\right)=2^{n-1}+2^{\frac{n}{2}}\cos\frac{\pi n}{4} \Rightarrow C_n^0+C_n^4+C_n^8+...=$$
$$=\frac{1}{2}\left(2^{n-1}+2^{\frac{n}{2}}\cos\frac{\pi n}{4}\right)$$

ga ega bo'lamiz;

b) 131- va 132-misollarga ko'ra: $\begin{cases} C_n^1+C_n^3+C_n^5+C_n^7+...=2^n \\ C_n^1-C_n^3+C_n^5-C_n^7+...=2^{n-1}\sin\frac{\pi n}{4} \end{cases}$

tengliklar o'rinli. Bu tengliklarni qo'shsak,

$$2\left(C_n^1+C_n^5+C_n^9+...\right)=2^n+2^{n-1}\sin\frac{\pi n}{4} \Rightarrow C_n^1+C_n^5+C_n^9+...=\frac{1}{2}\left(2^n+2^{n-1}\sin\frac{\pi n}{4}\right)$$

ekanligi kelib chiqadi;

c) 131- va 132-misollarga ko'ra,

$$\begin{cases} C_n^0+C_n^2+C_n^4+C_n^6+C_n^8+C_n^{10}+...=2^{n-1} \\ C_n^0-C_n^2+C_n^4-C_n^6+C_n^8-C_n^{10}+...=2^{\frac{n}{2}}\cos\frac{\pi n}{4} \end{cases}$$ ifodadagi birinchi

tenglikdan ikkinchisini ayirsak,

$$2\left(C_n^2+C_n^6+C_n^{10}+...\right)=2^{n-1}-2^{\frac{n}{2}}\cos\frac{\pi n}{4} \Rightarrow C_n^0+C_n^4+C_n^8+...=$$
$$=\frac{1}{2}\left(2^{n-1}-2^{\frac{n}{2}}\cos\frac{\pi n}{4}\right)$$

ga ega bo'lamiz;

d) 131- va 132-misollarga ko'ra, $\begin{cases} C_n^1+C_n^3+C_n^5+C_n^7+...=2^n \\ C_n^1-C_n^3+C_n^5-C_n^7+...=2^{n-1}\sin\frac{\pi n}{4} \end{cases}$

ifodadagi birinchi tenglikdan ikkinchisini ayirsak,

$$2\left(C_n^3+C_n^7+C_n^{11}+...\right)=2^n-2^{n-1}\sin\frac{\pi n}{4} \Rightarrow C_n^3+C_n^7+C_n^{11}+...=$$
$$=\frac{1}{2}\left(2^n-2^{n-1}\sin\frac{\pi n}{4}\right)$$ ekanligi kelib

chiqadi.

135. Quyidagi ifodani qaraymiz:

$$\left(1+\frac{1}{\sqrt{3}}i\right)^{n}=\left(\frac{2}{\sqrt{3}}\right)^{n}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^{n}=\left(\frac{2}{\sqrt{3}}\right)^{n}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{n}=$$

$$=\left(\frac{2}{\sqrt{3}}\right)^{n}\left(\cos\frac{\pi n}{6}+i\sin\frac{\pi n}{6}\right)\Rightarrow$$

$$\Rightarrow C_n^0+C_n^1\left(\frac{1}{\sqrt{3}}i\right)+C_n^2\left(\frac{1}{\sqrt{3}}i\right)^2+C_n^3\left(\frac{1}{\sqrt{3}}i\right)^3+C_n^4\left(\frac{1}{\sqrt{3}}i\right)^4+...=$$

$$=\left(\frac{2}{\sqrt{3}}\right)^{n}\left(\cos\frac{\pi n}{6}+i\sin\frac{\pi n}{6}\right)\Rightarrow$$

$$\Rightarrow\frac{1}{\sqrt{3}}\left(C_n^0+C_n^1 i-\frac{1}{\sqrt{3}}C_n^2-\frac{1}{3}C_n^3 i+\frac{1}{3\sqrt{3}}C_n^4+\frac{1}{9}C_n^5 i-...\right)=$$

$$=\left(\frac{2}{\sqrt{3}}\right)^{n}\left(\cos\frac{\pi n}{6}+i\sin\frac{\pi n}{6}\right)$$

Oxirgi tenglikning haqiqiy va mavhum qismlarini tenglashtirsak,

$$C_n^1-\frac{1}{3}C_n^3+\frac{1}{9}C_n^5+...=\frac{2^n}{\left(\sqrt{3}\right)^{n-1}}\sin\frac{\pi n}{6}\quad \text{va}\quad C_n^0-\frac{1}{3}C_n^2+\frac{1}{9}C_n^4-\frac{1}{27}C_n^6...=\left(\frac{2}{\sqrt{3}}\right)^{n}\cos\frac{\pi n}{6}$$

tengliklarga ega bo‘lamiz.

136.

$$\lim_{n\to\infty}\frac{1+2\sqrt{2}+3\sqrt{3}+...+n\sqrt{n}}{n^2\sqrt{n}}=\lim_{n\to\infty}\frac{x_{n+1}-x_n}{y_{n+1}-y_n}=\lim_{n\to\infty}\frac{\left(n+1\right)\sqrt{n+1}}{\left(n+1\right)^2\sqrt{n+1}-n^2\sqrt{n}}=$$

$$=\lim_{n\to\infty}\frac{\left(n+1\right)\sqrt{n+1}\left(n+1\right)^2\sqrt{n+1}+n^2\sqrt{n+1}}{\left(n+1\right)^5-n^5}=\lim_{n\to\infty}\frac{\left(n+1\right)^4+n^2\left(n+1\right)\sqrt{n^2+n}}{5n^4+10n^3+10n^2+5n+1}=$$

$$=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)^4+\left(1+\frac{1}{n}\right)\sqrt{1+\frac{1}{n}}}{5+\frac{10}{n}+\frac{10}{n^2}+\frac{5}{n^3}+\frac{1}{n^4}}=\frac{2}{5}.$$

137. $\lim\limits_{n\to\infty}\dfrac{1^p+2^p+3^p+...+n^p}{n^{p+1}}=\lim\limits_{n\to\infty}\dfrac{x_{n+1}-x_n}{y_{n+1}-y_n}=\lim\limits_{n\to\infty}\dfrac{\left(n+1\right)^p}{\left(n+1\right)^{p+1}-n^{p+1}}=$

$$=\lim_{n\to\infty}\frac{\left(n+1\right)^p}{n^{p+1}+\left(p+1\right)n^p+...-n^{p+1}}=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)^p}{p+1+o\left(\frac{1}{n}\right)}=\frac{1}{p+1}$$

138.

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2n}-2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...+\frac{1}{2n}\right)=\frac{1}{n+1}+\frac{1}{n+2}+..+\frac{1}{2n}\Rightarrow$$

$$\Rightarrow 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2n}-\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)=\frac{1}{n+1}+\frac{1}{n+2}+..+\frac{1}{2n}\Rightarrow$$

$$\Rightarrow \frac{1}{n+1}+\frac{1}{n+2}+..+\frac{1}{2n}=\frac{1}{n+1}+\frac{1}{n+2}+..+\frac{1}{2n}$$

$$\lim_{n\to\infty}\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{1}{2n-1}-\frac{1}{2n}\right)=\lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2n}\right)-$$

$$-\lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)=\ln 2n-\ln n=\ln 2.$$

139. $\left[a^{(\log_3 7)}\right]^{(\log_3 7)}=27^{(\log_3 7)} \Rightarrow a^{(\log_3 7)^2}=7^3;$

$\left[b^{(\log_7 11)}\right]^{(\log_7 11)}=49^{(\log_7 11)} \Rightarrow b^{(\log_7 11)^2}=11^2;\ \left[c^{(\log_{11} 25)}\right]^{(\log_{11} 25)}=\sqrt{11}^{(\log_{11} 25)} \Rightarrow c^{(\log_{11} 25)^2}=5;$

$a^{(\log_3 7)^2}+b^{(\log_7 11)^2}+c^{(\log_{11} 25)^2}=7^3+11^2+5=469.$

140. Quyidagilarni bajaramiz:

$$arctg\frac{1}{3}+arctg\frac{1}{4}=x-arctg\frac{1}{5}\Rightarrow tg\left(arctg\frac{1}{3}+arctg\frac{1}{4}\right)=tg\left(x-arctg\frac{1}{5}\right)$$

$$\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{12}}=\frac{tgx-\frac{1}{5}}{1+\frac{tgx}{5}}\Rightarrow tgx=\frac{23}{24}\Rightarrow x=arctg\frac{23}{24}$$

$$arctg\frac{23}{24}+arctg\frac{1}{n}=\frac{\pi}{4}\Rightarrow tg\left(arctg\frac{23}{24}+arctg\frac{1}{n}\right)=tg\left(\frac{\pi}{4}\right)\Rightarrow$$

$$\Rightarrow arctg\frac{1}{3}+arctg\frac{1}{4}=x-arctg\frac{1}{5}\Rightarrow tg\left(arctg\frac{1}{3}+arctg\frac{1}{4}\right)=tg\left(x-arctg\frac{1}{5}\right)$$

$$\frac{\frac{23}{24}+\frac{1}{n}}{1-\frac{23}{24n}}=1\Rightarrow n=47.$$

141. Oddiygina shakl almashtirish bajaramiz:

$$100^2+99^2-98^2-97^2+...+4^2+3^2-2^2-1^2=(100-98)(100+98)+$$
$$+(99-97)(99+97)+...+(4-2)(4+2)+(3-1)(3+1)=$$
$$=2(198+196+190+188+...+6+4)=$$
$$=2((196+188+180+...+4)++(198+190+182+...+6))=$$
$$=2\left(\frac{196+4}{2}\cdot 25+\frac{198+6}{2}\cdot 25\right)=50\cdot 202=10100=10000+100$$

$N=x(\bmod 1000)\Rightarrow x=100.$

142. Qavslarni ochamiz va Viet teoremasidan foydalanamiz:

$$(r+s)^3+(s+t)^3+(t+r)^3=2(r^3+s^3+t^3)+3(r^2s+r^2t+s^2t+s^2r+t^2s+t^2r)=$$

$$=(r+s+t)^3+r^3+s^3+t^3-6rst=\left|\begin{array}{l} r+s+t=0, r+s=-t \\ rst=-\dfrac{2008}{8}=-251 \\ rs+st+rt=\dfrac{1001}{8} \end{array}\right|=$$

$$=(r+s)(r^2-rs+s^2)+t^3-6rst=$$

$$=-t\left((r+s)^2-3rs\right)+t^3-6rst=-t^3+3rst+t^3-6rst=-3rst=753.$$

143. Matematik induksiya metodidan foydalanamiz:

$n=1$ da $a_2=\dfrac{1}{1+1}\cdot(a_1+1)=\dfrac{1}{2}$,

$n=2$ da $a_3=\dfrac{2}{1+2}\cdot(a_2+1)=\dfrac{2}{3}\cdot\left(\dfrac{1}{2}+1\right)=1=\dfrac{2}{2}$,

$n=3$ da $a_4=\dfrac{3}{3+1}\cdot(1+1)=\dfrac{3}{2}$,

$n=k-1$ da $a_k=\dfrac{k-1}{2}$ deb faraz qilamiz.

$n=k$ uchun isbotlaymiz. $a_{k+1}=\dfrac{k}{k+1}\cdot\left(\dfrac{k-1}{2}+1\right)=\dfrac{k}{k+1}\cdot\dfrac{k+1}{2}=\dfrac{k}{2}$.

Demak, $\forall n\in\mathbb{N}$ lar uchun $a_n=\dfrac{n-1}{2}$ tenglik o‘rinli ekan, bundan $a_{2014}=\dfrac{2013}{2}$ kelib chiqadi.

144. $f(x)=\dfrac{\ln x}{x}$ funksiyani qaraymiz $(x\geq 125)$ uning kamayuvchiligidan

$$3^5>5^3\ \Rightarrow\ f(3^5)<f(5^3)\ \Rightarrow\dfrac{\ln(3^5)}{3^5}<\dfrac{\ln(5^3)}{5^3}\ \Rightarrow 5^3\cdot\ln(3^5)<3^5\cdot\ln(5^3)\Rightarrow(3^5)^{5^3}<(5^3)^{3^5}.$$

145. $P_n=a_n-a_{n-1}$ deb belgilash kiritamiz. U holda masala shartiga ko‘ra, $P_n=P_{n-1}+1$ bundan P_n soni ayirmasi 1 ga teng bo‘lgan arifmetik progressiya tashkil etishi kelib chiqadi. Shuning uchun $P_n=P_2+n-2$. Endi

$$a_n=(a_n-a_{n-1})+(a_{n-1}-a_{n-2})+\ldots+(a_2-a_1)+a_1=P_n+P_{n-1}+\ldots+P_2+a_1=$$

$$=(n-1)P_2+(n-2)+(n-3)+\ldots+1+a_1=(n-1)(a_2-a_1)+\dfrac{(n-2)(n-1)}{2}$$

Demak, $a_n=(n-1)a_2-(n-2)a_1+\dfrac{(n-2)(n-1)}{2}$.

146. Quyidagicha shakl almashtirishlarni bajaramiz:

$$9\cdot 99\cdot 999\cdot\ldots\cdot\underbrace{999\ldots 9}_{9999ta}=(10-1)(100-1)(1000-1)\ldots\left(1\underbrace{00\ldots 0}_{9999ta}-1\right)=$$
$$=(10-1)((100-1)(1000A-1))=(1000-109)(1000A-1)=1000B+109$$

Javob: 109.

147. $\sqrt{\lg^2 x+\lg^2 y+\lg^2 z}=\sqrt{(\lg x+\lg y+\lg z)^2-2(\lg x\lg yz+\lg y\lg z)}=$
$=\sqrt{81^2-2\cdot 468}=\sqrt{5625}=75$

148. $b_{n+2}=5b_{n+1}-b_n,\ n>1$ munosabat o‘rinli bo‘ladigan q^n ketma-ketliklarning barchasini topamiz, bunda q-biror son. Tenglikni o‘rniga qo‘yib, $b_{n+2}=q^{n+2},\ b_{n+1}=q^{n+1},\ b_n=q^n$ bundan $q^2-5q+6=0$ ni hosil qilamiz. Shunday qilib, q ning 2 ta: $q_1=2$ va $q_2=3$ qiymatlarini topamiz.

Demak, 2^n va 3^n ketma-ketliklar yuqoridagi rekkurent munosabatni qanoatlantirar ekan. U holda $b_n=c_1\cdot 2^n+c_2\cdot 3^n$ ketma-ketlik ham yuqoridagi rekkurent munosabatni qanoatlantiradi, bundan $c_1,c_2\in R$. Endi c_1 va c_2 larni $b_1=1$ va $b_2=1$ bajariladigan qilib tanlash kerak.

Ushbu $\begin{cases}2c_1+3c_2=1\\4c_1+9c_2=1\end{cases}\Rightarrow\begin{cases}c_1=1\\c_2=-\dfrac{1}{3}\end{cases}$ tenglamalar sistemasidan c_1 va c_2 larni topib olamiz. Shunday qilib, $n\geq 1$da $a_n=2^n-3^{n-1}$ yechimga ega bo‘lamiz.

149. a_{n+1} ni almashtirirish bajarib quyidagicha yozib olamiz.

$$\frac{1}{a_{n+1}}=\frac{1}{a_n}+n=\frac{1}{a_{n-1}}+(n-1)+n=\frac{1}{a_{n-2}}+(n-2)+(n-1)+n=\ldots=$$
$$=\frac{1}{a_1}+1+2+\ldots+(n-1)+n=\frac{1}{a_{n+1}}+\frac{n(n+1)}{2}$$

Bundan $a_{n+1}=\dfrac{2a_1}{2+a_1n(n+1)}$ bo‘ladi endi a_{2014} ni hisoblaymiz.

$$a_{2014}=\frac{2}{2+2\cdot 2013(2013+1)}=\frac{1}{2013^2+2013+1}=\frac{1}{4054183}.$$

150. Ketma-ketlikning bir nechta hadlarini yozib olaylik.

$a_1=2,\ a_2=3,\ a_3=\frac{3}{2},\ a_4=\frac{1}{2},\ a_5=\frac{1}{3},\ a_6=\frac{2}{3},\ a_7=2,\ a_8=3,\ a_9=\frac{3}{2},\ldots$

Bundan ko‘rinib turibdiki ketma-ketlik 6 davrda aylanayapti (matematik induksiya metodi orqali isbotlash oson). $2014=6\cdot 335+4$ u holda $a_{2014}=\frac{1}{2}$.

Javob: $\frac{1}{2}$.

151. Matematik induksiya metodidan foydalanamiz.

$n=1$ da $a_2 = a_1 + \frac{1}{a_1^2} = 1 + \frac{1}{1^2} = 2 \Rightarrow a_2^3 = 8 > 3 \cdot 2,$

$n=2$ da $a_3 = a_2 + \frac{1}{a_2^2} = 2 + \frac{1}{2^2} = \frac{9}{4} \Rightarrow a_2^3 = \frac{729}{64} > 3 \cdot 3,$

$n=k$ da $a_k^3 > 3k$ to‘g‘ri deb faraz qilamiz. $n=k+1$ uchun isbotlaymiz.

$$a_{k+1}^3 = a_k^3 + \frac{3}{a_k^3} + 3 + \frac{1}{a_k^6} > a_k^3 + 3 > 3k + 3 = 3(k+1).$$

Demak, $\forall n \in N$ uchun

$$a_n^3 > 3n \Rightarrow a_{9000}^3 > 3 \cdot 9000 \Rightarrow a_{9000} > \sqrt[3]{3 \cdot 9000} = 30.$$

152. $x^2 - 2x + 2 + \frac{4}{x^2 - 2x + 2} = \sqrt{12 - x^2 + 4x}$ Bir tomondan, tenglikning chap qismi quyidagicha bo‘ladi. $x^2 - 2x + 2 + \frac{4}{x^2 - 2x + 2} \geq 4$ .O‘ng tomoni $\sqrt{12 - x^2 + 4x} = \sqrt{16 - (x-2)^2} \leq 4$. Demak bu tengliklar $x = 2$ da o‘rinli bo‘ladi.

153. $\sin^2 x \cdot \cos^4 x \cdot (2 - \sin^2 x) = \sin^2 x \cdot (1 - \sin^2 x) \cdot (2 - \sin^2 x) = \left|\sin^2 x = a\right| =$

$= a(2-a)(1-a)^2 = (2a - a^2)(1-a)^2 = f(a)$

$f'(a) = (2-2a)(1-a)^2 - 2(1-a)(2a-a^2) = 0 \Rightarrow (2-2a)(1 - 2a + a^2 - 2a + a^2 = 0) \Rightarrow$

$\Rightarrow a = 1, a = 1 \pm \frac{\sqrt{2}}{2} \Rightarrow f\left(1 - \frac{\sqrt{2}}{2}\right) = \left(2 - \sqrt{2} - 1 + \sqrt{2} - \frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{4}.$

154.

$$\left(2\sqrt[4]{a} + 2\sqrt[4]{b} + 2\sqrt[4]{c}\right) - \left(\sqrt{a} + \sqrt{b} + \sqrt{c}\right) = \left(\sqrt[4]{a} - 1\right)^2 + \left(\sqrt[4]{b} - 1\right)^2 + \left(\sqrt[4]{c} - 1\right)^2 + 3 \Rightarrow$$

$$\Rightarrow \left(2\sqrt[4]{a} + 2\sqrt[4]{b} + 2\sqrt[4]{c}\right) - \left(\sqrt{a} + \sqrt{b} + \sqrt{c}\right) \geq 0 .$$

Tenglik faqat $a = b = c = 1$ da bajariladi.

155. $n \cdot n! = (n+1)! - n!$ tenglikdan foydalaning. Javob: $(k+1)! - 1$

156. Shartga ko‘ra,

$$\begin{cases} a_1 - a_2 > 0 \\ a_2 - a_1 = 2(a_1 - a_0) \\ \dots\dots\dots\dots\dots\dots \\ a_{100} - a_{99} = 2(a_{99} - a_{98}) \end{cases}$$

bularga asosan $a_2 - a_1, a_3 - a_2, \dots, a_{100} - a_{99}$, ayirmalarning musbat ekanligi kelib chiqadi. Shu bilan birga

$\dfrac{a_2-a_1}{a_1-a_0}=\dfrac{a_3-a_2}{a_2-a_1}=...=\dfrac{a_{100}-a_{99}}{a_{99}-a_{98}}=2$. Bu 99 ta tenglikni bir-biriga ko‘paytirib, $\dfrac{a_{100}-a_{99}}{a_1-a_0}=2^{99}$ $\Rightarrow a_{100}=a_{99}+2^{99}\left(a_1-a_0\right)$ bundan esa $a_{99}>0$ va $a_1-a_0\geq 1$. Demak, $a_{100}>2^{99}$ tengsizlik o‘rinli.

157. $\sqrt{2014+\sqrt{2014+\sqrt{2014+...+\sqrt{2014}}}}=x$ deb belgilaymiz va ikkala tamonini kvadratga oshiramiz. So‘ngra quyidagi kvadrat tenglamaga ega bo‘lamiz:

$2014+x=x^2$ bundan $x_1=\left[\dfrac{1+\sqrt{1+4\cdot 2014}}{2}\right]$, $x_2=\left[\dfrac{1-\sqrt{1+4\cdot 2014}}{2}\right]$-bu chet ildiz hisoblanadi (chunki x musbat son). U holda $[x]=\left[\dfrac{1+\sqrt{1+4\cdot 2014}}{2}\right]=45$ yechimga ega bo‘lamiz.

158. a_n ni almashtirish bajarib quyidagi ko‘rinishga keltiramiz.

$$a_n=\frac{1}{(n+1)\sqrt{n}+n\sqrt{n+1}}=\frac{1}{\sqrt{n}\cdot\sqrt{n+1}\cdot\left(\sqrt{n+1}+\sqrt{n}\right)}=$$
$$=\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n}\cdot\sqrt{n+1}}=\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}$$

Bundan so‘ralgan ifodani hisoblaymiz:

$$a_1+a_2+...+a_{99}=\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{99}}-\frac{1}{\sqrt{100}}=1-0{,}1=0{,}9.$$

3-§. Mustaqil yechish uchun masalalar

1-variant

1. $A=(a_{ij})$ $a_{ij}\in\mathbb{R}$ bo‘lsa, $\left|\det A\right|^2\le\prod_{j=1}^{n}\sum_{i=1}^{n}a_{ij}^2$ ekanini isbotlang.

2. $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ tenglikdan foydalanib, $\int_0^1 \ln x\cdot\ln(x-1)dx$ ni hisoblang.

3. A–ikkinchi tartibli kvadrat matritsa va $k>2$ natural son. Agar $A^k=0$ bo‘lsa, u holda $A^2=0$ ekanini isbotlang.

4. Agar ABC uchburchakka tashqi chizilgan aylana markazi O va bu uchburchak medianalari kesishgan nuqta M bo‘lsa, ushbu $\overline{OM}=\frac{1}{3}(\overline{OA}+\overline{OB}+\overline{OC})$ tenglikni isbotlang.

5. $\int_0^{+\infty}\frac{\sin x}{x}dx$ xosmas integralni hisoblang.

2-variant

1. Quyidagi tengsizlikni isbotlang: $xy+yz+zx\ge\sqrt{3xyz(x+y+z)}$. Bu yerda $x\ge0,\ y\ge0,\ z\ge0$.

2. $\{x_n\}$ ketma-ketlik quyidagicha berilgan: $x_1=\frac{1}{2}$, $x_{n+11}=x_n-x_n^2$, $n\ge1$. $\lim_{n\to\infty}nx_n=1$ ekanini isbotlang.

3. $f(x)$ funksiya $[a,b]$ segmentda musbat, uzluksiz va monoton kamayuvchi bo‘lsa, u holda quyidagi tengsizlikni isbotlang. $\int_a^b xf^2(x)dx\le\frac{1}{2}\left(\int_a^b f(x)dx\right)^2$

4. $\begin{vmatrix}1 & n & n & \dots & n\\ n & 2 & n & \dots & n\\ n & n & 3 & \dots & n\\ \dots & \dots & \dots & \dots & \dots\\ n & n & n & \dots & n\end{vmatrix}$ determinantni hisoblang.

5. ABC uchburchakda AD mediana o‘tkazilgan. $\angle DAC+\angle ABC=90^0$ va $|AB|=|AC|$ bo‘lsa, $\angle BAC$ ni toping.

3-variant

1. $\lim_{n\to\infty}\frac{1+2^2+\dots+n^n}{n^n}$ limitni hisoblang.

2. $\int\limits_0^1 xf(x)dx = \int\limits_0^1 x^3 f(x)dx = \int\limits_0^1 x^5 f(x)dx = 0$ tenglik o‘rinli bo‘ladigan uchinchi darajali $f(x)$ ko‘phadni toping.

3. $(x+1)^m - x^m - 1$ ko‘phad m ning qanday qiymatlarida $x^2 + x + 1$ ga bo‘linadi?

4. $\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\frac{1}{C_n^k}\right)^n$ limitni hisoblang.

5. Quyidagi determinantni hisoblang:

$$\begin{vmatrix} x & a_1 & a_2 & ... & a_n \\ a_1 & x & a_2 & ... & a_n \\ a_1 & a_2 & x & ... & a_n \\ ... & ... & ... & ... & ... \\ a_1 & a_2 & a_3 & ... & x \end{vmatrix}$$

4-variant

1. $\sum\limits_{n=0}^{\infty}\frac{2^n}{n!}\frac{d^n}{dy^n}\left(\frac{1}{1+\sqrt{y}}\right)\Bigg|_{y=x} = \frac{1}{3}$ tenglamani yeching.

2. Quyidagi ayniyatni isbotlang.

a) $\sum\limits_{k=1}^{n} a_k^2 \sum\limits_{k=1}^{n} b_k^2 - \left(\sum\limits_{k=1}^{n} a_k b_k\right)^2 = \frac{1}{2}\sum\limits_{k=1}^{n}(a_k b_j - a_j b_k)^2$;

b) $\sum\limits_{k=1}^{n} a_k^2 \sum\limits_{k=1}^{n} b_k^2 - \left(\sum\limits_{k=1}^{n} a_k b_k\right)^2 = \sum\limits_{k=1}^{n} a_k^2 \sum\limits_{k=1}^{n}\left(a_k \frac{a_1 b_1 + ... + a_n b_n}{a_1^2 + ... + a_n^2}\right)^2$;

3. Aylananing AB vatarining o‘rtasi bo‘lgan C nuqtadan ikkita KL va MN vatarlar o‘tkazilgan (K va M nuqtalar AB vatardan bir tomonda yotadi). Agar Q nuqta AB va KN hamda P nuqta AB va ML vatarlarning kesishish nuqtalari bo‘lsa, $QC = CP$ tenglikni isbotlang.

4. Agar $a_0 x^n + a_1 x^{n-1} + ... + a_{n-1}x + a_n$ ko‘phadning barcha ildizlari haqiqiy bo‘lsa, u holda $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ ko‘phadning ham barcha ildizlari haqiqiy bo‘lishini isbotlang.

5. Agar $A_n = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}^2 = \|a_{ij}(n)\|$ $A_n = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}^n = \|a_{ij}(n)\|$ bo‘lsa, quyidagi $\frac{a_{12}(n)}{a_{22}(n)}$ nisbatning $n \to \infty$ da limiti mavjudligini isbotlang va uni toping.

5-variant

1. $f(x)$ funksiya $[0,1]$ kesmada uzluksiz bo‘lsa, $\int_0^1 af(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ tenglikni isbotlang.

2. $\lim_{n\to\infty}\frac{1}{n}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}+...+\sqrt{1+\frac{n}{n}}\right)$ ni hisoblang.

3. Quyidagi ketma-ketliklar uchun x_n ning faqat n bog‘liqlik formulasini keltirib chiqaring.

1) $x_n = x_{n-1} - 4x_{n-2}$ bu yerda, $x_1 = 1,\ x_2 = 19$;

2) $x_n = x_{n-1} + x_{n-2}$ bu yerda, $x_1 = 1,\ x_2 = 2$ bu ketma-ketlik Fibonachchi sonlarini ifodalaydi:

3) $x_n = 6x_{n-1} - 9$ bu yerda, $x_1 = 3$;

4. A matristaning xos qiymatlarini bilgan holda $f(A)$ matristaning determinantini hisoblang. Bunda $f(A) = a_nA^n + a_{n-1}A^{n-1} + ... + a_0$ matritsaviy ko‘phad.

5. To‘rt yoqli burchakning ixtiyoriy tekis burchagi qolgan uchtasining yig‘indisidan kichik bo‘lishini isbotlang.

6-variant

1. $f'(x) = f\left(\frac{1}{x}\right)$ $x > 0$ tenglamani yeching.

2. Agar $f(x) = \int_x^{x+1}\sin t^2 dt$ bo‘lsa, $xf(x)$ funksiyani $x \to +\infty$ da yuqori va quyi limitini hisoblang.

3. ABC uchburchak ichidan ixtiyoriy M nuqta olingan. Agar uchburchakning yuzi S ga teng bo‘lsa, quyidagini isbotlang:

$$4S \le AM\cdot BC + BM\cdot AC + CM\cdot AB$$

4. $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} - \ln n$ ketma-ketlikning yaqinlashuvchi ekanini isbotlang.

5. $f(x)$ va $g(x)$ ko‘phadlar berilgan. Agar $\deg f < \deg g$ va $g(x_i) = 0,\ i = \overline{1,n}$ bo‘lib, $x_1, x_2, ..., x_n$ lar turli haqiqiy sonlar bo‘lsa, $\frac{f(x)}{g(x)} = \sum_{j=1}^{n}\frac{f(x_j)}{g'(x_j)}\cdot\frac{1}{x - x_j}$ tenglikni isbotlang.

7-variant

1. $\overline{\lim_{n\to\infty}} x_n = \lim_{n\to\infty}\sup_{k>n}\{x_k\}$ va $\underline{\lim_{n\to\infty}} x_n = \lim_{n\to\infty}\inf_{k>n}\{x_k\}$ munosabatlarni isbotlang.

2. $ctg^2\frac{\pi}{2n+1}+ctg^2\frac{2\pi}{2n+1}+\ldots+ctg^2\frac{n\pi}{2n+1}=\frac{n(2n-1)}{3}$ ayniyatni isbotlang.

3. $\lim\limits_{x\to\infty}\frac{\int\limits_0^x \cos t^2 dt}{x}$ limitni hisoblang.

4. Ellipsning yarim o'qlarini toping. $45x^2-72xy+80y^2=4$

5. Darajasi 2013 dan oshmaydigan $P(x)$ ko'phadning 2013-darajasi oldidagi koeffitsiyenti 1 ga teng. $P(0)=2012,\ P(1)=2011,\ldots,P(2012)=0$ bo'lsa, $P(2013)$ ni toping.

8-variant

1. Agar natural n sonini $n=\frac{x^2-1}{y^2-1}$ ko'rinishida tasvirlab bo'lmasa, uni "Sehrli" son deb ataymiz, bu yerda $x,y\in\mathbb{N}$ va $x,y>1$. "Sehrli" sonlar cheksiz ko'pmi?

2. $x>0$ bo'lganda, $a^x>ax$ tengsizlikni qanoatlantiruvchi barcha musbat a larni toping.

3. Agar $A(x_1,y_1),\ B(x_2,y_2),\ C(x_3,y_3)$ nuqtalar bir to'g'ri chiziqda yotsa, quyidagi tenglikni isbotlang:

$$\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}=0$$

4. Bir idishda (probilkada) oq, ikkinchisida qizil ichimlik bor. Qizil ichimlikdan oqiga bir tomchi tomizamiz. So'ngra hosil bo'lgan aralashmadan qizil ichimlikka bir tomchi qaytaramiz. Qizil ichimlikdagi oq ichimlik ko'pmi yoki oqidagi qizilmi?

5. $ABCDA_1B_1C_1D_1$ parallelopipedda AC_1 diagonal BDA_1 tekislikni K nuqtada kesib o'tadi. Ushbu $\frac{|AK|}{|AC_1|}$ nisbatni toping.

9-variant

1. Limitni hisoblang. $\lim\limits_{n\to\infty}(n!e-[n!e])$

2. A, B, C, D — $(n \times n)$ matritsalar uchun $AD^T - BC^T = E$, bunda E—birlik matritsa AD^T va BC^T matritsalar simmetrik bo'lsa, u holda $A^TD - C^TB = E$ ekanligini isbotlang.

3. $f(x) = \sum_{y=2}^{2013} \frac{1}{x^y}$ bo'lsa, $\sum_{k=2}^{\infty} f(k)$ ni toping.

4. ABC uchburchakning medianalari O nuqtada kesishsa, quyidagi tenglikni isbotlang: $AB^2 + BC^2 + CA^2 = 3(OA^2 + OB^2 + OC^2)$

5. Sistemani yeching. $\begin{cases} x_1 + x_2 + ... + x_n = 1 \\ x_1^2 + x_2^2 + ... + x_n^2 = \dfrac{1}{n} \end{cases}$

10-variant

1. $P(x)$ ko'phad faqat haqiqiy ildizlarga ega va a soni $P'(x)$ ko'phadning karrali ildizi bo'lsa, u holda $P(a) = 0$ ekanligini isbotlang.

2. Ellipsoidda yotmaydigan biror nuqta orqali ellipsga o'tkazilgan barcha urinmalar bilan ellipsning kesishish nuqtalari to'plami qandaydir tekislikda yotishini isbotlang.

3. $\{x_n\}$ $x = tgx$ tenglama manfiymas yechimlarining mos ravishda o'sib boruvchi yechimlari ketma-ketligi bo'lsa, $x_n - x_{n+1}$ ketma-ketlikning limitini toping.

4. a^b ifoda natural son bo'ladigan a va b irratsional sonlar mavjudligini isbotlang.

5. λ ning shungay qiymatlarini topingki, natijada rang$\begin{pmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix}$ eng kichik bo'lsin.

11-variant

1. $(n \times n)$, $n > 1$ nolmas diagonal matritsani, determinanti birga teng ikki matritsaning yig'indisi ko'rinishda yozish mumkinligini isbotlang.

2. $[a, b]$ kesmada $f(x), g(x) > 0$ bo'lsa, $\int_a^b f(x)g(x)dx \le \int_a^b f(x)dx \cdot \int_a^b g(x)dx$ ni isbotlang.

3. a, b, c uchburchakning tomonlari bo‘lsa, $a + b + c < 2(\sqrt{ab} + \sqrt{bc} + \sqrt{ac})$ tengsizlikni isbotlang.

4. $x, y > 0$, $f(xy)\dfrac{f(x)}{y} + \dfrac{f(y)}{x}$ ning barcha differensiallanuvchi yechimlarini toping.

5. Tenglamani yeching. $\left[\dfrac{x^2 - 3x + 3}{2}\right] + 1 = x$. Bu yerda $[a]$-ifoda a sonning butun qismi.

12-variant

1. $P(x)$ n-daralaji ko‘phad n ta turli $x_1, x_2, ..., x_n$ ildizlarga ega bo‘lsa, quyidagi tenglikni isbotlang: $\dfrac{1}{P'(x_1)} + \dfrac{1}{P'(x_2)} + ... + \dfrac{1}{P'(x_n)} = 0$.

2. Shunday eng kichik natural n sonini topingki, natijada quyidagi tenglik o‘rinli bo‘lsin: $\lim\limits_{n\to\infty} \dfrac{1}{2^n}\begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

3. Yig‘indini toping. $C_n^1 + C_n^4 + C_n^7 + ...$

4. Agar $\{x_n\}$ ketma-ketlik uchun $x_1 = a,\ x_2 = b,\ x_n = \dfrac{x_{n-1} + x_{n-2}}{2}$ tengliklar o‘rinli bo‘lsa, $\lim\limits_{n\to\infty} x_n$ ni hisoblang.

5. Isbotlang. $trAB = trBA$. Bu yerda trX ifoda X matritsaning izi.

13-variant

1. $ABCD$ qavariq to‘rtburchakda $AB < AD$, AC diaganali $\angle BAD$ burchakning bissektrisasi. $\angle ABD = 103^0$ va AD kesmadagi E nuqta uchun $\angle BAD = 40^0$, $BC = CD = DC$ bo‘lsa $\angle ACE - ?$

2. 99999 ga bo‘linadigan 10 xonali raqamlari har xil bo‘lgan sonlar nechta?

3. $ABCD$ parallelogrammning AD va DC tomonlarining o‘rtalarimosravishda M, N nuqtalar bo‘lsin. CM va BN kesmalar O nuqtada kesishadi. U holda $\dfrac{BO}{ON} \cdot \dfrac{CO}{OM} = ?$

4. Funksional qatorning absolyut yaqinlashish sohasini toping:

$$\sum_{n=1}^{\infty} \frac{n!}{(x^2 + 1)(x^2 + 2)...(x^2 + n)}$$

5. $f(\alpha) = \int\limits_{e^{-\alpha}}^{e^{\alpha}} \ln(1 + \alpha^2 x^2)dx$, $f'(\alpha)$ ni toping.

14-variant

1. $\angle ABC$ burchagi to‘g‘ri bo‘lgan $ABCD$ qavariq to‘rtburchakda $AB=2$, $BC=3$, $AD=6$, $CD=7$ va bu to‘rtburchakka aylana ichki chizish mumkin bo‘lsa, uning radiusini aniqlang.

2. Musbat x,y,z va birdan kichik bo‘lmagan a,b,c haqiqiy sonlar uchun

$$\begin{cases} a^x+b^y+c^z=4 \\ a^x+b^y+c^z=4 \\ x^2a^x+y^2b^y+z^2c^z=9 \end{cases}$$ bo‘lsa, c ning eng katta qiymatini toping.

3. $(a_1,a_1),(a_2,a_2),(a_3,a_3),...$ nuqtalar uchun $(a_{n+1},b_{n+1})=(\sqrt{3}a_n-b_n,\sqrt{3}b_n-a_n)$ barcha $n=1,2,...$ da $(a_{100},b_{100})=(2,4)$ bo‘lsa (a_1+b_1) ni toping.

4. ABC uchburchakda E,F nuqtalar AC va AB dan olingan. BE va CF lar X nuqtada kesishadi. $\frac{AF}{FB}=\left(\frac{AE}{EC}\right)^2$ va X nuqta BE kesmaning o‘rtasi bo‘lsa $\frac{CX}{XF}-?$

5. $a_n=\sum_{k=1}^{n}\frac{1}{C_n^k}$ bo‘lsa, limitni hisoblang $\lim_{n\to\infty} a_n^n=?$

15-variant

1. Aylanaga ichki chizilgan $ABCDEF$ oltiburchakda $AB=BC=2$, $CD=DE=9$, $EF=FA=12$ bo‘lsa aylana radiusini toping.

2. $x,y\in R^+$, $\alpha\neq\frac{\pi n}{2}$ n-butun son uchun $\frac{\sin\alpha}{x}=\frac{\cos\alpha}{y}$ va $\frac{\sin^4\alpha}{y^4}+\frac{\cos^4\alpha}{x^4}=\frac{97\sin\alpha}{x^3y+xy^3}$ bo‘lsa $\frac{x}{y}+\frac{y}{x}$ ni toping.

3. $a>0$ va $P(x)$-butun koeffisiyentli ko‘phad uchun $P(1)=P(3)=P(5)=P(7)=a$ va $P(2)=P(4)=P(6)=P(8)=-a$ bo‘lsa, a ni toping.

4. $x=3t-t^3$, $y=3t^2$, $z=3t+t^3$ egri chiziqning qaysi nuqtasidagi urinma $3x+y+z+2=0$ tekislikka parallel bo‘ladi.

5. Uchinchi tartibli matritsaning elementlari 0 va 1 sonlaridan iborat. Shu matritsaning determinantining eng katta va eng kichik qiymatini toping.

16-variant

1. a,b,c,d,e-natural sonlar uchun $a+b+c+d+e=2010$, M-mumkin bo‘lgan eng katta yig‘indi $a+b,b+c,c+d,d+e$ sonlar uchun. M ning mumkin bo‘lgan eng kichik qiymatini toping

2. ABC uchburchakda $AB=65, BC=33, AC=56$ uchun G aylana AC, BC va ABC uchburchakka tashqi chizilgan aylanaga urinadi. G aylananing radiusini toping.

3. A matritsa uchun $A^k=0$ tenglik o‘rinli bo‘lsa, u holda $(E-A)^{-1}=E+A+A^2+...+A^{k-1}$ bo‘lishini isbotlang. Bu yerda E birlik matritsa.

4. $\{a_n\}$ ketma-ketlik $a_1=a>0,\ a_2=a^2,\ a_n=\sum_{i=1}^{n-1}a_ia_{n-i}$, $(n\geq 3$ larda) ko‘rinishida rekkurent berilgan. $\lim_{n\to\infty}\sqrt[n]{a_n}$ ni hisoblang.

5. Integralni hisoblang: $\int_0^{\frac{\pi}{2}}\left(\frac{\sin n\varphi}{\sin\varphi}\right)^2 d\varphi,\ \ n\in\mathbb{N}$.

17-variant

1. Integralni hisoblang: $\int_0^{\frac{\pi}{2}}\frac{dx}{1+(tgx)^{\sqrt{2}}}$

2. $\{a_n\}$-geometrik progressiyada $a_1=\sin x, a_2=\cos x$ va $a_3=tgx$, $x\in\mathbb{R}$ da. $a_n=1+\cos x$ bo‘lsa n-ni toping.

3. $\{a_n\}$ va $\{b_n\}$ ketma- ketliklar butun koeffisiyentli arifmetik progressiya tashkil qiladi. $a_1=b_1=1<a_2\leq b_2$ va $a_nb_n=2010$ bo‘lsa $n\in\mathbb{N}$ -ni toping.

4. ABC uchburchakda $AB=1$, $BC=\sqrt{7}$ va $CA=\sqrt{3}$. ℓ_1 to‘g‘ri chiziq A nuqtadan o‘tib, AB perpendikulyar. ℓ_2 to‘g‘ri chiziq esa B nuqtadan o‘tib, AC ga perpendikular. ℓ_1 va ℓ_2 to‘g‘ri chiziqlar P nuqtada kesishadi, u holda PC ni toping.

5. Limitni hisoblang: $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt[p]{n^p+ka}}$

18-variant

1. Agar $a,b,c\in\mathbb{R}$ bo‘lsa, u holda limitni hisoblang:

$$\lim_{x\to\infty}(a\sqrt{x+1}+b\sqrt{x+2}+c\sqrt{x+3})$$

2. ABC uchburchak va uning BC tomonida A_1 nuqta berilgan. AC tomondan o‘tgan to‘g‘ri chiziqda B_1 nuqta AB tomondan to‘g‘ri chiziqda esa C_1 nuqta shunday tanlanganki $AA_1 \| BB_1 \| CC_1$ (o‘zaro parallel) $A_1B_1C_1$ uchburchak yuzining ABC uchburchak yuziga nisbatini toping.

3. Agar $\psi_1(x) = 2x^2 - 1$ bo‘lsa, u holda $\psi_{n+1}(x) = \psi_1(\psi_n(x))$ tenglamaning yechimini toping. Limitni hisoblang: $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+k}}$.

4. $x^3 + 3xy^2 = R(x^2 + y^2)$, $R > 0$, $x \neq 0$ egri chiziqning asimptotasini toping.

5. Agar $a, b, c > 0$ bo‘lsa, u holda $\frac{(1+a^2)(1+b^2)(1+c^2)}{(1+a)(1+a)(1+c)} \geq \frac{1+abc}{2}$ tengsizlikni isbotlang.

19-variant

1. $ABCD$ -trapetsiyada $AD \| BC$, $BD = 1$, $\angle DBA = 23^0$ va $\angle BDC = 46^0$, $BC / AD = 9/5$. bo‘lsa CD ?

2. Quyidagi shart bilan aniqlangan ketma-ketliklar limitini toping:

$$x_{n+1} = \frac{x_n + \frac{a}{x_n}}{2}, \quad x_1 > 0, \quad a > 0, \quad n \geq 1$$

3. a va d musbat sonlar, A_n va G_n lar $a, a+d, a+2d, \dots, a+(n-1)d$ sonlarning mos ravishda o‘rta arifmetigi va o‘rta geometrigi bo‘lsin. U holda $\lim_{n\to\infty} \frac{G_n}{A_n} = \frac{2}{e}$ bo‘lishini ko‘rsating.

4. n-hadi $\frac{\sin x}{1} + \frac{\sin 2x}{2} + \dots + \frac{\sin nx}{n} + \dots$ qatorning n- qismiy yig‘indisiga teng bo‘lgan cheksiz ketma-ketlik $x = \frac{\pi}{n+1}$ bo‘lganda noldan farqli limitga ega bo‘lishini isbotlang.

5. $f(x)$ kvadrat uchhadni har bir qadamda $x^2 \cdot f\left(1 + \frac{1}{x}\right)$ ga yoki $(x-1)^2 f\left(\frac{1}{1-x}\right)$ ga o‘tkazish mumkin. Ma’lum bir sondagi qadamlar bilan $x^2 + 4x + 3$ ni $x^2 + 10x + 9$ ga o‘tkazish mumkinmi?

20-variant

1. $ABCD$ trapetsiyada $AB \parallel CD$, $AB = 11$, $BC = 5$, $CD = 19$ va $DA = 7$. $\angle A$ va $\angle B$ ning bissektrisalari P nuqtada, $\angle B$ va $\angle C$ niki esa Q nuqtada kesishsa S_{ABQCDP} ni toping.

2. Markazi O boʻlgan aylanada AB va CD oʻzaro perpendikular vatarlar E nuqtada kesishadi. N va T nuqtalar, mos ravishda, AC va BD kesmalarning oʻrtalari boʻlsa, $ENOT$-toʻrtburchak parallelogramm ekanligini isbotlang.

3. $y = 1 + \left(\frac{1}{2}\right)^2 z + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 z^2 + ... + \left(\frac{1\cdot 3\cdot ...\cdot (2n-1)}{2\cdot 4\cdot ...\cdot 2n}\right)^2 z^n + ...$ qator

$$z(1-z)\frac{d^2y}{dz^2} + (1-2z)\frac{dy}{dz} - \frac{1}{4}y = 0$$

differensial tenglamani qanoatlantirishini koʻrsating.

4. Agar $\int\limits_0^1 \frac{\ln(1+x)dx}{x} = \frac{\pi^2}{12}$ boʻlsa, unda $\int\limits_0^1 \frac{\ln(1-x^{2012})dx}{x}$ ni toping

5. $P(x)$-koeffisiyentlari haqiqiy boʻlgan koʻphadda $P(0) = 1$, $P(2) + P(3) = 125$ va ixtiyoriy x uchun $P(x)P(2x^2) = P(2x^3 + x)$ boʻlsa $P(5) = ?$

21-variant

1. $2^N = a! + b! + c!$ tenglamani natural sonlarda yeching.

2. Toʻrtta aylana bir-biri bilan tashqi urinadi. Ularning urinish nuqtalari A, B, C, D bir aylanada yotishini isbotlang.

3. $f: R \to R$ funksiya berilgan. $f^*(x)$ orqali quyidagi munosabatlarni belgilaymiz: $f^*(x) = \lim\limits_{h\to 0}\frac{f(x+2h) - f(x-h)}{3h}$. Agar $f^*(x) \geq 0$ boʻlsa, $f(x)$ funksiya kamaymaydigan funksiya boʻlishini isbotlang.

4. $f(x)$ funsiya $(0, +\infty)$ da uzluksiz, qatʼiy oʻsuvchi musbat funksiya boʻlsin, bundan tashqari $\lim\limits_{x\to\infty}\frac{f(x)}{x\ln x} = 1$ boʻlsin. $\varphi(x)$ bilan $f(x)$ ga teskari funsiyani belgilaymiz. Quyidagi tenglikni isbotlang: $\lim\limits_{x\to\infty}\frac{\varphi(x)}{x\ln x} = 1$

5. $\{x_n\}$ ketma-ketlik quyidagi, $x_1 = \frac{1}{2}$, $x_{n+1} = x_n - x_n^2$ $(n \geq 1)$ koʻrinishda berilgan, u holda $\lim\limits_{n\to\infty} nx_n$ mavjudligi isbotlang va limitini toping.

22-variant

1. Radiusi $2\sqrt{3}$ ga teng boʻlgan yarim sharga ichki chizilgan konuslarning eng kichik hajmli konusning hajmini toping.

2. $x_n = y_n e^{-\frac{1}{12n}}$, $y_n = n!\, n^{-n-\frac{1}{2}} e^n$ bo‘lsin. (x_1, y_1), (x_2, y_2), (x_3, y_3),... intervallar biri ikkinchisining ichida yotishini ko‘rsating, ya’ni har biri keyingisini o‘zining qismi sifatida o‘z ichiga oladi.

3. $x_1 = a > 0$ va $x_{n+1} = \dfrac{x_1 + x_2 + ... + nx_n}{n}$, $n \in \mathbb{N}$ bo‘lsa, u holda bu ketma-ketlikning limitini hisoblang.

4. $\left[\sqrt{20102011}\right]$ ni toping ($[x] - x$ ning butun qismi).

5. $9 \cdot 99 \cdot 999 \cdot ... \cdot \underbrace{999...9}_{9999\, ta} = x \pmod{1000}$ bo‘lsa, $x = ?$

23-variant

1. Uchburchakka ichki chizilgan aylananing radiusi 1 ga teng. Uchburchakning tomonlari butun son ekanligi ma’lum bo‘lsa, uning tomonlarini toping.

2. $a, b, c, d \in \mathbb{N}$, $a > b > c > d$, $a + b + c + d = 2010$
$a^2 - b^2 + c^2 - d^2 = 2010$. a ning mumkin bo‘lgan qiymatlarini toping?

3. $P(x)$-haqiqiy koeffisiyentli kvadrat uchhad uchun

$$x^2 - 2x + 2 \le P(x) \le 2x^2 - 4x + 3, \ \forall x \in \mathbb{R}$$

da $P(11) = 181$ bo‘lsa $P(16)$ ni toping.

4. (a, b, c) soni $\begin{cases} x^3 - xyz = 2 \\ y^3 - xyz = 6 \\ z^3 - xyz = 20 \end{cases}$ sistemaning ildizlari. $a^3 + b^3 + c^3 = \dfrac{m}{n}$ ko‘rinishida bo‘lsa (bunda m, n –o‘zaro tub sonlar), u holda $(m + n) = ?$

5. ABC uchburchakda $\angle C = 60^0$, $BC = 4$. D nuqta BC kesmaning o‘rtasi bo‘lsa, $tg(\angle BAD)$ ni mumkin bo‘lgan eng katta qiymatini toping.

24-variant

1. Agar $a_1 > 0$ va $a_{n+1} = a_n + \dfrac{n}{a_n}$, $n \ge 1$ bo‘lsa, $n \ge 2$ bo‘lganda, $a_{2014} \ge 2014$ ekanini isbotlang.

2. $\cos\left(\dfrac{1}{tgx}\right) = x$ bo‘lsa, x^2 ni toping.

3. ABC uchburchakda tomonlari $AB = 6\sqrt{2}$, $BC = 8$, $AC = 14$ ga teng. Uchburchakning B uchidan mediana va balandlik o‘tkazilgan. C uchidan o‘tkazilgan p to‘g‘ri chiziq mediananing davomini K nuqtada, balandlikning davomini M nuqtada va BA ning davomini N nuqtada kesadi. $\frac{KM}{CN}$ nisbat katta bo‘lishi uchun p va AC to‘g‘ri chiziqlar orasidagi burchak nimaga teng bo‘lishi kerak?

4. $y = x^4$ egri chiziqning $O(0,0)$ nuqtasidagi egriligini hisoblang.

5. $\frac{x^2}{p} + \frac{y^2}{q} = 1$ $(p + q = 1)$ chiziqlar oilasi o‘ramasini toping.

25-variant

1. ABC uchburchakning A va B uchlaridan o‘tuvchi aylana BC tomonni D nuqtada kesadi. B va C uchlaridan o‘tuvchi aylana esa AB tomonni E nuqtada va birinichi aylanani boshqa F nuqtada kesib o‘tadi. A, E, D, C nuqtalar markazi O nuqtada bo‘lgan aylanaga tegishli ekanligi ma’lum bo‘lsa, $\angle BFO$ ni toping.

2. $a_1, a_2, ..., a_{2013}$ butun sonlar bo‘lsa,

$$\left|(a_1 - a_2)(a_2 - a_3)(a_3 - a_4)...(a_{2012} - a_{2013})(a_{2013} - a_1)\right|$$

ko‘paytmaning juft son ekanini isbotlang (bu yerda $|x|$- sonning moduli).

3. Limitni hisoblang: $\lim_{n\to\infty} \int_0^{\pi} \cos x^n dx$.

4. 10-tartibli kvadrat matritsaning barcha elementlari butin sonlar bo‘lib, ulardan kamida 92 tasi toq sonlar bo‘lsa, u holda bu matritsa determinanti juft son ekanligini isbotlang.

5. $f(x)$ funksiya $[0;2\pi]$ kesmada uzluksiz va botiq bo‘lsa, u holda quyidagini isbotlang: $\int_0^{2\pi} f(x)\cos x\, dx \geq 0$.

26-variant

1. Fazoda $x = 0$, $y = 0$, $z = 0$ va $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ tekisliklar bilan chegaralangan piramidaga ichki chizilgan shar radiusini toping.

2. Aytaylik, $a_1, ..., a_n$ musbat sonlar bo‘lsin. U holda $x^n - a_1 x^{n-1} - ... - a_n$ ko‘phad yagona musbat ildizga ega ekanligini isbotlang.

3. Quyidagi shartni qanoatlantiruvchi barcha $f : [0;1] \to R$ uzluksiz funksiyalarni toping: $\int_0^1 f(x)dx = 2\int_0^1 (f(x^4))^2 dx + \frac{2}{7}$.

4. a_n ketma-ketlik uchun $a_1 = c$ va $a_n = \frac{n+1}{4^n} a_1$ bo'lsa, $\sum_{n=1}^{\infty} a_n$ yig'indini hisoblang.

5. Tenglamaning barcha butun yechimlari sonini toping.

$$\cos\left(\frac{\pi}{8}(3x - \sqrt{9x^2 + 160x + 800})\right) = 1$$

27-variant

1. $\sqrt{b^5}(8x - x^2 - 16) + \frac{\sqrt{b}}{8x - x^2 - 16} \geq -\frac{2}{3}b|\cos \pi x|$ tengsizlik bir yechimga ega bo'ladigan b ning eng katta qiymatin toping.

2. Integralni hisoblang. $I = \int_0^{\pi/2} \cos(2012x) \cdot (\cos x)^{2010} dx$.

3. ABC to'g'ri burchakli uchburchakda AB va BC katetlar mos ravishda 3 va 4 ga teng. BC katetda shunday D nuqta olinganki, $BD : DC = 3 : 2$ ga teng. $\overrightarrow{AB}$ va $\overrightarrow{AD} + \overrightarrow{AC}$ vektorlarning skalyar ko'paytmasini toping.

4. $EKUK(a,b,c) = 2^{2011}$ bo'lsa, $EKUB(ab, bc, ca)$ ifoda nechta qiymat qabul qila oladi?

5. $x_1, x_2, ..., x_n$ nomanfiy sonlari uchun $\frac{x_1}{\sqrt{1}} + \frac{x_2}{\sqrt{2}} + ... + \frac{x_n}{\sqrt{n}} = 1$ bo'lsa, $x_1^2 + x_2^2 + ... + x_n^2$ ning eng katta qiymatini toping.

28-variant

1. $\frac{x+2}{3} = \frac{y+1}{-2} = \frac{z}{5}$ to'g'ri chiziq orqali o'tuvchi va $3x + 3y - z + 1 = 0$ tekislikka perpendykular tekislikning tenglamasini tuzing.

2. $\int_2^{+\infty} \frac{dx}{x^{n+1}} = \frac{1}{n \cdot 2^n}$ dan foydalanib, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 2^2} + ... + \frac{1}{n \cdot 2^n} ...$ hisoblang.

3. Integralni hisoblang. $I = \int_2^4 \frac{\sqrt{\ln(9-x)}dx}{\sqrt{\ln(9-x)} + \sqrt{\ln(3+x)}}$

4. 8 kishilik talabalar u gruhida 3 tasi Toshkentga 2 tasi Samarqandga, qolganlari Buxoroga sayohat qiladigan bo'ldilar. Sardor va Rustam ismli 2 talaba bir shaharga boradigan guruhda bo'lmasligi sharti bilan nechta guruh hosil qilish mumkin?

5. $y^2 = 3x$ parabolaga muntazam uchburchak ichki chizilgan. Bu uchburchakning yuzini hisoblang.

4-§. Olimpiada testlari

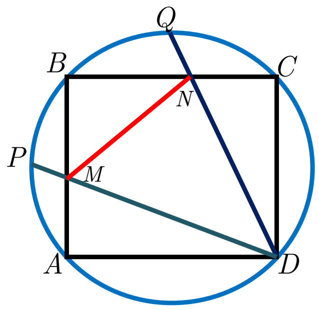
1. 2^{2013} sonining oxirgi 2 ta raqamini toping.

A) 32 B) 72 C) 92 D) 12

2. Noto‘g‘ri javobni toping

A) $(shx)' = chx$ B) $(chx)' = shx$ C) $(tgx)' = \sec^2 x$ D) $(ctgx)' = \cos ec^2 x$.

3. Tomoni 4 ga teng $ABCD$ kvadratga tashqi chizilgan aylanadagi AB va BC yoylar o‘rtalari, mos ravishda, P va Q bo‘lsin. Agar DP va DQ kesmalar AB va BC ni mos ravishda M va N da kessa $MN = ?$



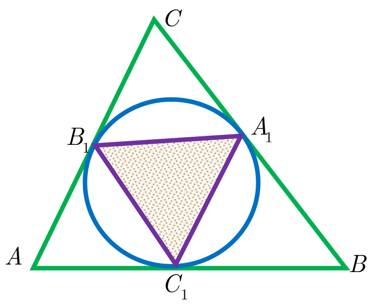
A) $8(\sqrt{2}-1)$

B) $4(\sqrt{2}-1)$

C) $4(\sqrt{2}+1)$

D) $6(\sqrt{2}-1)$

4. $\triangle ABC$ da $AB = 7, BC = 6$ va $CA = 5$ bo‘lsin. ABC uchburchakka ichki chizilgan aylana AB, BC, CA tomonlarga mos ravishda C_1, A_1, B_1 nuqtalarda urinsa $A_1B_1C_1$ uchburchak yuzini toping.



A) $\frac{8\sqrt{6}}{7}$

B) $\sqrt{6}$

C) $\frac{48\sqrt{6}}{35}$

D) $\frac{47\sqrt{6}}{35}$

5. Muntazam uchburchakning ichida olingan nuqtadan uning uchlarigacha bo‘lgan masofalar $3, 4$ va $\sqrt{13}$ ga teng bo‘lsa, uning tomoni nechaga teng?

A) 6 B) $\sqrt{37}$ C) $2\sqrt{13}$ D) $\sqrt{38}$

6. Hisoblang : $\int_0^{2008} x(x-4)(x-8)\cdot \ldots \cdot (x-2008)\,dx$.

A) 0 B) -1 C) 1 D) 4.

7. $P(x) = x^5 - 2x^4 + x^3 - x + 3$ ko'phadni $x^2 + 2$ ga bo'lganda qanday qoldiq qoladi?

A) $x + 2$ B) $2x + 1$ C) $x - 5$ D) $x + 3$

8. Agar $x = a\cos t$, $y = a\sin t$ bo'lsa y' ni toping

A) $y' = -tgt$ B) $y' = -ctgt$ C) $y' = -a\cos t$ D) $y' = -a\sin t$

9. $y = 8 - x^2$ va $y = x^2$ parabolalar qanday burchak ostida kesishadi?

A) $arctg(5/7)$ va $arctg(-5/7)$

B) $arctg(15/11)$ va $arctg(-15/11)$

C) $arctg(8/15)$ va $arctg(-8/15)$

D) aniqlab bo'lmaydi

10. $\int\limits_{-\pi/4}^{\pi/4} \frac{x^{2n+1} + x^{2n-1} + ... + x^5 + 3x^3 - x + 1}{\cos^2 x} dx = ?$

A) 2 B) 1 C) 0 D) 8

11. Integralni hisoblang: $\int\limits_0^1 \frac{\ln(1+x)}{x} dx$.

A) $\frac{\pi^2}{12}$ B) $\frac{\pi^2}{6}$ C) $\frac{\pi^2}{4}$ D) $\frac{\pi^2}{24}$

12. $u = \begin{cases} \frac{1}{x+y} e^{-\frac{1}{|x+y|}}, & agar \ x + y \neq 0, \\ a, & agar \ x + y = 0, \end{cases}$ funksiya $\mathbb{R}^2$ da uzluksiz bo'ladigan a ning qiymatini toping.

A) 2 B) 1 C) 0 D) har qanday qiymatida

13. $T = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 3 & 0 \end{pmatrix}$ bo'lsa, $(I - T)^{-1}$ ni T orqali ifodalang (I-birlik matritsa).

A) $I + T$ B) $I + T + T^2$ C) $I - 3T + 3T^2$ D) $I - T$

14. $\varphi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$ tenglik bilan A to'plamning xarakteristik funksiyasi kiritiladi. $\varphi_{A\cup B}(x)$ funksiyani $\varphi_A(x)$ va $\varphi_B(x)$ funksiyalar orqali ifodalang.

A) $\varphi_A(x) + \varphi_B(x)$ B) $\varphi_A(x) \cdot \varphi_B(x)$

C) $\max\left(\varphi_A(x),\varphi_B(x)\right)$ D) $\min\left(\varphi_A(x),\varphi_B(x)\right)$

15. Tomoni a ga teng bo‘lgan $ABCD$ kvadrat berilgan. BC va CD tomonlarda M va N nuqtalar, mos ravishda, shunday olinganki, bunda $BM = 3MC$, $2CN = ND$. AMN uchburchakka ichki chizilgan aylana radiusini toping.

A) $\frac{3a}{2(5+\sqrt{13})}$ B) $\frac{3a}{2(5-\sqrt{13})}$ C) $\frac{3a}{2(2+\sqrt{13})}$ D) $\frac{3a}{\sqrt{5}}$

16. $\sqrt{a^2+b^2}+\sqrt{(a-\sqrt{2010})^2+(b-\sqrt{15})^2}$ ifodaning eng kichik qiymatini toping.

A) 45 B) 46 C) 48 D) 50

17. To‘g‘ri burchakli Dekart koordinatalari sistemasida $M\left(1;-\sqrt{3}\right)$ nuqta berilgan. Agar qutb koordinatalar boshi bilan qutb o‘qi abssissa o‘qinining musbat yo‘nalishi bilan ustma-ust tushsa, M nuqtaning qutb koordinatalarini toping.

A) $(2;\pi/3)$ B) $(2;5\pi/3)$ C) $(2;3\pi/4)$ D) $(2;\pi/6)$

18. Determinantni hisoblang.

$$\begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 2 & 3 & 0 & 0 \\ 0 & 4 & 3 & 4 & 0 \\ 0 & 0 & 5 & 4 & 5 \\ 0 & 0 & 0 & 6 & 5 \end{vmatrix}$$

A) 270 B) 350 C) 640 D)1200

19. Ketma-ketlik limitini toping. $x_1 = \frac{\pi}{2},\ x_n = x_n + \cos x_n - \frac{1}{2},\ n \in \mathbb{N}$.

A) 1 B) $\pi/6$ C) $\pi/3$ D) 0

20. Quyidagini ko‘paytuvchilarga ajrating,

$(ab+bc+ca)(a+b+c)-abc$

A) $(a+b)(b-c)(c+a)$ B) $(a+b)(b+c)(c-a)$

C) $(a-b)(b-c)(c+a)$ D) $(a+b)(b+c)(c+a)$

21. $y = 2^x$ bo‘lsa $y^{(n)}$ ni toping.

A) $y^{(n)} = 2^x \ln^n 2$, B) $y^{(n)} = 0$ C) $y^{(n)} = 2^x$ D) $y^{(n)} = x^n 2^x$

22. Quyidagini limitni hisoblang: $\lim\limits_{x\to+\infty}\dfrac{\int\limits_0^x (arctgt)^2\,dt}{\sqrt{x^2+1}}$

A) $\dfrac{\pi^2}{4}$ B) $\dfrac{\pi}{4}$ C) $\dfrac{\pi}{3}$ D) $\dfrac{\pi^2}{2}$

23. To‘g‘ri burchakli ABC uchburchakning C uchi to‘g‘ri burchak bo‘lib BC kateti D va E nuqtalar orqali teng uch bo‘lakka bo‘lingan. Agar $BC=3AC$ bo‘lsa, $\angle ADC$ va $\angle ABC$ burchaklarning yig‘indisini toping.

A) 60^0 B) 45^0 C) 30^0 D) 90^0

24. $\dfrac{4x^2+4y^2+x^2y^2}{x^4+y^4+16}\geq 1$ tengsizlikni qanoatlantiruvchi barcha (x,y) sonlar juftliklari nechta?

A) 1 B) 2 C) 3 D) 4

25. Noto‘g‘ri tasdiqni aniqlang.

A) Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz, $(a;b)$ intervalda differentsiallanuvchi va $f(a)=f(b)$ bo‘lsa u holda $(a;b)$ intervalda shunday $x=c$ nuqta topiladiki $f'(c)=0$ bo‘ladi;

B) Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz va$(a;b)$ intervalda differensiallanuvchi bo‘lsa, u holda bu intervalda shunday $x=c$ nuqta topiladiki $f(b)-f(a)=f'(c)(b-a)$ bo‘ladi;

C) Agar $f(x)$ va $g(x)$ funksiyalar $[a;b]$ kesmada uzluksiz, $(a;b)$ intervalda differensiallanuvchi va $g'(x)\neq 0$ bo‘lsa, u holda $(a;b)$ intervalda shunday $x=c$ nuqta topiladiki, $\dfrac{f(b)-f(a)}{g(b)-g(a)}=\dfrac{f'(c)}{g'(c)}$ bo‘ladi

D) Noto‘g‘ri tasdiq keltirilmagan.

26. a, b, c lar arifmetik progressiyani hosil qiladi. Tomonlari a, b, c bo‘lgan uchburchakka ichki chizilgan aylana radiusini toping.

A) $r=1$ B) $r=\dfrac{h_a+h_b-h_c}{2}$

C) $r=\dfrac{1}{3}h_b$ D) masala yechimga ega emas

27. Balandliklari $4,6,4$ bo‘lgan uchburchakning yuzini toping.

A) 24 B) 12 C) $12\sqrt{3}$ D) $9\sqrt{2}$

28. $\overrightarrow{CA} = \vec{a}$ va $\overrightarrow{CB} = \vec{b}$ vektorlar berilgan. P - A nuqtaning BC to‘g‘ri chiziqdagi ortogonal proyeksiyasi. $\overrightarrow{CP}$ vektorni toping.

A) $\dfrac{\left(\vec{a}, \vec{b}\right) \cdot \vec{b}}{\vec{b}^2}$ B) $\dfrac{\left(\vec{a}, \vec{b}\right)}{\vec{a}^2}$ C) $\dfrac{\left(\vec{a}, \vec{b}\right)}{\vec{b}^2}$ D) $\dfrac{\left(\vec{a}, \vec{b}\right) \cdot \vec{b}}{\vec{a}^2}$

29. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ ellips berilgan. $\left(-2;1\right)$ nuqta orqali shu nuqtada teng ikkiga bo‘linuvchi vatar o‘tkazilsin.

A) $9x - 8y + 25 = 0$ B) $9x + 8y + 10 = 0$

C) $8x + 9y - 26 = 0$ D) $8x - 9y + 25 = 0$

30. λ ning qanday qiymatida $x^3 - \lambda x + 2$ va $x^2 + \lambda x + 2$ ko‘phadlar umumiy ildizga ega bo‘ladi?

A) $\lambda_1 = 3, \quad \lambda_2 = -1$, B) $\lambda_2 = 3, \quad \lambda_1 = 1$,

C) $\lambda_1 = 1, \quad \lambda_2 = -3$, D) $\lambda_1 = -3, \quad \lambda_2 = -1$.

31. Noto‘g‘ri tasdiqni aniqlang.

A) Agar $f'(x_0) > 0$ bo‘lsa, $f(x)$ funksiya x_0 nuqtada o‘suvchi bo‘ladi;

B) Agar $f'(x_0) < 0$ bo‘lsa, $f(x)$ funksiya x_0 nuqtada kamayuvchi bo‘ladi;

C) Agar $f''(x_0) > 0$ bo‘lsa, $f(x)$ funksiya x_0 maksimumga erishadi;

D) Agar $f'(x_0) = 0$ bo‘lsa, $f(x)$ funksiya x_0 nuqtada ekstremumga erishadi.

32. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ matritsaga teskari matritsaning izini toping.

A) $\dfrac{ad}{ad - bc}$; B) $\dfrac{bc}{ad - bc}$; C) $\dfrac{ad + bc}{ad - bc}$; D) $\dfrac{a + d}{ad - bc}$;

33. $x^2 + y^2 + ay = 0$ $\left(a > 0\right)$ aylana markazidan $y = 2\left(a - x\right)$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

A) $\dfrac{a\sqrt{5}}{4}$ B) $\dfrac{a\sqrt{3}}{2}$ C) $\dfrac{a\sqrt{5}}{2}$ D) $\dfrac{\sqrt{5}}{2a}$

34. Ushbu halqa butunlik sohasi bo‘lmaydi.

A) $C\left[-1;1\right]$ B) $\mathbb{Z}$ C) $\mathbb{Q}$ D) $\mathbb{R}$

35. Funksiyaning qavariqlik oralig‘ini aniqlang: $y = x^5 + 5x - 6$

A) $(-\infty;0)$ B) $(0;+\infty)$ C) $(-\infty;+\infty)$ D) $(-4/5;+\infty)$

36. Quyidagi aniq integralni hisoblang: $\int_0^6 [x]\sin\frac{\pi x}{6}\,dx$.

A) $\frac{3}{\pi}$ B) $\frac{\pi}{3}$ C) $\frac{30}{\pi}$ D) $\frac{\pi}{30}$

37. $\frac{x+4}{x+1} > 2-x$ tengsizlikni yeching.

A) (-6;-3) B) (0;1) C) $(-\infty;+\infty)$ D) $(-1;+\infty)$

38. Integralni hisoblang: $\int_0^{+\infty} x^{2n}e^{-x^2}dx$ n-butun musbat son.

A) $\frac{(2n-1)!!}{2^{n+1}}\sqrt{\pi}$; B) $\frac{(2n-1)!!}{2^{n}}\sqrt{\pi}$;

C) $\frac{(2n+1)!!}{2^{n+1}}\sqrt{\pi}$; D) $\frac{(2n-1)!}{2^{n+1}}\sqrt{\pi}$.

39. Limitni hisoblang: $\lim\limits_{n\to\infty}\sum_{k=1}^{n}\frac{k!k}{(n+1)!}$

A) 1 B) e C) $\frac{1}{e}$ D) ∞

40. Limitni hisoblang: $\lim\limits_{n\to\infty}(n^2+n-\sum_{k=1}^{n}\frac{2k^3+8k^2+6k-1}{k^2+4k+3})$

A) $\frac{5}{12}$ B) ∞ C) 0 D) $\frac{2}{5}$

41. Agar $A=\begin{pmatrix}1&1&2\\1&3&1\\4&1&1\end{pmatrix}$ va E uchinchi tartibli birlik matritsa bo‘lsa, $2A^2+3A+5E$ matritsa ko‘phadning qiymatini hisoblan

A) $\begin{pmatrix}1&1&8\\1&27&1\\64&1&1\end{pmatrix}$ B) $\begin{pmatrix}28&15&16\\19&36&15\\30&19&28\end{pmatrix}$ C) $\begin{pmatrix}21&13&12\\15&24&18\\24&13&31\end{pmatrix}$

D) $\begin{pmatrix}21&13&12\\0&-24&18\\24&0&-31\end{pmatrix}$

42. Toq funksiyani ko‘rsating.

A) $f(x)=-2\cos(x/3)+1$

B) $f(x) = tg3x + \cos 4x$

C) $f(x) = \lg \cos 2x$

D) $f(x) = x^4 \sin 7x$

43. $u = \frac{y}{x} \text{tg} \frac{x}{x+y}$ bo'lsa, $\lim\limits_{x\to 0} \lim\limits_{y\to 0} u$, $\lim\limits_{y\to 0} \lim\limits_{x\to 0} u$, $\lim\limits_{\substack{x\to 0 \\ y\to 0}} u$ larni toping.

A) 0; 1; mavjud emas;

B) mavjud emas; mavjud emas; 0;

C) 0; mavjud emas; 0;

D) 1; 0; mavjud emas;

44. Quyidagini aniqmas integralni hisoblang:

$$\int \frac{dx}{\sin(x+5)\cos(x+7)}$$

A) $\frac{1}{\cos 2} \ln \left| \frac{\sin(x+5)}{\sin(x+7)} \right|$ B) $\frac{1}{\cos 2} \ln \left| \frac{\sin(x+5)}{\cos(x+7)} \right|$

C) $\frac{2}{\cos 2} \ln \left| \frac{\sin(x+5)}{\cos(x+7)} \right|$ D) $\frac{1}{\sin 2} \ln \left| \frac{\sin(x-5)}{\sin(x+7)} \right|$

45. $f(x)$ funksiya $[0,+\infty]$ da uzluksiz. $f(f(f(x))) = x^3$ bo'lsa, $f(x)$ funksiyani toping.

A) x^3 B) $\sqrt[3]{x}$ C) $x^{\sqrt{3}}$ D) $x^{\sqrt[3]{3}}$

46. Quyidagini aniqmas integralni hisoblang:

$$\int \frac{dx}{\cos(x+a)\cos(x+b)}$$

A) $\frac{1}{\sin(a-b)} \ln \left| \frac{\cos(x+b)}{\cos(x+a)} \right|$ B) $\frac{1}{\sin(a-b)} \ln \left| \frac{\cos(x+a)}{\cos(x+b)} \right|$

C) $\frac{1}{\sin(b-a)} \ln \left| \frac{\sin(x+b)}{\sin(x+a)} \right|$ D) $\frac{1}{\sin(a-b)} \ln \left| \frac{\cos(x-b)}{\cos(x+a)} \right|$

47. $y = -x^3$ funksiyani abssissasi $x = 1/2$ bo'lgan nuqtasidagi egriligini hisoblang

A) $3/2$ B) $192/125$ C) $321/257$ D) 1

48. $r = i\cos t + j\sin t + k$ vektor funksiya hosilasini toping.

A) $-i\sin t + j\cos t$ B) $i\sin t + j\cos t + k$

C) $-it - j\sin t - k$ D) 0

49. $K - ABCD$ kvadrat AB tomonining o'rtasi, l nuqta AC diagonalni $AL:LC = 3:1$, $AL:LC = 3:1$ kabi nisbatta bo'ladi. KLD burchakni toping.

A) 135^0 B) 105^0 C) 90^0 D) 75^0

50. $r = t^2 i + t^3 j + t^6 k$ egri chiziqni urinma vektorini toping.

A) $\tau = i/3 - j/3 - k/3$

B) $\tau = -3i/5 + 3j/5 + 2k/5$

C) $\tau = -i/11 + 4j/11 + 3k/11$

D) $\tau = 2i/7 + 3j/7 + 6k/7$

51. $x = a\cos t, y = a\sin t, Z = \sqrt{R^2 - a^2 t}$, $R > a > 0$, vint chizig'ining boshnormali topilsin.

A) $v = -i\cos t - j\sin t$

B) $v = i\cos t + j\sin t + k$

C) $v = i\cos t - j\sin t - k$

D) $v = i\cos t - j + k\sin t$

52. Kvadratga tashqi parallelogramm chizilgan. Parallelogrammning uchidan kvadratning tomoniga tushirilgan perpendikularlar qanday figurani hosil qiladi.

A) Parallelogramm B) Kvadrat

C) Trapetsiya D) Romb

53. $y = \frac{9}{\pi}\arccos\frac{3\sqrt{2} + \sin x - \cos x}{4\sqrt{2}}$ funksiyaning qiymatlar sohasini aniqlang.

A) $[0;3)$ B) $[0,1;3)$ C) $[0;3]$ D) $(0;3)$

54. $a_n = \frac{1}{(n+1)\sqrt{n} + n\sqrt{n+1}}$ bo'lsa, $a_1 + a_2 + a_3 + ... + a_{99}$ ni hisoblang.

A) $\frac{9}{10}$ B) 1 C) $\frac{1}{10}$ D) 10

55. Shunday n- natural sonni topingki, shu songacha bo'lgan toq sonlar yig'indisidan 50 marta kichik bo'lsin.

A) 100 B) 200 C) 300 D) 250

56. $\bigcap_{k=1}^{\infty}\left[\frac{1}{k},1\right]$ yig'indini toping.

A) $\{1\}$. B) $[0,1]$ C) $\varnothing$ D) $(0,1]$

57. $2^{99}+2^{9}$ sonini 49 ga bo'lgandagi qoldiqni toping.

A) 0 B) 40 C) 39 D) 1

58. $\int_0^3 \operatorname{sgn}(x-x^3)dx$ integralni hisoblang.

A) 10 B) -3 C) -1 D) 0

59. Quyidagini aniq integralni hisoblang, $[x]$ -soning butun qismi

$$\int_1^{2013} \ln[x]dx$$

A) $\ln 2013!$ B) $2013!$ C) $\ln 2012!$ D) $2012!$

60. Quyidagini aniq integralni hisoblang: $\int_0^{\pi} x\operatorname{sgn}(\cos x)dx$

A) $-\frac{\pi^2}{4}$ B) $-\frac{\pi}{4}$ C) $-\frac{\pi}{3}$ D) $-\frac{\pi^2}{2}$

61. $a=\log_9 10$ va $b=\lg 11$ sonlar orasidagi munosabatni aniqlang.

A) $a>b$ B) $a<b$ C) $a=b$ D) $a=b+1$

62. Hisoblang: $\cos\frac{\pi}{7}-\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}$

A) $\frac{1}{2}$ B) 0 C) 1 D) $\frac{1}{\sin\frac{\pi}{7}}$

63. $500!=13^m$ (m-natural son). m ning eng katta qiymatini toping.

A) 40 B) 41 C) 42 D) 43

64. ABC uchburchakda BH balandlik va BM medianalar. Agar $AB=1, BC=2$ va $AM=BM$ bo'lsa, $\angle MBH$ burchakni hisoblang.

A) $\arccos 0,8$ B) $\arcsin 0,8$ C) $\arcsin 0,6$ D) $\arccos 0,6$

65. Ushbu $(x-y)^3+(y-z)^3+(z-x)^3=30$ tenglamani qanoatlantiruvchi (x,y,z) butun sonlar uchliklari nechta.

A) 0 B) 3 C) 4 D) 2

66. Uchburchakka tashqi chizilgan aylana radiusi 2 ga teng bo‘lsa, shu uchburchak medianalari kvadratlari yig‘indisining eng katta qiymatini toping.

A) 27 B) 24 C) 16 D) 4

67. m ning qanday qiymatida $P(x)=x^{5n}-3x^{3n}+mx^{2n}+x^n+2$ ko‘phad x^n+2 ga qoldiqsiz bo‘linadi.

A) 2 B) 1 C) 5 D) 3

68. Burchaklaridan hech biri o‘tmas bo‘lmagan uchburchakka tashqi chizilgan aylana radiusi 5 ga teng bo‘lsa, u holda shu uchburchak medianalari yig‘indisining eng kichik qiymatini toping.

A) 20 B) 10 C) $22,5$ D) 5

69. Funksiyani aniqlanish sohasini toping: $u=\arcsin(x/y^2)$

A) $D=\{(x;y):-1\le x\le 1,-1\le y^2<0,0<y^2\le 1\}$

B) $D=\{(x;y):0\le x/y^2\le 1\}$

C) $D=\{(x;y):-y^2\le x\le y^2\}/\{(0;0)\}$

D) $D=\{(x;y):-1\le x\le 1,-1\le y^2\le 1\}$

70. Yig‘indini hisoblang: $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+...+\frac{99}{100!}$

A) $1-\frac{1}{100!}$ B) $1+\frac{1}{100!}$ C) $\frac{1}{2}+\frac{1}{100!}$ D) $\frac{1}{2}-\frac{1}{100!}$

71. Uchburchakka tashqi chizilgan aylana radiusi $\sqrt{24}$ ga teng bo‘lsa, ichki chizilgan aylana radiusining eng katta qiymatini toping.

A) $\sqrt{6}$ B) $3\sqrt{2}$ C) $2\sqrt{3}$ D) $2\sqrt{2}$

72. Limitni hisoblang: $\lim\limits_{n\to\infty}\sqrt[n]{\frac{3^{3n}(n!)^3}{(3n)!}}$

A) 1 B) 0 C) ∞ D) e

73. Limitni hisoblang: $\lim\limits_{n\to\infty}(n^2+n-\sum_{k=1}^{n}\frac{2k^3+8k^2+6k-1}{k^2+4k+3})$

A) $\frac{5}{12}$ B) $\propto$ C) 0 D) $\frac{2}{5}$

74. Limitni hisoblang: $\lim_{n\to\infty}\frac{\ln(1+x+x^2+...+x^n)}{nx}$, $n\in\mathbb{N}$

A) $\frac{1}{n}$ B) n C)1 D) 0

75. Limitni hisoblang: $\lim_{n\to\infty}\frac{a+\sqrt{a}+\sqrt[3]{a}+...+\sqrt[n]{a}-n}{\ln n}$

A) $\ln a$ B) 1 C) e D) $\ln\frac{1}{a}$

76. Ushbu $y=\cos(x\sqrt{2})+\cos\frac{x}{\sqrt{2}}$ funksiyaning eng kichik davrini toping.

A) 2π B) 3π C) $2\pi\sqrt{2}$ D) $3\pi\sqrt{3}$

77. $\varphi(n)$ orqali n dan kichik va n bilan o'zaro tub sonlarning sonini belgilasak, $\varphi(2013)$ni toping.

A) 1000 B) 1100 C) 1200 D) 1260

78. ''Al-jabr va al muqobala'' asari muallifining ismi kim?

A) Muso B) Ahmad C)Muhammad D) Al-Xorazmiy

79. Qaysi biri katta? π^e yoki e^π

A) $\pi^e > e^\pi$ B) $\pi^e < e^\pi$ C) $\pi^e = e^\pi$ D) $\pi^e \approx e^\pi$

80. $u=x^2-3xy-4y^2-x+2y+1$. $\frac{du}{dx}$ va $\frac{du}{dy}$ xususiy hosilalar hisoblansin

A) $\frac{du}{dx}=-3x+2, \frac{du}{dy}=-3y-1$

B) $\frac{du}{dx}=2x-1, \frac{du}{dy}=8y+2$

C) $\frac{du}{dx}=2x-3y-1, \frac{du}{dy}=-3x-8y+2$

D) $\frac{du}{dx}=-3x-4y-1, \frac{du}{dy}=2x-3y-4$

81. Agar $k=2010^2+2^{2010}$ bo'lsa, (k^2+2^k) ning oxirgi raqamini toping.

A) 0 B) 2 C) 4 D) 6

82. $z=\ln\left(x^2+y^2\right)$ funksiyadan $M(3;4)$ nuqtada z funksiyaning gradienti yo‘nalishidagi hosilasini hisoblang.

A) $1/2$ B) $1/3$ C) $2/3$ D) $3/4$

83. ABC uchburchakda $AC=3, BC=4$ va $AB=5$. D nuqta AB kesmada olingan bunda CD kesma o‘tkir burchak bissektrissasi. ADC va BCD uchburchaklarning ichkichizilgan aylana radiuslari r_a va r_b bo‘lsa, u holda $\frac{r_a}{r_b}$ ni hisoblang.

A) $\frac{1}{28}\left(10-\sqrt{2}\right)$ B) $\frac{3}{56}\left(10-\sqrt{2}\right)$ C) $\frac{1}{14}\left(10-\sqrt{2}\right)$ D) $\frac{3}{28}\left(10-\sqrt{2}\right)$

84. $z=x^2-2xy+y^2-x+2y$ sirtning $M(1;1;1)$ nuqtasidan o‘tgan urinma tekislik tenglamasini ko‘rsating.

A) $x-2y+z=0$ B) $2x+y-3z=0$

C) $x+y+z-3=0$ D) $2x-2y-3z+3=0$

85. $f(x)$ funksiya $[0;2]$ va $[0;1]$ oraliqda aniqlangan bo‘lsa, $g(x)=1-f(x+1)$ funksiya qaysi oraliqda aniqlangan?

A) $[-1,1],[-1,0]$ B) $[-1,1],[0,1]$ C) $[0.2],[-1.0]$ D) $[1,3],[0,1]$

86. $z=x^2+xy+y^2-3x-6y$ funksiyaning statsionar nuqtalarini toping.

A) $(1;2)$ va $(2;1)$ B) $(2;5)$

C) $(0;5)$ D) Statsionar nuqtasi mavjud emas

87. Agar x va y erkli o‘zgaruvchilar $2x+3y-5=0$ tenglama bilan bog‘langan bo‘lsa z=xy funksiyaning eng katta qiymatini toping.

A) $Z_{\max}=1$

B) $Z_{\max}=25/24$

C) $Z_{\max}=17/15$

D) $Z_{\max}=1,2$

88. $7^{x+7}=8^x$ tenglama ildizi $x=\log_b 7^7$ bo‘lsa, b-ni toping.

A) $7/15$ B) $7/8$ C) $8/78$ D) $15/8$

89. Noto‘g‘ri tenglikni ko‘rsating. $\int\frac{f'(x)}{f(x)}dx=\ln\left|f(x)\right|+C$

A) $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + C$ B) $\int \frac{A}{x-a}dx = A\ln|x-a| + C$

C) $\int \frac{f'(x)}{f^2(x)}dx = \ln|f^2(x)| + C$

90. Aniqmas integralni hisoblang: $\int \frac{dx}{\sin^4 x}$

A) $-ctgx - \frac{1}{3}ctg^3x + C$ B) $tgx + \frac{1}{3}tg^3x + C$

C) $ctgx - \frac{1}{3}tg^3x + C$ D) $tgx + \frac{1}{3}ctg^3x + C$

91. ABC -uchburchakda $\cos(2A - B) + \sin(A + B) = 2$ va $AB = 4$ bo‘lsa, BC ni toping.

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 2 D) $2\sqrt{2}$

92. Noto‘g‘ri tenglikni ko‘rsating.

A) $\int_a^b f(x)dx = -\int_b^a f(x)dx$ B) $\int_a^a f(x)dx = 0$

C) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

D) $\int_a^b [f(x) \cdot g(x)]dx = \int_a^b f(x)dx \cdot \int_a^b g(x)dx$

93. Xosmas integralni hisoblang: $\int_{-\infty}^{+\infty} \frac{dx}{1+x^2}$

A) π B) $\pi / 2$ C) 0 D) hosmas integral uzoqlashuvchi

94. $ABCD$ - to‘rtburchakda $AB = 5, BC = 17, CD = 5, DA = 9$ AB=5 va BD butun son bo‘lsa uni toping.

A) 11 B) 12 C) 13 D) 14

95. $i + 2i^2 + 3i^3 + ... + ni^n = 48 + 49i$ bo‘lsa $n \in \mathbb{N}$ ni toping (bunda $i^2 = -1$).

A) 24 B) 48 4 C) 49 D) 97

96. $\{a_n\}$ ketma-ketlikda $a_1 = 1$ va $a_2 = \frac{1}{\sqrt{3}}, a_{n+2} = \frac{a_n + a_{n+1}}{1 - a_n a_{n+1}}$, $|a_{2009}|$ ni toping.

A) 0 B) $2 - \sqrt{3}$ C) $\frac{1}{\sqrt{3}}$ D) $2 + \sqrt{3}$

97. a va b haqiqiy sonlar uchun, $a * b = (a-b)^2$ bo'lsa, $(x-y)^2 * (y-x)^2$ ni toping.

A) 0 B) x^2+y^2 C) $2x^2$ D) $2y^2$

98. Ushbu $\sqrt{x^2-6x+13}+\sqrt{x^2-14x+58}$ ifodaning eng kichik qiymatini toping.

A) $\sqrt{41}$ B) 51 C) $\sqrt[3]{25}$ D) $\sqrt{43}$

99. ABC -uchburchakda $A=(3,0), B=(0,3)$ va C uchi $x+y=7$ to'g'ri chiziqda bo'lsa, S_{ABC} ni toping.

A) 6 B) 8 C) 10 D) 12

100. Ushbu $\frac{1}{x}+\frac{1}{y}=\frac{1}{2013}$ tenglama nechta natural yechimga ega?

A) 9 B) 10 C) 20 D) 27

101. a, b, c, d, e -lar bir-biridan farqli butun sonlar $(6-a)(6-b)(6-c)(6-d)(6-e)=45$ bo'lsa, $a+b+c+d+e$?

A) 5 B) 17 C) 25 D) 27

102. nechta (a,b) butun sonlar uchun $EKUB(a,b)=1$ va $\frac{a}{b}+\frac{14b}{9a}$ butun son bo'ladi?

A) 4 B) 6 C) 9 D) 12

103. Aylanaga $ABCD$ to'rtburchak ichki chizilgan. Agar $AB=3, BC=4, CD=7, AD=5$ bo'lsa, uning diagonallari ko'paytmasini toping.

A) 47 B) 43 C) 41 D) 42

104. $\int_{-\pi}^{\pi} \sin^7 x \cos^7 x dx$ nimaga teng?

A) $-1/7$ B) 0 C) $1/7$ D) $1/49$

105. $|\sin x| > |\cos x|$ tengsizlikni yeching.

A) $(\frac{\pi}{4}+\pi n; \frac{3\pi}{4}+\pi n)$ B) $(\frac{\pi}{4}+2\pi n; \frac{3\pi}{4}+2\pi n)$

C) $(-\frac{\pi}{4}+\pi n; \frac{\pi}{4}+\pi n)$ D) $(\pi n; \frac{\pi}{4}+\pi n)$

106. $\int_0^1 f(x)dx = a$ bo‘lsa $\int_0^1 xf(x^2)dx$ ni hisoblang.

A) $a/2$ B) a C) a^2 D) $2a$

107. Agar $ax = by = cz = k$ va $x + y + z = k^2$ bo‘lsa, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = ?$

A) k^2 B) $\frac{1}{k}$ C) k D) $\frac{1}{k^2}$

108. Tenglama butun sonlarda nechta yechimga ega? $x^2 - y^2 = 1985$

A) 12 ta B) 8 ta C) 10 ta D) 9 ta

109. Diagonallari o‘zaro perpendikular bo‘lgan trapetsiyaning balandligi 4 ga teng. Agar bu trapetsiyaning diaganallaridan birining uzunligi 5 ga teng bo‘lsa, uning yuzini toping.

A) $16\frac{2}{3}$ B) 15 C) $15\frac{2}{3}$ D) $16\frac{1}{3}$

110. Tekislikning biror nuqtasidan to‘g‘ri to‘rtburchakning uchlarigacha bo‘lgan masofalar 5, 12, 13 ga teng. To‘g‘ri to‘rtburchakning yuzini toping.

A) 65 B) 55 C) $\sqrt{780}$ D) 60

111. Quyidagi ifodaning butun qismini toping:

$$\sqrt{2010 + \sqrt{2010 + \sqrt{2010 + ... + \sqrt{2010 + \sqrt{2010}}}}}$$

A) 43 B) 44 C) 45 D) 46 E) 47

112. Ifodani soddalashtiring: $\frac{a^3(c-b) + b^3(a-c) + c^3(b-a)}{a^2(c-b) + b^2(a-c) + c^2(b-a)}$

A) $a - b + c$ B) $a - b - c$ C) $a + b - c$ D) $a + b + c$

113. 2^{p^2} sonini 18 ga bo‘lgandagi qoldiqni toping ($p > 3$ -tub son)

A) 2 B) 3 C) 5 D) 2 va 3

114. Tenglamani yeching: $(x^2 - 2x)^3 + x\sqrt{x(x-2)^3} = 2$

A) $1 + \sqrt{1 + \sqrt[3]{4}}$ B) $1 + \sqrt{2}$ C) $1 - \sqrt{2}$ D) $1 \pm \sqrt{2}$

115. To‘g‘ri burchakli uchburchakning yuzi $\frac{\sqrt{3}}{12}(a^2 + 3b^2)$ kvadrat birlikka ega. Uning burchaklarini toping.

A) $30^0, 60^0, 90^0$ B) $45^0, 45^0, 90^0$ C) $15^0, 75^0, 90^0$ D) $40^0, 50^0, 90^0$

116. Hisoblang: $1+4\cdot 2+7\cdot 2^2+...+67\cdot 2^{22}$

A) $2^{28}+1$ B) $2^{29}-1$ C) $2^{29}-5$ D) $2^{29}+5$

117. $\left[\frac{1^2}{2010}\right];\left[\frac{2^2}{2010}\right];...;\left[\frac{2009^2}{2010}\right]$ ketma-ketlikda nechta turli sonlar uchraydi ($[x]$- x ning butun qismi)

A) 2010 B) 1005 C) 2009 D) 2011

118. Uchburchak tomonlarining uzunliklari: a, b, c. Uchburchakning ichidagi biror nuqtadan o‘tuvchi 3 ta kesishuvchi to‘g‘ri chiziq uchburchakning tomonlariga parallel. Kesishuvchi to‘g‘ri chiziqlarning tomonlari bilan kesilishdan hosil bo‘lgan kesmalarning har biri x ga teng bo‘lsa, x ni toping.

A) $(ab+bc+ca)/(a+b+c)$ B) $abc/(ab+bc+ca)$

C) $2abc/(a^2+b^2+c^2)$ D) $2abc/(ab+bc+ca)$

119. $ABCD$ qavariq to‘rtburchakda ABC, BCD, CDA, DAB uchburchaklarning og‘irlik markazlari ketma-ket tutashtirilgan. Agar $ABCD$ to‘rtburchakning yuzi S bo‘lsa, u holda hosil bo‘lgan to‘rtburchakning yuzini toping.

A) S/3 B) S/5 C) S/9 D) S/6

120. Tomoni n ga teng bo‘lgan kvadrat vertikal va gorizontal bo‘lgan n^2 ta har xil kvadratlarga bo‘lingan. Hosil bo‘lgan chizmada nechta kvadrat sanash mumkin?

A) $[n(n+1)(2n+1)]/6$ B) $n(n+1)/3$

C) $n(n+1)(n+2)/6$ D) $[n(n+1)]/2$

121. $|\vec{a}|=4, |\vec{b}|=8$ va $|\vec{a}-\vec{b}|=10$ bo‘lsa, $|\vec{a}+\vec{b}|$ ning qiymatini toping.

A) $\sqrt{26}$ B) $\sqrt{39}$ C) $2\sqrt{15}$ D) $2\sqrt{9}$

122. p-fazodagi kesmaning uzunligi a, b, c-lar esa uning koordinata tekisliklardagi proyeksiyalari bo‘lsin. $[a+b+c]/p$ nisbatning mumkin bo‘lgan eng katta qiymatini toping.

A) $\sqrt{3}$ B) $\sqrt{6}$ C) $\sqrt{5}$ D) $\sqrt{2}$ E) 2

123. 994/49995 oddiy kasrning o'nli kasrga yoyilmasida 2010 o'rinda qaysi raqam turadi?

A) 0 B) 1 C) 9 D) 8

124. a, b, c sonlar $\left(0, \frac{\pi}{2}\right)$ intervaldan olingan va $\cos a = a$, $\sin(\cos b) = b$, $\cos(\sin c) = c$ tengliklarni qanoatlantirsa, bu sonlarni o'sish tartibida joylashtiring.

A) $a < b < c$ B) $b < a < c$ C) $b < c < a$ D) $a < c < b$

125. tengsizlikni yeching: $\left|\sin x - \sin y\right| + \sin x \cdot \sin y \le 1$

A) $\left(\frac{\pi n}{4}, \frac{\pi m}{4}\right)$ B) $\left(\frac{\pi n}{3}, \frac{\pi m}{3}\right)$ C) $\left(2\pi n, 2\pi m\right)$ D) $\left(\frac{\pi n}{2}, \frac{\pi m}{2}\right)$ $\left(n, m \in \mathbb{Z}\right)$

126. Agar $a = \frac{15}{16} + \frac{24}{25} + \frac{35}{36} + \frac{48}{49}$ bo'lsa, $\frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{49} = ?$

A) $a^2 - 1$ B) $4a^2 - a$ C) $4 - a$ D) $a - 4$

127. $2010!$ songa 19 qanday daraja bilan kiradi?

A) 110 B) 101 C) aniqlab bo'lmaydi D) 120

128. Agar S_n arifmetil progressiyaning birinchi n ta hadining yig'indisi bo'lsa,

$\left(S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n\right)$ ni hisoblang.

A) 1 B) S_n C) n D) 0

129. Agar $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $\left(\left|x\right| \ge 2\right)$ bo'lsa, $f\left(x\right)$ni toping.

A) $f(x) = x^2$ B) $f(x) = (x+1)^2$ C) $f(x) = x^2 - 2$ D) $f(x) = x2$

130. $x^4 + 4x^3 + \frac{5}{2}x^2 - 2x - \frac{3}{16} = 0$ tenglamaning haqiqiy ildizlari joylashgan oraliqni toping.

A) $\left(-2, 1\right)$ B) $\left(-5, 5\right)$ C) $\left(-\frac{3}{4}, \frac{1}{2}\right)$ D) $\left(-\frac{3}{4}, \frac{1}{4}\right)$

131. Berilgan kvadrat teng 49 ta kvadratchalarga bo'lindi. Hosil bo'lgan shaklda nechta kvadrat hosil bo'ladi?

A) 50 B) 140 C) 343 D) 344

132. Hisoblang: $\cos\frac{\pi}{7}-\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}$.

A) $\frac{1}{2}$ B) 0 C) 1 D) $\frac{1}{\sin\frac{\pi}{7}}$

133. $\varphi(n)$ orqali $n$dan kichik va n bilan o'zaro tub sonlar sonini belgilasak, $\varphi(2010)$ ni toping.

A) 2009 B) 528 C) 1005 D) 1001

134. AC va BD diognallari o'zaro perpendikular bo'lgan $ABCD$ to'rtburchakka radiusi 2 ga teng bo'lgan aylana tashqi chizilgan, agar $AB=3$ bo'lsa, CD ni toping

A) $\sqrt{3}$ B) 3 C) $\sqrt{5}$ D) $\sqrt{7}$

135. $\begin{pmatrix}\cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi\end{pmatrix}^{2013}$ matritsaning izini toping.

A) $2\sin\varphi$ B) $\cos\varphi$ C) $\sin\varphi$ D) $2\cos\varphi$

136. ABC uchburchakning medianalari 9 va 12 ga teng medianalari bu uchburchakni 3 ta uchburchakka va 1 ta to'rburchakka bo'ladi. Hosil bo'lgan to'rburchakning yuzi 16 ga teng bo'lsa, berilgan uchburchak yuzini toping.

A) 32 B) 24 C) $\frac{32}{3}$ D) 48

137. ABC uchburchakning medianalari $5, 6, 7$ bo'lsa, bu uchburchak yuzini toping.

A) $8\sqrt{6}$ B) $6\sqrt{3}$ C) $8\sqrt{3}$ D) $6\sqrt{6}$

138. $y=\arccos x$ funksiyaning juft va toq funksiyalar yig'indisi ko'rinishida yozilganda, hosil bo'lgan juft funksiyadan hosila oling.

A) $\frac{1}{\sqrt{1-x^2}}$ B) $-\frac{1}{\sqrt{1-x^2}}$ C) 0 D) $\arcsin x$

139. Quyidagi funksiyani eng kichik qiymatini toping.

$f(x,y)=2^x+2^{1-x-y}+2^y$

A) 2 B) 6 C) 4 D) 5

140. $\bar{a},\bar{b},\bar{c}$ lar birlik vektorlar bo‘lib, $\bar{a}+\bar{b}+\bar{c}=0$ bo‘lsa,

$\bar{a}\cdot\bar{b}+\overline{a\cdot c}+\bar{b}\cdot\bar{c}$

ni toping.

A) 0 B) $\frac{3}{2}$ C) $-\frac{3}{2}$ D) -1

141. Agar $f(x)=x^2+14x+42$ bo‘lsa, $f(f(f(f(x))))=0$ tenglamani yeching.

A) ildizi yo‘q B) $\pm\sqrt[16]{7}-7$ C) $\pm\sqrt[32]{7}+7$ D) $\pm\sqrt[16]{7}+7$

142. $y=3x-9$, $y=0$, $x=5$ chiziqlar bilan chegaralangan figurani, OX o‘qi atrofida aylanishdan hosil bo‘lgan jismni hajmini toping.

A) 26π B) 33π C) $34\frac{2}{3}\pi$ D) 24π

143. λ- soning $\bar{a}=(\lambda+2\sqrt{2},0,2)$, $\bar{b}=(4,\lambda-2\sqrt{2},0)$, $\bar{c}=(0,\lambda,1)$ vektorlar komplanar bo‘ladigan bo‘ladigan barcha qiymatlari ko‘paytmasini toping.

A) $2\sqrt{2}-2$ B) -8 C) $2\sqrt{2}+2$ D) 0

144. Agar $(x-2\sqrt{2})(x+2\sqrt{2})=\frac{x^2}{1-x}$ bo‘lsa, u holda $\frac{2x^2}{x-1}$ ni toping.

A) 2 B) $\frac{8}{2\sqrt{2}-1}$ C) 16 D) 8

145. $\sqrt{x}^{\,x}=x^{\sqrt{x}}$ tenglama yechimlarini yig‘indisini toping.

A) 1 B) 4 C) 3 D) 5

146. Soddalashtiring: $\lg tg1^0+\lg tg2^0+\lg tg3^0+...+\lg tg89^0$

A) 1 B) 0 C) -1 D) $\frac{\sqrt{2}}{2}$

147. $|x|<1$ bo‘lsa $1+2x+3x^2+4x^3+...$ ni toping.

A) $\frac{1}{(1-x)^2}$ B) $-\frac{1}{(1-x)^2}$ C) $\ln(1-x)$ D) $\ln(1+x)$

148. Hisoblang: $1+2\cdot3+3\cdot3^2+...+2010\cdot3^{2009}$

A) $1003\cdot3^{2010}-1$ B) $\frac{1}{2}(1003\cdot3^{2010}-1)$ C) $\frac{1}{4}(4019\cdot3^{2010}+1)$ D) $3^{2010}-1$

149. 13^{20} ni 19 ga bo‘lgandagi qoldiqni toping.

A) 5 B) 12 C) 13 D) 17

150. ABC uchburchakning medianalari $3, 4$ va 5 bo‘lsa, uchburchak yuzi nimaga teng?

A) 5 B) 10 C) 8 D) 25

151. Agar $f(x)=\dfrac{x}{\sqrt{1+x^2}}$ bo‘lsa $f(f(f(...f(2010)...)))$ ni hisoblang.

A) 0 B) $\dfrac{1}{\sqrt{2011}}$ C) $\dfrac{2010}{\sqrt{1+2010^3}}$ D) $\dfrac{2010}{\sqrt{1+2010^2}}$

152. Agar x^3-mx^2+nx-5 ko‘phad x^2 - 1 ga bo‘linsa, $m+n$ toping.

A) -6 B) -5 C) -7 D) -14

153. Hisoblang: $\sum_{n=1}^{2010} i^n$ (i-mavhum birlik)

A) 0 B) 2010 C) $1005i$ D) $i-1$

154. $a, b \in Z^+, a^2-b^4=2009 \Rightarrow (a+b)-?$

A) 43 B) 47 C) 49 D) aniqlab bo‘lmaydi

155. $(1+ix)^{2010}$ ning haqiqiy koeffisiyentlari yig‘indisi S bo‘lsa $\log_2 S$ ni toping.

A) 1005 B) 2010 C) 1004 D) 0

156. $tgx+tgy=4,\ \cos(x+y)=\dfrac{1}{5}$ bo‘lsa, $tg(x+y)$ ni hisoblang.

A) 20 B) $\dfrac{4}{5}$ C) 4 D) 1

157. $a, b, c \in \mathbb{Z}$ uchun $x^2+ax+b,\ x^2+bx+c$ ko‘phadlar $x+1$ ga bo‘linsa x^3 - $4x^2+x+6$ ko‘phad $x^2+ax+b,\ x^2+bx+c$ ko‘phadlarga bo‘linadi. $a+b+c$ ni toping.

A) -5 B) -6 C) -7 D) -8

158. a, b, c uchlik x^3 - $x+1=0$ ning ildizlari bo‘lsa,

$\dfrac{1}{a+1}+\dfrac{1}{b+1}+\dfrac{1}{c+1}$ ni toping.

A) 2 B) -2 C) 3 D) -3

159. x butun sonni toping, agar quyidagi o‘rinli bo‘lsa,

$$\lim_{n\to\infty}\frac{n^{2012}}{n^x-(n-1)^x}=\frac{1}{2013}$$

A) 2012 B) 2013 C) 2014 D) 2015

160.

$a,b,x,y\in R\Rightarrow ax+by=3, ax^2+by^2=7, ax^3+by^3=16, ax^4+by^4=42\Rightarrow$ $ax^5+by^5-?$

A) 10 B) 20 C) 30 D) 40

161. $x+y=3, x^2+y^2=6, x^4=y^4+24\Rightarrow(x-y)-?$

A) $\frac{4}{5}$ B) $\frac{5}{4}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$

162. $\log_{2010}\frac{1}{2}\log_{2009}\frac{1}{3}...\log_2\frac{1}{2010}$ ni hisoblang.

A) 1 B) -1 C) 0 D) $\frac{1}{2010}$

163. $P(x)$-ko‘phad uchun $P(x)=(27x^{27}+2x^2+1)=2010x^{2010}$ bo‘lsa, ko‘phadning barcha koeffitsiyentlari yig‘indisini toping.

A) 30 B) 2 C) 5 D) 67

164. $a=5^{56}, b=10^{51}, c=17^{35}, d=31^{28}$ o‘sib borish tartibida joylashtiring.

A) a,c,d,b B) a,d,c,b C) a,d,b,c D) a,b,c,d

165. $\sin x=\frac{x}{8}$ tenglamaning nechta haqiqiy ildizi bor?

A) 5 B) 7 C) 8 D) 10

166. Hisoblang: $11^2\text{-}1^2+12^2\text{-}2^2+13^2\text{-}3^2+...+20^2\text{-}10^2$

A) 2100 B) 2010 C) 1000 D) 1005

167. Uzunliklari $|\vec{a}|=3, |\vec{b}|=4$ bo‘lgan $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\frac{\pi}{3}$ bo‘lsa, $|\vec{c}|=3\vec{a}+2\vec{b}$ vektorni uzunligini hisoblang.

A) $\sqrt{217}$ B) 12 C) $\sqrt{221}$ D) 13

168. x va y sonlari uchun $x^2+y^2-3y-1=0$ bo‘lsa $(x+y)_{\max}$ ni toping.

A) aniqlab bo‘lmaydi B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{26}+3}{2}$ D) $\frac{8}{15}$

169. $ABCD$ trapetsiyada $(AD \parallel BC)$ $\angle A = \angle D = 45^0$, $\angle B = \angle C = 135^0$, $AB = 6$ va $S_{ABCD} = 30$ bo'lsa BC – ?

A) $2\sqrt{2}$ B) $3\sqrt{2}$ C) $4\sqrt{3}$ D) $2\sqrt{3}$

170. $ABCD$ parallelogramning uchta uchining koordinatalari (1,2),(3,8),(4,1) bo'lsa, to'rtinchi uchinning mumkin bo'lsa, abssissalari yig'indisini toping.

A) 8 B) 6 C) 2 D) 0

171. $ABCD$ to'g'ri to'rtburchak ichidagi P nuqta uchun $PA = 2, PB = 3, PC = 10$ bo'lsa PD ni toping.

A) $\sqrt{105}$ B) 11 C) $\sqrt{91}$ D) $\sqrt{95}$

172. $|x| + |y| + |x + y| \le 1$ sohaning yuzini toping.

A) $\frac{3}{4}$ B) $\frac{4}{3}$ C) $\frac{3}{2}$ D) $\frac{2}{3}$

173. $ABCD$-trapetsiyada $(AD \parallel BC)$ $AB = 6, BC = 7, CD = 8, AD = 17$ va AB va DC lar E nuqtada kesishsa $\angle AED$ ni toping (gradusda).

A) 60 B) 90 C) 150 D) 120

174. Hisoblang: $\frac{d}{da}\int_a^b \sin x^2 dx$

A) $-\sin a^2$ B) $\sin a^2$ C) $\sin a^3$ D) 0

175. Agar $\lg 2 = m, \lg 5 = n$ va $\lg 1400 = p$ bo'lsa, $\lg 7$ ni hisoblang.

A) $p + 3m + 2n$ B) $p - 3m - 2n$ C) $p - 2m - 3n$ D) $p + 2m + 3n$

176. Limitni hisoblang: $\lim_{n\to\infty}\left(\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + ... + \frac{1}{n(n+1)(n+2)}\right)$

A) 0.5 B) 0.15 C) 0.25 D) 0.75

177. Noldan farqli a, b, c sonlar uchun $a + b + c = 26$, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 28$ bo'lsa, $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} + \frac{a}{c} + \frac{c}{b} + \frac{b}{a}$ ni toping.

A) 725 B) 726 C) 727 D) 728

178. Hisoblang: $2010 + \frac{1}{2}\left(2009 + \frac{1}{2}\left(2008 + ... + \frac{1}{2}\left(3 + \frac{1}{2}\cdot 2\right)...\right)\right)$

A) 2010 B) 2009 C) 4020 D) 4018

179. Hisoblang: $\frac{tg^2 20^0 - \sin^2 20^0}{tg^2 20^0 \sin^2 20^0}$

A) -1 B) $\frac{1}{2}$ C) 1 D) to‘g‘ri javob keltirilmagan

180. p, q -natural sonlar uchun $\frac{2008}{2009} < \frac{p}{q} < \frac{2009}{2010}$ bo‘lsa $p_{\min}$ ni toping.

A) aniqlab bo‘lmaydi B) 4019 C) 4018 D) 4017

181. 17^{17} ning oxirgi ikkita raqamini toping.

A) 57 B) 77 C) 87 D) 67

182. $f(x) = \frac{1}{1-x}, f^{k+1}(x) = f(f^k(x)), f^1(x) = f(x)$ bo‘lsa $f^{2010}(2010)$ ni toping.

A) $\frac{2009}{2010}$ B) 2010 C) 2009 D) $\frac{2010}{2009}$

183. Hisoblang: $\cos\left(\frac{2\pi}{18}\right) + \cos\left(\frac{4\pi}{18}\right) + ... + \cos\left(\frac{34\pi}{18}\right)$

A) 0 B) -1 C) 1 D) aniqlab bo‘lmaydi

184. Nechta p, q –tub sonlari uchun $p^2 + pq + q^2$ aniq kvadrat bo‘ladi?

A) 1 B) 2 C) 3 D) 4

185. $\{a_n\}$ketma-ketlikda $a_0 = \frac{6}{7}$, va $a_{n+1} = \begin{cases} 2a_n, a_n < \frac{1}{2} bo'lsa \\ 2a_n - 1, a_n \geq \frac{1}{2} bo'lsa \end{cases}$ u holda a_{2010} ni toping.

A) $\frac{6}{7}$ B) $\frac{5}{7}$ C) $\frac{3}{7}$ D) aniqlab b’lmaydi

186. 1000 dan oshmaydigan 21 ta bo‘luvchisi bor sonni toping.

A) aniqlab bo‘lmaydi B) 720 C) 576 D) 432

187. ABC uchburchakda $AB = 13, BC = 15$ va $CA = 14$ bo‘lsin. D nuqta BC ning, E nuqta AD ning , F nuqta BE ning va G nuqta DF ning o‘rtalari bo‘lsa EFG uchburchakning yuzini toping.

A) 21 B) $\frac{21}{2}$ C) $\frac{21}{4}$ D) $\frac{21}{8}$

188. $f(x) = \sin^6\left(\frac{x}{4}\right) + \cos^6\left(\frac{x}{4}\right)$ funksiyaning 2010 –tartibli hosilasining $x = 0$ nuqtadagi qiymatini toping (ya’ni $f^{(2010)}(0)$ ni).

A) $\frac{5}{8}$ B) $\frac{3}{8}$ C) $-\frac{5}{8}$ D) $-\frac{3}{8}$

189. $a,b,c,d\in\mathbb{R}$ sonlar uchun $a\ge b\ge c\ge d$, $a^2+d^2=1, b^2+c^2=1,$ $ac+bd=\frac{1}{3}$ bo‘lsa, $ab-cd$ ni toping.

A) $\frac{\sqrt{3}}{2}$ B) $\frac{2\sqrt{3}}{3}$ C) $-\frac{2\sqrt{2}}{3}$ D) $\frac{3\sqrt{3}}{2}$

190. Nechta (a,b) butun sonlar juftligi $\begin{cases} a^2+b^2<16 \\ a^2+b^2<8a \\ a^2+b^2<8b \end{cases}$ sistemani qanoatlantiradi?

A) 2 B) 4 C) 6 D) 8

191. $(2010!)!\vdots((n!)!)!$ shartni qanoatlantiruvchi n-natural sonni toping.

A) 5 B) 10 C) 3 D) 6

192. Hisoblang: $\left[\frac{2010^3}{2008\cdot 2009}-\frac{2008^3}{2009\cdot 2010}\right]$ (bunda $[x]-x$ ning butun qismi).

A) 8 B) 0 C) 1 D) 7

193. Hisoblang: $\frac{1}{2[\sqrt{1}]+1}+\frac{1}{2[\sqrt{2}]+1}+...+\frac{1}{2[\sqrt{100}]+1}$ (bunda $[x]-x$ ning butun qismi).

A) $\frac{190}{21}$ B) $\frac{189}{21}$ C) 9 D) $\frac{180}{21}$

194. ABC uchburchakning a va b tomonlari berilgan. Agar bu tomonlarga tushirilgan medianalar o‘zaro perpendikular bo‘lsa, uchburchakning uchinchi tomonini toping.

A) $\frac{a^2+b^2}{5}$ B) $\frac{ab}{3}$ C) $\sqrt{\frac{2ab}{5}}$ D) $\sqrt{\frac{a^2+b^2}{5}}$

195. $x^4+(2-x)^4=34$ tenglama ildizlari yig‘indisini toping.

A) 1 B) 2 C) 4 D) 8

196. $x,y,k\in\mathbb{R}^+$ sonlari uchun $k^2\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)+k\left(\frac{x}{y}+\frac{y}{x}\right)=3$ bo‘lsa, k_{max} ni toping.

A) $\frac{\sqrt{7}-1}{2}$ B) $\frac{\sqrt{7}+1}{2}$ C) $\frac{\sqrt{5}-1}{2}$ D) $\frac{\sqrt{5}+1}{2}$

197. x haqiqiy son uchun $x^3+4x=8$ bo‘lsa, u holda x^7+64x^2 ifoda nimaga teng bo‘ladi?

A) 16 B) 64 C) 128 D) -64

198. Hisoblang $\frac{\sin 10^0+\sin 20^0+...+\sin 70^0+\sin 80^0}{\cos 5^0 \cos 10^0 \cos 20^0}$

A) $4\sqrt{3}$ B) $8\sqrt{3}$ C) $4\sqrt{2}$ D) $8\sqrt{2}$

199. Qanday n natural sonlarda n^3+2n^2+9n+8 ifoda aniq kub bo‘ladi?

A) 5 B) 6 C) 8 D) 7

200. Taqqoslang: $a=\log_{2008}2009$ va $a=\log_{2009}2010$.

A) $a>b$ B) $a<b$ C) $a=b$ D) aniqlab bo‘lmaydi

201. Quyidagi limitni hisoblang: $\lim\limits_{n\to+\infty}\frac{\left(\int_0^x e^{t^2}dt\right)^2}{\int_0^x e^{2t^2}dt}$

A) 0 B) $\frac{1}{4}$ C) $\frac{1}{e}$ D) $\frac{e}{2}$

202. ACE uchburchakda B nuqta AC kesmada, D nuqta CE kesmada shunday olinganki, bunda $AE \parallel BD$ va AE kesmadaki Y nuqta uchun CE va BD kesmalar X nuqtada kesishadi. $CX=5$ va $CY=8$ bo‘lsa $\frac{S_{ABDE}}{S_{BCD}}$ nisbatni aniqlang.

A) $\frac{25}{9}$ B) $\frac{64}{25}$ C) $\frac{39}{25}$ D) $\frac{169}{64}$

203. Aylanadagi A,B,C,D nuqtalar uchun $AB=11, CD=19$, AB kesmadagi P nuqta uchun $AP=6$ va CD kesmadigi Q nuqta uchun $CQ=7, P$ va Q nuqtalardan o‘tuvchi to‘g‘ri chiziq a7ylanani X va Y nuqtalarda kesadi. Agar $PQ=27$ bo‘lsa XY ni toping.

A) 29 B) 30 C) 31 D) 32

204. ABC uchburchakda B burchagi to‘g‘ri burchak va BC tomondagi nuqta uchun $3<BAD=<BAC, AC=2, CD=1$ bo‘lsa BD ni toping.

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$

205. Limitni hisoblang. $\lim\limits_{n\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+...+\sin\frac{(n-1)\pi}{n}\right)$

A) $\frac{2}{\pi}$ B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{4}{\pi}$

206. $1\cdot2+2\cdot3+3\cdot4+...+2009\cdot2010$ ni hisoblang.

A) $\frac{2009\cdot2010\cdot2011}{3}$ B) $\frac{2009\cdot2010\cdot2008}{2}$ C) $\frac{4021\cdot2010\cdot2009}{6}$ D) t.j.y

207. $x^3+x^2+x+1=0$ bo'lsa $x^4+2x^3+2x^2+2x+1$ nimaga teng bo'ladi?

A) 1 B) 0 C) $\sqrt{3}$ D) 2

208. $100!$ ni 101 ga bo'lgandagi qoldiqni toping.

A) 0 B) 1 C) 9 D) 100

209. $\left(a+\frac{1}{a}\right)^2=3$ bo'lsa $\left(a+\frac{1}{a}\right)^3$-?

A) $3\sqrt{3}$ B) $\pm3\sqrt{3}$ C) $-3\sqrt{3}$ D) $\sqrt{3}$

210. Ushbu $\vec{a}=2\vec{i}+\vec{j}$ va $\vec{b}=-2\vec{j}+\vec{k}$ vektorlardan yasalgan parallelogrammning diagonallari orasidagi burchakni toping.

A) $\arccos\frac{1}{\sqrt{21}}$ B) $\frac{\pi}{2}$ C) $\arccos\frac{2}{\sqrt{21}}$ D) $\arccos\frac{3}{\sqrt{21}}$

211. Tenglamani yeching: $x=\sqrt{x-\frac{1}{x}}+\sqrt{1-\frac{1}{x}}$

A) $\frac{1+\sqrt{5}}{2}$ B) $\frac{\sqrt{5}-1}{2}$ C) $\frac{\sqrt{3}+1}{2}$ D) $\frac{\sqrt{3}-1}{2}$

212. $231m^2=130n^2$ tenglama butun sonlar to'plamida nechta yechimga ega?

A) 3 B) 1 C) cheksiz ko'p D) aniqlab bo'lmaydi

213. Hisoblang: $2^{\sqrt{\log_2 3}}-3^{\sqrt{\log_3 2}}$

A) 0 B)1 C)-1 D)3

214. $\frac{2010!}{2010^n}$ butun son bo'ladigan n ning eng katta qiymatini toping.

A) 201 B) 15 C) 30 D) 45

215. x_1, x_2, x_3 sonlari $x^3 - ax^2 + ax - a = 0$ tenglamaning ildizlari bo‘lsa, $x_1^3 + x_2^3 + x_3^3 - 3x_1x_2x_3$ ni toping.

A) $a^3 - 3a$ B) $a^3 - 3a$ C) $a^3 + 3a^2$ D) $a^3 - 3a^2$

216. $7^{2048} - 1 \vdots 2^n$ bo‘ladigan $n_{\max}$ ni toping.

A) 14 B) 16 C) 18 D) 20

217. $a, b, c > 0$ sonlar bo‘lsa, $\left[\frac{a+b}{c}\right] + \left[\frac{b+c}{a}\right] + \left[\frac{c+a}{b}\right]$ ifodaning eng kichik qiymatini toping.

A) 3 B) 4 C) 5 D) 6

218.
$3a + 2b + 4d = 10, 6a + 5b + 4c + 3d + 2e = 8, a + b + 2c + 5e = 3, 2c + 3d + 3e = 4$ va $a + 2b + 3c + d = 7$ bo‘lsa, $a + b + c + d + e$ ifodaning son qiymatini toping.

A) 4 B) 5 C) 6 D) 7

219. $ABCD$ qavarriq to‘rtburchakda $AB = BC = 7, CD = 5, AD = 3$ va $\angle ABC = 60^0$ bo‘lsa $BD - ?$

A) 8 B) 12 C) 10 D) 15

220. Quyidagini aniqmas integralni hisoblang: $\int \frac{dx}{\sin(x+a)\sin(x+b)}$

A) $\frac{1}{\sin(a-b)} \ln\left|\frac{\sin(x+b)}{\sin(x+a)}\right|$ B) $\frac{1}{\sin(a-b)} \ln\left|\frac{\sin(x+a)}{\sin(x+b)}\right|$

C) $\frac{1}{\sin(b-a)} \ln\left|\frac{\sin(x+b)}{\sin(x+a)}\right|$ D) $\frac{1}{\sin(a-b)} \ln\left|\frac{\sin(x-b)}{\sin(x+a)}\right|$

221. ABC uchberchakda A burchagi to‘g‘ri. AB tomondagi D nuqta uchun $CD = 1$. AE gipotenuzaga tushurilgan balandlik(E nuqta gipotenuzada) uchun $BD = BE = 1$ bo‘lsa $AD - ?$

A) 2 B) $\sqrt[3]{2} - 1$ C) $\sqrt{2} - 1$ D) $\frac{1}{2}$

222. Limitni hisoblang: $\lim\limits_{x \to \infty}\left(\cos\frac{x}{2}\cos\frac{x}{4}\ldots\cos\frac{x}{2^n}\right)$

A) $\frac{\sin x}{x}$ B) 1 C) $\frac{\sin x}{4}$ D) $\frac{\sin x}{2x}$

223. $ABCD$ to‘rtburchakda E,F,G,H nuqtalar, mos ravishda, AB, BC, CD, DA tomonlarining o‘rtalari. $EG=12, FH=15$ bo‘lsa, $\left(S_{ABCD}\right)_{\max}$ ni toping.

A) 150 B) 160 C) 170 D) 180

224. Hisoblang: $arctg\frac{1}{3}+arctg\frac{1}{5}+arctg\frac{1}{7}+arctg\frac{1}{8}$.

A) $\frac{\pi^2}{4}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi^2}{2}$

225. $f(x)=x^4+ax^3+bx^2+cx+d$ ko‘phadning barcha ildizlari nomusbat butun va $a+b+c+d=2010$ bo‘lsa d ni toping.

A) 2010 B) 528 C) 2011 D) 0

226. $x^3-9x^2+11x-1=0$ tenglamaning ildizlari a,b,c. $s=\sqrt{a}+\sqrt{b}+\sqrt{c}$ bo‘lsa, $s^4-18s^2-8s-?$

A) -36 B) -37 C) -38 D) -39

227. Hisoblang: $\dfrac{\sqrt{31+\sqrt{31+\sqrt{31+...}}}}{\sqrt{1+\sqrt{1+\sqrt{1+....}}}}$

A) $\frac{1+\sqrt{5}}{2}$ B) $\frac{1+5\sqrt{5}}{2}$ C) $6-\sqrt{5}$ D) $\frac{1+5\sqrt{5}}{4}$

228. $a\neq b\in\mathbb{R}$ sonlar uchun $a^2+b^2=8ab$ shart bajarilsa, $\left|\frac{a+b}{a-b}\right|$ ni toping.

A) $\frac{\sqrt{15}}{3}$ B) $\frac{\sqrt{10}}{3}$ C) $\frac{2\sqrt{5}}{3}$ D) $\frac{3\sqrt{5}}{2}$

229. Hisoblang: $\sqrt[2010]{3\sqrt{5}-2\sqrt{11}}\cdot\sqrt[4020]{89+12\sqrt{55}}$

A) 2 B) 1 C) 3 D) 5

230. $a,b\in\mathbb{Z}^+$ sonlari uchun $a+\sqrt{b}=\sqrt{15+\sqrt{216}}$ shart bajarilsa, $\frac{a}{b}$ ni yoping.

A) $\frac{3}{2}$ B) $\frac{9}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$

231. Ushbu $pq+qr+pr-pqr=2$ tenglamaning nechta (p,q,r) musbat butun yechimlari uchliklari bor?

A)7 B)5 C)9 D)3

232. p,q,r sonlar $x^3+ax^2+bx+c=0$ tenglamaning ildizlari bo‘lsa, $(pq)^2+(qr)^2+(rp)^2$ ifodani a,b,c lar orqali ifodalang.

A) b^2+abc B) b^3-3abc C) b^2-2ac D) b^3+ac

233.Quyidagini limitni hisoblang: $\lim\limits_{x\to 0} \sqrt[1-\cos x]{1+x^2e^x}$

A) $\frac{e^2}{4}$ B) $\frac{e}{4}$ C) e^2 D) $\frac{e^2}{2}$

234. Quyidagini aniqmas integralni hisoblang: $\int \operatorname{sgn} x dx$

A) x B) $-x$ C) $|x|$ D) 0

235. Musbat a,b,c sonlari uchun $\log_a b+\log_b c+\log_c a=0$ bo‘lsa, $(\log_a b)^3+(\log_b c)^3+(\log_c a)^3$ ni toping.

A) 3 B) 0 C) 1 D) aniqlab bo‘lmaydi

236. Hisoblang: $\left[\frac{2010!+2007!}{2009!+2008!}\right]$ bunda ($[x]-x$ ning butun qismi, $n!=1\cdot 2\cdot\ldots\cdot n$)

A) 2010 B) 2011 C) 2009 D) 2008

237. $ABCD$ to‘rtburchakda AD va BC tomonlari AB ga va AC diaganali DC ga perpendikular. $AB=4, BC=3$ bo‘lsa $CD-?$

A) 7 B) $\frac{4}{3}$ C) $\frac{20}{3}$ D) 5

238. ABC uchburchakning AB va BC tomonlari olingan D va E nuqtalar uchun $AC=25, DB=6, BE=20$ va $AD=EC=x$ bo‘lib, $S_{DBE}=S_{ADEC}$ shart bajarilsa x ni toping.

A) 4 B) 5 C) 6 D) 7

239. Uchlari $A(-2,-4), B(2,8)$ va $C(10;2)$ nuqtalarda bo‘lgan uchburchak yuzini toping.

A) 60 B) 50 C) 70 D) 80

240. To‘g‘ri burchakli Dekart koordinatalari sistemasida $M(1;-\sqrt{3})$ nuqta berilgan. Agar qutb koordinatalar boshi bilan, qutb o‘qi abssissa o‘qining musbat yo‘nalishi bilan ustma-ust tushsa M nuqtaning qutb koordinatalarini toping.

A) $(2;\pi/3)$ B) $(2;5\pi/3)$ C) $(2;3\pi/4)$ D) $(2;\pi/6)$

241. Uzunligi c bo‘lgan kesmaning bir uchi abssissa o‘qi boshqa uchi ordinata o‘qi bo‘yicha siljimoqda. Kesma o‘rtasi hosil qilgan chizig‘ining tenglamasini ko‘rsating.

A) $xy = c^2/9$ B) $x^2 + y = c/2$ C) $x^2 + y^2 = c^2/4$ D) $x/y = c$

242. Koordinatalari ushbu $x = a\cos t,\ y = a\sin t$ tenglamalar bilan berilgan chiziqni aniqlang.

A) Parabola B) Giperbola C) Aylana D) To‘g‘ri chiziq

243. $y = -3x + 7$ va $y = 2x + 1$ to‘g‘ri chiziqlar orasidagi burchakni aniqlang.

A) $\pi/2$ B) $\pi/4$ C) 0 D) $\pi/3$

244. $x^2 + y^2 = 49$ aylanani teng ikkiga bo‘luvchi va $A(1;2)$ nuqtadan o‘tuvchi vatar tenglamasini toping.

A) $x + 2y - 5 = 0$ B) $x - 2y + 5 = 0$ C) $x + 2y + 5 = 0$ D) $2x + y - 4 = 0$

245. Fokal radiuslari ayirmasi 6.4 bo‘lgan $x^2/25 + y^2/9 = 1$ ellipsning nuqtalarini ko‘rsating.

A) $(4,1.8),(4,-1.8),(-4,1.8),(-4,-1.8)$

B) $(5,3),(5,-3),(-5,3),(-5,-3)$

C) (3, 2.4), (3, -2.4), (-3, 2.4), (-3, -2.4)

D) (0, 3), (0, -3)

246. To‘g‘ri chiziq $M(0;-1)$ nuqta va $3x^2 - 4y^2 = 12$ giperbolaning o‘ng uchi orqali o‘tadi. To‘g‘ri chiziq va giperbola kesishgan ikkinchi nuqtani toping.

A) $(4;3)$

B) $(4;-3)$

C) $(-4;3)$

D) $(-4;-3)$

247. $y^2 = 8x$ parabolaning direktricadan 4 birlik uzoqlashgan nuqtasini toping.

A) $(0;0)$

B) $(1;2\sqrt{2})$, $(1; -2\sqrt{2})$

C) $(2,4),(2,-4)$

D) (8, 8), (8,-8)

248. Koordinatalar sistemasi $\alpha = \pi / 6$ burchakka burildi. $M\left(\sqrt{3};3\right)$ nuqtaning yangi koordinatalarini aniqlang.

A) $\left(3;\sqrt{3}\right)$

B) $\left(0;3\sqrt{3}\right)$

C) $\left(3\sqrt{3};0\right)$

D) $(2;3)$

249. $x^2 - 2xy + y^2 - 10x - 6y + 25 = 0$ tenglama bilan berilgan chiziqni aniqlang

A) Aylana B) Ellips C) Parabola D) Giperbola

250. Determinantlar uchun noto‘g‘ri tasdiqni ko‘rsating

A) Biror satr elementlarining umumiy ko‘paytuvchisini determinant belgisidan tashqariga chiqarish mumkin

B) Agar satrning elementlari boshqa satrning mos elementlari bilan teng bo‘lsa, u holda determinant qiymati 0 ga teng

C) ikkita satr o‘rinlarini o‘zaro almashtirish natijasida determinant qiymati o‘zgarmaydi

D) Determinant barcha satrlarini mos ustunlarga almashtirish natijasida determinant qiymati o‘zgarmaydi

251. $a = 2i + 3j + 5k$ va $b = 6i - 2j + 6k$ vektorlarga perpendikular vektorni ko‘rsating.

A) $c = 14i - 42j - 21k$

B) $c = 18i + 21j - 11k$

C) $c = i - j + 5k$

D) $c = -i + 2j + k$

252. Uchlari $A(2,\ 2,\ 2)$, $B(4,\ 3,\ 3)$ $C(4,\ 5,\ 4)$ va $D(5,5,\ 6)$ nuqtalarda bo‘lgan uchburchakli piramida hajmini hisoblang.

A) 2 B) 3 / 4 C) 7 / 6 D) 5 / 3

253. M(3, 5, -8) nuqtadan $6x - 3y + 2z - 28 = 0$ tekislikkacha masofani toping.

A) 6 B) 12 / 23 C) 55 / 9 D) 41 / 7

254. $(x + 1) / 2 = (y - 1) / (-1) = (z - 2) / 3$ to‘g‘ri chiziq orqali o‘tuvchi va $x / (-1) = (y + 2) / 2 = (z - 3) / 3$ to‘g‘ri chiziqqa parallel tekislik tenglamasini toping.

A) $x - y - z + 4 = 0$

B) $2x - y + 2z - 1 = 0$

C) $x + y - 2z + 3 = 0$

D) $x + 2y + 6 = 0$

255. $4x^2 + 9y^2 + 36z^2 - 8x - 18y - 72z + 13 = 0$ tenglamani kanonik ko‘rinishga keltiring

A) $x^2 / 9 + x^2 / 16 + z^2 = 1$

B) $x^2 + y^2 + z^2 / 9 = 1$

C) $x^2 - y^2 + z^2 / 9 = 1$

256. Agar $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix}$ va E uchinch tartibli birlik matritsa bo‘lsa, $2A^2 + 3A + 5E$ 2matritsa ko‘phadning qiymatini hisoblang.

A) $\begin{pmatrix} 1 & 1 & 8 \\ 1 & 27 & 1 \\ 64 & 1 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 28 & 15 & 16 \\ 19 & 36 & 15 \\ 30 & 19 & 28 \end{pmatrix}$

C) $\begin{pmatrix} 21 & 13 & 12 \\ 15 & 24 & 18 \\ 24 & 13 & 31 \end{pmatrix}$ D) $\begin{pmatrix} 21 & 13 & 12 \\ 0 & -24 & 18 \\ 24 & 0 & -31 \end{pmatrix}$

257. $A=\begin{pmatrix}1&0&2&0&0\\0&1&0&2&0\\2&0&4&0&0\end{pmatrix}$ matritsaning rangini va bazis minorlari sonini aniqlang.

A) rangA $=3$; 6 ta

B) rangA $=3$; 4 ta

C) rangA $=1$; 6 ta

D) rangA $=2$; 8 ta

258. Tenglamalar sistemasini yeching. $\begin{cases}x_1+5x_2+4x_3+3x_4=1\\2x_1-x_2+2x_3-x_4=0\\5x_1+3x_2+8x_3+x_4=1\end{cases}$

A) $x_1=-u+4v/17-3/17, x_2=-u/17+v/17-3/17, x_3=u, x_4=v$;

B) $x_1=-6u/11+8v/11-1/11, x_2=-6u/11+7v/11+2/11, x_3=u, x_4=v$

C) $x_1=u+5v/7-4/7, x_2=-2u/7-4v/7+1/7, x_3=u, x_4=v$;

D) Yechimga ega emas

259. Ildizlari x_1 va x_2 bo'lgan, hamda $x_1\cdot x_2=4, \dfrac{x_1}{x_1-1}+\dfrac{x_2}{x_2-1}=\dfrac{2}{5}$ shartlarni qanoatlantiruvchi kvadrat tenglamani toping.

A) $x^2-10x+4$ B) x^2-8x+4 C) x^2+8x+4 D) x^2-6x+4

260. Noto'g'ri tasdiqni ko'rsating.

A. Uchta nokomplanar a, b va c vektorlar chiziqli erkli bo'ladi

B. Uch o'lchovli fazoda har qanday 4ta vektor chiziqli bog'liq bo'ladi

C. Agar $x_1, x_2, ..., x_n$ vektorlar chiziqli bog'liq bo'lsa u holda $x_1, x_2, ..., x_n, x_{n+1}$ vektorlar ham chiziqli bog'liq bo'ladi

D. $a=3i+2j+2k$, $b=i+3j+k$ va $c=5i+3j+4k$ vektorlar chiziqli bo'g'liq

261. R^4 chiziqli fazodagi $\begin{cases} x_1 + x_2 - x_3 + x_4 = 0 \\ x_1 - x_2 + x_3 - x_4 = 0 \\ 3x_1 + x_2 - x_3 + x_4 = 0 \\ 3x_1 - x_2 + x_3 - x_4 = 0 \end{cases}$ tenglamalar sistemasi yechimlaridan iborat qismfazoning o'lchamini va bazis vektorlarini ko'rsating.

A) O'lchami 2, bazis vektorlari $f = (0;1;1;0)$, $g(0;-1;0;1)$

B) O'lchami 3, bazis vektorlari

C) $f = (1;0;1;0)$, $g(0;-1;0;1), h(1;1;0;1)$

D) O'lchami 1, bazis vektorlari $f = (0;0;1;0)$

E) O'lchami 0, yani yechimga ega emas

262. Agar $f(x) = x^2$ bo'lsa, $\dfrac{f(b) - f(a)}{b - a}$ ni hisoblang.

A) $b^2 - a^2$

B) $b - a$

C) $a + b$

D) $a - b$

263. Toq funksiyani ko'rsating.

A) $f(x) = 2\cos(x / 3) + 1$

B) $f(x) = tg3x + \cos 4x$

C) $f(x) = \lg \cos 2x$

D) $f(x) = x^4 \sin 7x$

264. Ketma-ketlikning limitini toping. $\frac{7}{3}, \frac{10}{5}, \frac{13}{7}, \ldots, \frac{3n+4}{2n+1}, \ldots$

A) 2 B) $3/2$ C) $4/3$ D) $5/3$

265. Limitni hisoblang. $\lim\limits_{x \to 0} \dfrac{1 - \cos 5x}{x^2}$

A) $10/7$ B)15/11 Cc)$25/2$ D) 15

266. Funksiyani uzilish nuqtasini aniqlang. $y = \dfrac{x^3 - 6x^2 + 11x - 6}{x^2 - 3x + 2}$

A) $x = 1, x = 2$

B) $x = 1$

C) funksiya uzluksiz

D) $x = 1, x = 3$

267. Nechta nomanfiy butun (a,b,c) uchligi $2^a + 2^b = c!$ tenglamani qanoatlantiradi? Bunda $(n! = 1 \cdot 2 \cdot \ldots \cdot n)$

A) 5 B) 6 C) 7 D) 9

268. Tenglama nechta nomanfiy butun yechimga ega?

$x_1 + x_2 + x_3 + x_4 = 11$

A) 365 B) 326 C) 374 D) 364

269. $0 \le a \le b \le c \le d \le e \le 100$ bo'lsa, $\left(\frac{1}{5}(a+b+c+d+e) - c\right)_{\max}$ ni toping.

A) 60 B) 40 C) 100 D) aniqlab bo'lmaydi

270. $m \circ n = \dfrac{m+n}{mn+4}$ bo'lsa, $(((\ldots(2010 \circ 2009) \circ 2008) \circ \ldots \circ 1) \circ 0$ ni toping.

A) $\dfrac{1}{2010}$ B) $\dfrac{1}{12}$ C) $\dfrac{1}{10}$ D) aniqlab bo'lmaydi

271. $a + b + c = 5, (1 \le a, b, c \le 2)$ bo'lsa, $\left(\dfrac{1}{a+b} + \dfrac{1}{b+c}\right)_{\min}$ ni toping

A) 2 B) $\dfrac{1}{2}$ C) $\dfrac{8}{15}$ D) $\dfrac{4}{7}$

272. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} arctg(e^x)dx$ ni hisoblang.

A) $\dfrac{\pi^2}{2}$ B) $\dfrac{\pi^2}{4}$ C) $\dfrac{\pi^2}{3}$ D) $\dfrac{3\pi^2}{4}$

273. 2^{2010} sonini $(2^7 - 1)$ ga bo'lgandagi qoldiqni toping.

A) 16 B) 8 C) 4 D) 2

274. Nechta $(a,b,c,d,e) \in \mathbb{Z}^+$ sonlari uchun

$abcde \le a + b + c + d + e \le 10$ tengsizlik bajariladi?

A) 116 B) 115 C) 106 D) 105

275. Limitni hisoblang: $\lim\limits_{x \to +0} (x^x - 1)\ln x$

A) 1 B) $\dfrac{1}{e}$ C) 0 D) e

276. a, b, c, x, y, z sonlar uchun $a=\frac{b+c}{x-2}, b=\frac{c+a}{y-2}, c=\frac{a+b}{z-2} (x, y, z \neq 2)$ va $xy+yz+zx=67,\ x+y+z=2010$ bo‘lsa xyz ni toping.

A) 5892 B)-5892 C) 0 D) 1

277. $f(x)=|x-1|+|2x-1|+|3x-1|+...+|119x-1|$ bo‘lsa $f_{\min}$ ni toping.

A)49 B)50 C)51 D)52

278. $(x_1, y_1), (x_2, y_2)(x_3, y_3)$ sonlari $x^3+3xy^2=2010$ va $y^3+3x^2y=2009$ tenglamaning ildizlari bo‘lsa, $\left(1-\frac{x_1}{y_1}\right)\left(1-\frac{x_2}{y_2}\right)\left(1-\frac{x_3}{y_3}\right)$ ni toping.

A) $\frac{1}{2009}$ B) $\frac{1}{2010}$ C) $\frac{-1}{2009}$ D) $\frac{-1}{2010}$

279. $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{4\cdot 5\cdot 6}+\frac{1}{7\cdot 8\cdot 9}+...=?$

A) $\frac{\pi\sqrt{3}}{12}-\frac{\ln 2}{2}$ B) $\frac{\pi\sqrt{3}}{6}-\frac{1}{4}$ C) $\frac{\pi\sqrt{2}}{12}-\frac{\ln 3}{4}$ D) $\frac{\pi\sqrt{3}}{12}-\frac{\ln 3}{4}$

280. $P(x)=\left(1+x+x^2+...+x^{27}\right)\left(1+x+x^2+...+x^{14}\right)^2$ ko‘phadning x^{28} oldidagi koeffitsiyentini toping.

A) 405 B) 434 C) 224 D) 378

281. Ushbu $(1+x+x^2+x^3+...+x^{100})^3$ ko‘phadning yoyilmasidagi x^{100} oldidagi koeffitsiyentni toping.

A) 5151 B) 5150 C) 5050 D) 5051

282. $\frac{1\cdot 3!}{3}+\frac{2\cdot 4!}{3^2}+...+\frac{n\cdot(n+2)!}{3^n}$ yig‘indini toping.

A) $\frac{(n+3)!}{3^n}+6$ B) $\frac{(n+3)!}{3^n}-12$ C) $\frac{(n+3)!}{3^{n+1}}-6$ D) $\frac{(n+3)!}{3^n}-6$

283. Limitni hisoblang: $\lim_{n\to\infty}\sum_{k=1}^{n} arctg\frac{1}{2k^2}$

A) $\frac{3\pi}{4}$ B)0 C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$

284. $\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+...+\sqrt{1+\frac{1}{2011^2}+\frac{1}{2012^2}}=?$

A) $2010\frac{1005}{2012}$ B) $2012\frac{1005}{2012}$ C) $2010\frac{1010}{2012}$ D) $2011\frac{1005}{2012}$

285. $\vec{u}$ vektori OY va OZ oqlari bilan mos, mos ravishda, 120^0 va 45^0 burchak hosil qiladi va $|\vec{u}| = 6$ bo‘lsa u holda $\vec{u}$ vektorining koordinatalari topilsin.

A) $\vec{u} = \left(-3\sqrt{2}, 3, -3\right)$ va $\vec{u} = \left(3; -3; -3\sqrt{2}\right)$

B) $\vec{u} = \left(3; -3\sqrt{2}, 3\right)$ va $\vec{u} = \left(-3; -3; -3\sqrt{2}\right)$

C) $\vec{u} = \left(3; -3; 3\sqrt{2}\right)$ va $\vec{u} = \left(-3; -3; 3\sqrt{2}\right)$

D) $\vec{u} = \left(3; -3; 3\sqrt{2}\right)$ va $\vec{u} = \left(-3; -3; -3\sqrt{2}\right)$

286. Arximed spirali tenglamasi to‘g‘ri berilgan javob:

A) $r = a\varphi$ B) $r = \frac{a}{\varphi}$ C) $r = ae^{\varphi}$ D) to‘g‘ri javob yo‘q

287. $z = \frac{\sqrt{2+\sqrt{3}}}{2} + i \cdot \frac{\sqrt{2-\sqrt{3}}}{2}$ soning qanday eng kichik musbat darajasi 1 ga teng.

A) 24 B) 36 C) 12 D) 4

288. $\lim\limits_{x\to\infty} x\left[\frac{1}{x}\right]$ limitni hisoblang ($[a] - a$ sonining butun qismi).

A) aniqlanmagan B) ∞ C) 1 D) 0

289. $\frac{2\cdot 2}{1\cdot 4} + \frac{2\cdot 2^3}{3\cdot 4^3} + \frac{2\cdot 2^5}{5\cdot 4^5} + ... + \frac{2\cdot 2^{2n-1}}{(2n-1)\cdot 4^{2n-1}}$ qatorning yig‘indisini toping.

A) e B) $\ln 2$ C) $\ln 3$ D) $\ln 4$

290. $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 0 & -4 & -1 \end{pmatrix}$ bo‘lsa, quyidagi ko‘phadlarning qaysi birisi uchun $P(A) = 0$ bo‘ladi.

A) $P(x) = x^3 - 2x^2 + x + 1$

B) $P(x) = x^3 - 3x^2 + x + 10$

C) $P(x) = x^3 - 2x^2 - x + 10$

D) $P(x) = x^3 - 2x^2 - x - 1$

291. $f(x)$ uzluksiz va irratsional qiymat qabul qilmaydi. Agar $f\left(\frac{1}{2011}\right) = 2012$ ga teng bo‘lsa, $f\left(\frac{1}{2012}\right)$ ni toping.

A) aniqlab bo‘lmaydi B) 2011 C)2012 D)2013

292. Matritsaning rangini toping. $A=\begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$

A) 4 B)3 C)2 D)5

293. Yig‘indini hisoblang $\left[\frac{1}{3}\right]+\left[\frac{2}{3}\right]+\left[\frac{2^2}{3}\right]+...+\left[\frac{2^{1000}}{3}\right]$ (bunda $[a]-a$ ning butun qisimi).

A) $\frac{2^{1001}-2}{3}-500$ B) $\frac{2^{1001}-2}{3}+500$ C) $\frac{2^{1001}-1}{3}-500$ D) $\frac{2^{1001}+1}{3}-500$

294. Taqqoslang: $\sin 1$ kattami yoki $\log_3\sqrt{7}$ $\log_3\sqrt{7}$ mi?

A) $\sin 1 > \log_3\sqrt{7}$ B) $\sin 1 < \log_3\sqrt{7}$ C) $\sin 1 = \log_3\sqrt{7}$

D) $\sin 1 \approx \log_3\sqrt{7}$

295. Hisoblang: $1+\frac{(1+2)}{2!}+\frac{(1+2+2^2)}{3!}+...=$

A) 2 B) $\frac{e+1}{2}$ C) e^2-e D) e

296. Hisoblang: $\int\limits_0^{\infty}\frac{x^2+1}{x^4+1}dx$

A) $\frac{\pi}{2}$ B) $\frac{\pi}{\sqrt{2}}$ C) $\frac{\pi}{2\sqrt{2}}$ D) $\frac{\pi}{3\sqrt{2}}$

297. Limitni hisoblang: $\lim\limits_{x\to 0}(\cos x)\frac{1}{\sin^2 x}$

A) $\frac{1}{\sqrt{e}}$ B) $\sqrt{e}$ C) $\frac{1}{x}$ D) 1

298. A va Blar n- tartibli kvadrat matritsalar bo‘lib, $\det(AB)\neq 0$ bo‘lsa, u holda quyidagilardan qaysi biri noto‘gri.

1) $(AB)^{-1}=B^{-1}A^{-1}$ 2) $(AB)^T=B^TA^T$ 3) $rangA=rangA^T$

4) $(AB)^T-A^TB^T$

A) 1 va 2 B) 1 va 4 C) 4 D) 3

299. $Y=|X-1|+|X-2|+...+|X-2012|$ ifodaning eng kichik qiymatini toping.

A) $1005 * 1006$ B) 1005^2 C) 1006^2 D) $1006^2 - 1005$

300. Agar $0 < x < \dfrac{\pi}{2}$ бўлса, $(tgx)^{\sin x} + (ctgx)^{\cos x}$ ifodaning eng kichik qiymatini toping.

A) 2 B) 3 C) 1 D) $\dfrac{3}{2}$

301. Tenglamani yeching. $\left[\sqrt[3]{1}\right] + \left[\sqrt[3]{2}\right] + ...\left[\sqrt[3]{x^3 - 1}\right] = 400$

A) 5 B) 4 C) 3 D) 2

302. $x^2 + px - \dfrac{1}{2p^2}$ ko'phadning ildizlari x_1 va x_2 bo'lsa, $\min\left(x_1^4 + x_2^4\right) - ?$ bunda $p \in R, p \neq 0$.

A) $2 + \sqrt{2}$ B) $2 - \sqrt{2}$ C) 2 D) $\sqrt{2}$

303. $\lim\limits_{n\to\infty} \underbrace{\sin \sin \ldots \sin x}_{nta}$ ni hisoblang.

A) 1 B) 0 C) $\dfrac{1}{2}$ D) $\dfrac{1}{3}$

304. $A = \begin{pmatrix} 3 & -1 & 0 \\ 6 & -3 & 2 \\ 8 & -6 & 5 \end{pmatrix}$ matritsaning barcha xos qiymatlarining yig'indisini toping.

A) –5 B) 1 C) –1 D) 5

305. Limitni hisoblang. $\lim\limits_{x\to\infty}\left[\dfrac{x^{n+1}}{(n+1)!} + \dfrac{x^{n+2}}{(n+2)!} + ... + \dfrac{x^{2n}}{(2n)!}\right]$

A) aniqlanmagan B) ∞ C) 1 D) 0

306. $A = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$ bo'lsa, $\det e^A$ ni toping.

A) e^3 B) $2e^4$ C) e^4 D) $-2e^2$

307. Agar $[x]\cdot\{x\} = 100$ bo'lsa, u holda $[x^2] - [x]^2$ ifodaning qiymatini toping. Bunda $[x]$ belgi x ning butun, $\{x\}$ belgi x ning kasr qisimini bildiradi.

A) aniqlab bo'lmaydi B) 0 C) 100 D) 200

308. A $n \times n$ kvadrat matritsaning rangi $n - 3$ ga teng bo'lsa, A^T matritsaning rangini toping.

A) n B) $n-1$ C) $n-3$ D) aniqlab bo‘lmaydi

309. Quyidagi munosabatlarning qaysi biri noto‘g`ri.

A) $o\left(o\left(f\right)\right)=o\left(f\right)$ B) $O\left(O\left(f\right)\right)=O\left(f\right)$

C) $o\left(f\right)+O\left(f\right)=O\left(f\right)$ D) $O\left(o\left(f\right)\right)=O\left(f\right)$

310. $A=\begin{pmatrix} 4 & -15 & 6 \\ 1 & -4 & 2 \\ 1 & -5 & 3 \end{pmatrix}$ bo‘lsa, $\det\left(\ln A\right)$ ni toping.

A) $\ln 4$; B) $\ln 2$; C) 1; D) 0.

Test kalitlari

	0	1	2	3	4	5	6	7	8	9
0		C	D	A	C	D	A	C	B	C
1	A	A	C	B	D	A	A	B	C	C
2	D	A	A	B	D	D	C	D	A	D
3	A	C	D	C	A	A	C	D	A	A
4	A	B	D	B	B	D	A	B	A	C
5	D	A	B	C	A	B	A	A	C	C
6	A	A	A	A	A	A	A	A	C	C
7	A	A	A	A	A	A	C	C	C	B
8	C	B	D	D	A	A	C	B	C	C
9	A	C	D	A	C	D	D	A	A	A
10	D	C	A	C	B	A	A	C	B	A
11	D	C	D	A	D	A	D	B	B	C
12	A	B	A	B	B	D	C	A	D	C
13	A	B	A	B	D	D	D	A	C	C
14	C	B	D	B	D	D	B	A	C	D
15	C	C	A	D	B	A	A	B	B	B
16	B	C	B	D	B	A	A	A	C	A
17	C	D	A	B	A	B	C	A	D	C
18	D	B	B	B	B	A	C	B	D	C
19	C	C	A	A	D	C	A	C	C	D
20	A	A	C	C	C	A	A	B	D	B
21	B	A	B	A	C	D	A	B	A	A
22	A	B	D	D	B	D	B	C	A	B
23	D	A	C	C	D	A	C	C	A	A
24	B	C	C	B	A	A	D	C	A	C
25	C	A	C	D	A	B	B	D	B	A
26	D	A	C	D	B	C	A	A	D	B
27	B	D	B	D	A	C	B	A	C	D
28	B	A	D	D	A	C	A	A	C	D
29	C	C	C	C	B	C	B	A	B	A
30	A	A	A	B	D	D	C	D	C	D
31	A									

5-§. Turli yillarda olimpiadalarda taklif qilingan masalalar

2006-yil 15-16-may(UrDU)

1-tur (yozma ish)

1. $[a,b]$ da uzluksiz bo'lgan funksiya uchun $\frac{1}{b-a}\int_a^b f(x)dx = 1, \frac{1}{b-a}\int_a^b f^2(x) = 1$ tengliklar bajarilsa, $f(x)=1$ ekanligini isbotlang.

2. $f(x) = (x^2+1)e^x$ bo'lsa, $\lim_{n\to\infty} n\int_0^1 (f(\frac{x^2}{n})-1)dx$ ni hisoblang.

3. Agar $I + AB$ matritsaning teskarisi mavjud bo'lsa, $I + AB$ martirsaning ham teskarisi mavjud bo'lishini isbotlang (I-birlik matritsa).

4. $x_1, x_2, ..., x_n$ sonlari $[a,b]$ kesmada yotadi, bunda $0 < a < b$. Quyidagi tengsizlikni isbotlang.

$$(x_1 + x_2 + ... + x_n)(\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n}) \le \frac{n^2(a+b)^2}{4ab}$$

5. ABC uchburchak O ixtiyoriy nuqta. Quyidagi tengsizlikni isbotlang.

$$AB^2 + BC^2 + CA^2 \le 3(OA^2 + OB^2 + OC^2)$$

2-tur (og'zaki)

Aniq integral uchun o'rta qiymat haqidagi teorema.

3-tur (ijodiy ish)

Ikkinchi tartibli egri chiziqlar va ularning klassifikatsiyasi.

4-tur (test)

1. $\sin 1^0, \sin 10^0, \sin 100^0, \sin 1000^0, ..$ ketma-ketlikning nechta hadi musbat?

A) 3 ta B) 4 ta C) barchasi D) cheksiz ko'p E) 5 ta

2. $f(x)$ funksiya $[0,+\infty]$ da uzluksiz. $f(f(f(x))) = x^3$ bo'lsa, $f(x)$ funksiyani toping.

A) $\sqrt[3]{x}$ B) x^3 C) $x^{\sqrt[3]{3}}$ D) $x^{\sqrt{3}}$ E) $x^{\sqrt[3]{2}}$

3. $f(x)=\sum_{i=1}^{5}\frac{|x-i|}{x-i}$ funksiyaning qiymatlar sohasi nechta butun sondan iborat?

A) 11 B) 10 C) 8 D) 6 E) 7

4. $\int_0^1 f(x)dx = a$ bo'lsa, $\int_0^1 xf^2(x)dx$ ni hisoblang.

A) a B) $2a$ C) $\frac{a}{2}$ D) a^2 E) $\sqrt{a}$

5. $T=\begin{pmatrix}0&0&0\\1&0&0\\2&3&0\end{pmatrix}$ bo'lsa, $(I-T)^{-1}$ ni T orqali ifodalang (I-birlik matritsa).

A) $I+T$ B) $I+T+T^2$ C) $I-3T+3T^2$ D) $I-T$ E) $T-T^2$

6. $f(x)$ va ko'phadlarning haqiqiy ildizlari to'plami, mos ravishda, X_f va X_g bo'lsin. $f^2(x)+g^2(x)$ ko'phadning haqiqiy ildizlari to'plami quyidagilarning qaysi biriga teng?

A) $X_f\cap X_g$ B) $X_f\cup X_g$ C) $X_f\Delta X_g$ D) X_f E) X_g

7. $\phi_A(x)=\begin{cases}1, x\in A\\0, x\notin A\end{cases}$ tenglik bilan A to'plamning tavsifiy funksiyasi kiritiladi. $\phi_{A\cup B}(x)$ funksiyani $\phi_A(x)$ va $\phi_B(x)$ funksiyalar orqali ifodalang.

A) $\phi_A(x)+\phi_B(x)$ B) $\phi_A(x)\cdot\phi_B(x)$ C) $\max(\phi_A(x),\phi_B(x))$

D) $\min(\phi_A(x),\phi_B(x))$ E) ifodalash mumkin emas

8. $\int_{-4}^{4} x^3|x|dx$ integralni hisoblang.

A) 0 B) $\frac{1}{4}$ C) $\frac{1}{16}$ D) $-\frac{1}{4}$ E) $-\frac{1}{16}$

9. Agar $y=F(x)$ funksiya $y=f(x)$ funksiya uchun boshlang'ich funksiya bo'lsa, $y=4f(-4x)$ $y=4f\left(-4x\right)$ funksiyaning boshlang'ich funksiyasini toping.

A) $y=-4F(-4x)$ B) $y=-F(-4x)$ C) $y=-\frac{1}{4}F(-4x)$

D) $y=F(-4x)$ E) $y=4F(-4x)$

10. $(x-1)(x-2)+(x-2)(x-3)-(x-3)(x-1)=2$ tenglamani yeching.

A) 1;2 B) 2 C) -2 D) $\forall x \in R$ E) yechimga ega emas

11. $x^3(x^3+1)(x^3+2)(x^3+3)$ ifodaning eng kichik qiymatini toping.

A) -1 B) 2 C) -2 D) 1 E) to‘g‘ri javob yo‘q

12. $x^{100}-2x^{51}+1$ ko‘phadni x^2-1 ga bo‘lgandagi qoldiqni toping.

A) -4 B) 4 C) $-4x$ D) $4x$ E) to‘g‘ri javob yo‘q

13. $\lim\limits_{n\to\infty}\dfrac{1^3+2^3+...+n^3}{5n^4}$ ni hisoblang.

A) 2 B) $\dfrac{1}{20}$ C) $\dfrac{1}{25}$ D) $\dfrac{5}{4}$ E) 0

14. $\left|\vec{a}\right|=11, \left|\vec{b}\right|=23$ va $\left|\vec{a}-\vec{b}\right|=30$ bo‘lsa, $\left|\vec{a}+\vec{b}\right|$ ni toping.

A) 16 B) 15 C) 20 D) 18 E)14

15. $2\log_2(x-1)>\log_2(5-x)+1$ tengsizlikni yeching.

A) (3;5) B) (-2;4) C) yechimga ega emas D) [5;6) E) $(-\infty;\infty)$

16. $\dfrac{x+4}{x+1}>2-x$ tengsizlikni yeching.

A) (-6;-3) B) (0;1) C) $(-\infty;\infty)$ D) yechimga ega emas E) $(-1;\infty)$

17. $P^2(x+1)=P(x^2)+2x+1$ ayniyatni qanoatlantiradigan $P(x)$ ko‘phadni toping.

A) $P(x)=x$ B) $P(x)=1$ C) $P(x)=-x$ D) $P(x)=x^2+1$

E) $P(x)=x^2-1$

18. $\begin{cases} x+y=2 \\ xy-z^2=1 \end{cases}$ sistema nechta haqiqiy ildizga ega?

A) 2ta B) 1 ta C) sistema yechimga ega emas D) 3 ta E) 4 ta

19. $a_0=0, a_{n+1}=\sqrt{6+a_n}$ ketma-ketlikning a_{2006}-hadini toping.

A) 1 B) 2 C) 3 D) 4 E) to‘g‘ri javob yo‘q

20. $a_0=-2, a_2=1, a_{n+2}=\dfrac{1}{2}(a_n+a_{n+1})$ ketma-ketlikning limitini toping.

A) 1 B) 0 C) -1 D) $\dfrac{1}{2}$ E) aniqlab bo‘lmaydi

2007-yil 16-17-may(UrDU)

1-tur (yozma ish)

2-variant

1. $[a,b]$ segmentda aniqlangan $f(x)$ funksiya uzluksiz differensiallanuvchi bo‘lib, $\Delta_n = \int\limits_a^b f(x)dx - \frac{b-a}{n}\sum\limits_{v-1}^{n} f(a+v\frac{b-a}{n})$ bo‘lsa, u holda $\lim\limits_{n\to\infty} n\Delta_n$ ni hisoblang.

2. $f(x)$ va $g(x)$ funksiyalar $[a,b]$ segmentda uzluksiz bo‘lib, $\left|\int\limits_a^b f(x)g(x)dx\right|^2 = \int\limits_a^b |f(x)|^2\,dx\int\limits_a^b |g(x)|^2\,dx$ tenglikni qanoatlantirsa, u holda $g(x)\equiv cf(x)$ ekanligini isbotlang.

3. Ushbu $x^2+x+1=py$ tenglama p parametrning cheksiz ko‘p tub qiymatlarida butun yechimlarga ega bo‘lishini isbotlang.

4. ABC uchburchakning AB tomoniga teng tomonli ABC_1 uchburchak shunday yasalganki, uning C va C_1 uchlari AB to‘g‘ri chiziqning bir tomonida joylashgan. Ushbu $|CC_1|^2 = \frac{1}{2}(a^2+b^2+c^2) - 2S\sqrt{3}$ tenglikni isbotlang. Bu yerda a,b,c uchburchak tomonlari va S uchburchak yuzasi.

5. $SABC$ tetraedrning har bir qirrasi va bu qirralarga qarama-qarshi bo‘lgan qirralar o‘rtalaridan tekisliklar o‘tkazilgan. Bu o‘tkazilgan barcha tekisliklar bir nuqtada kesishishini isbotlang. Bu tasdiqlarni kesishish nuqtasini K bilan belgilab, $\overrightarrow{SK}$ vektorni $\overrightarrow{SK}=\vec{a}, \overrightarrow{SB}=\vec{b}, \overrightarrow{SC}=\vec{c}$ vektorlar orqali ifodalang.

2-tur (og‘zaki)

1-variant

Uzluksiz funksiyalarni oraliqda nolga aylanishi haqidagi Bolsano-Koshi teoremasi.

3-tur (ijodiy ish)

3-variant

Ayqash to‘g‘ri chiziqlar orasidagi masofa va uning formulasi.

4-tur (test)

2-variant

1. $x^2 + y^2 + ay = 0 \quad (a > 0)$ aylana markazidan $y = 2(a - x)$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

A) $\frac{a\sqrt{5}}{4}$ B) $\frac{a\sqrt{3}}{4}$ C) $\frac{a\sqrt{5}}{2}$ D) $\frac{\sqrt{5}}{2a}$

2. $\int \frac{dx}{\sin x}$ integralni hisoblang.

A) $\ln\frac{1+\cos x}{1+\sin x} + C$ B) $\frac{1}{2}\ln\frac{1-\cos x}{1+\sin x} + C$ C) $\ln\frac{1+\cos x}{\sin x} + C$

D) $\ln \sin x + C$

3. $|\sin x| > |\cos x|$ tengsizlikni yeching.

A) $\frac{\pi}{4} + \pi n; \frac{3\pi}{4} + \pi n$ B) $\frac{\pi}{4} + 2\pi n; \frac{3\pi}{4} + 2\pi n$ C) $-\frac{\pi}{4} + \pi n; \frac{3\pi}{4} + \pi n$

D) $\pi n; \frac{3\pi}{4} + \pi n$

4. $\lim\limits_{n\to\infty} \frac{1^3 + 2^3 + ... + n^3}{5n^4}$ ni hisoblang.

A) $\frac{1}{10}$ B) 2 C) $\frac{1}{20}$ D)0

5. $\int \frac{dx}{e^x - 1}$ ni hisoblang.

A) $\ln\left|\frac{e^x - 1}{e^x}\right| + C$ B) $\ln\left|\frac{e^x}{e^x - 1}\right| + C$ C) $\ln\left|\frac{e^x + 1}{e^x - 1}\right| + C$

D) $\ln\left|\frac{e^x - 1}{e^x + 1}\right| + C$

6. $f(x) = (x^2 + x)^{100}$ ko‘phadning barcha koeffitsiyentlarini toping.

A) 200 B) 3 C) 2^{100} D) 2^{200}

7. $f(x)$ ko‘phadni $x - c$ ga bo‘lgandagi qoldiqni toping.

A) 0 B) $f(0)$ C) $f(-c)$ D) $f(c)$

8. Agar $\vec{a}, \vec{b}, \vec{c}$ lar birlik vektorlar bo‘lib, $\vec{a} + \vec{b} + \vec{c} = 0$ bo‘lsa, $\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{a}\vec{c}$ ni qiymatini toping.

A) 0 B) $\frac{3}{2}$ C) $-\frac{3}{2}$ D) -1

9. $\vec{a}$ vektor $\vec{b}=(1;2;3)$ va $\vec{c}=(-2;4;1)$ vektorlarga perpendikulyar bo‘lib, ushbu $\vec{a}\cdot(\vec{i}-2\vec{j}+\vec{k})=6$ shartni qanoatlantirsa, $\vec{a}$ ni toping.

A) $\vec{a}=(1;2;-1)$ B) $\vec{a}=(5;\frac{7}{2};-4)$ C) $\vec{a}=(5;1;-4)$ D) $\vec{a}=(5;\frac{7}{2};-4)$

10. $\frac{x^2}{9}+\frac{y^2}{4}=1$ ellipsning $x-y+1=0$ to‘g‘ri chiziqqa parallel bo‘lgan urinmalarini toping.

A) $x-y+\sqrt{3}=0, x-y-\sqrt{3}=0$ B) $x-y+3=0, x-y-3=0$

C) $x-y+\sqrt{10}=0, x-y-\sqrt{10}=0$ D) $x-y+10=0, x-y-10=0$

11. AC va BD diagonallari o‘zaro perpendikular bo‘lgan $ABCD$ to‘rtburchakka radiusi 2 ga teng bo‘lgan aylana tashqi. Agar $AB=3$ bo‘lsa, CD ni toping.

A) $\sqrt{3}$ B) $\sqrt{10}$ C) $\sqrt{5}$ D) $\sqrt{7}$

12. $(x^2-x-3)^4$ ifoda yoyilmaida $x$ning juft darajalari oldidagi koeffitsiyentlar yig‘indisini toping.

A)40 B) 41 C)42 D) 43

13. $\lim\limits_{n\to\infty}\sin^2(\pi\sqrt{n^2+n})$ ni hisoblang.

A)0 B) 1 C)2 D) 3

14. $f(x)=x^3-3x+\lambda$ ko‘phad λ ning qanday qiymatlarida karrali ildizga ega?

A) ± 3 B) ± 1 C) ± 2 D) ± 4

15. $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}^{10}$ matritsa elementlari yig‘indisini toping.

A)12 B) 13 C)14 D)15

16. $z=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ bo‘lsa, $z^4=?$

A)-1 B)4 C)-2 D)-4

17. $\int\limits_0^1 f(x)dx=a; \int\limits_0^1 xf(x^2)dx$ ni hisoblang.

A)$\frac{a}{2}$ B)$\sqrt{a}$ C) a^2 D) $2a$

18. $f(x)=\sum_{i=1}^{5}\frac{|x-i|}{x-i}$ funksiyaning qiymatlar sohasi nechta butun sondan iborat bo‘ladi?

A)4 B)5 C)6 D)7

19. (a,b,c) nuqtadan, koordinata o‘qlaridan a, b va c uzunlikdagi kesmalar ajratuvchi tekislikkacha bo‘lgan masofa topilsin.

A) $\frac{abc}{\sqrt{a^2b^2+a^2b^2+b^2c^2}}$ B) $\frac{abc}{2\sqrt{a^2b^2+a^2b^2+b^2c^2}}$ C) $\frac{4abc}{\sqrt{a^2b^2+a^2b^2+b^2c^2}}$

D) $\frac{2abc}{\sqrt{a^2b^2+a^2b^2+b^2c^2}}$

20. $\arccos x$ funksiyani juft va toq funksiyalar yig‘indisi ko‘rinishida yozib, juft qismini ko‘rsating.

A) $\frac{\pi}{2}$ B) $\frac{\arccos x+\arcsin x}{2}$ C) $\arcsin x$ D) $\frac{\arccos x-\arcsin x}{2}$

2008-yil 16-17-may (UrDU)

1-tur (yozma ish)

4-variant

1. Haqiqiy sonlardan tuzilgan chegaralangan $\{a_n\}_{n=1}^{\infty}$ ketma-ketlik uchun quyidagi $\lim_{n\to\infty}(a_n-2a_{n+1}+a_{n+2})=0$ tenglik bajarilishi ma’lum bo‘lsa, u holda ushbu $\lim_{n\to\infty}(a_n-a_{n+1})=0$ tenglik bajarilishini isbotlang.

2. $f(x)$ funksiya $[0,1]$ kesmada Riman ma’nosida integrallanuvchi bo‘lib, $|f(x)|\le 1$ bo‘lsa, quyidagi tengsizlikni isbotlang:

$$\int_0^1\sqrt{1-f^2(x)}dx\le\sqrt{1-\left(\int_0^1 f(x)dx\right)^2}.$$

3. $ABCD$ qavariq to‘rtburchakning BC va DA qarama-qarshi tomonlarida M va N nuqtalar shunday olinganki, bunda ushbu

$$\frac{|BM|}{|MC|}=\frac{|AN|}{|ND|}=\frac{|AB|}{|CD|}$$

tenglik o‘rinli. MN to‘g‘ri chiziq AB va CD tomonlar yordamida hosil qilingan burchak bissektrisasiga parallel bo‘lishini isbotlang.

4. Aytaylik A va B n-tartibli kvadrat matritsalar va A teskarilanuvchi. $AB - BA = A$ tenglik o‘rinli bo‘lishi mumkinmi?

5. $x^2 - y^2 = 1$ giperbolaning $(1,0)$ va $(-1,0)$ nuqtalaridan farqli bo‘lgan nuqtadan $x^2 + y^2 = 1$ aylanaga ikkita urinma o‘tkazilgan. Aylananing urinish nuqtalaridan o‘tuvchi to‘g‘ri chiziq giperbolaga urinishini isbotlang.

2-tur (og‘zaki)

2-variant

Chekli orttirmalar haqidagi Lagranj teoremasi.

3-tur (ijodiy ish)

1-variant

Chiziqli tenglamalar sistemasining birgalikdaligi (birgalashganligi) haqidagi Kroneker-Kopelli teoremasi.

4-tur (test)

2-variant

1. Determinantni hisoblang. $\begin{vmatrix} 1 & 0 & 1+i \\ 0 & 1 & i \\ 1-i & -i & 1 \end{vmatrix}$

A)3 B)-1 C)-2 D)2

2. Hisoblang. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}^2$

A) $\begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ B) $\begin{pmatrix} 2 & 0 & 3 & 9 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ C) $\begin{pmatrix} 0 & 0 & 4 \\ 0 & 0 & 9 \\ 0 & 0 & 0 \end{pmatrix}$ D) $\begin{pmatrix} 9 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

3. $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ matritsaga teskari matritsaning determinantini toping.

A)-2 B) $-\frac{1}{2}$ C)-3 D)-5

4. $A=\begin{pmatrix} -1 & 3 \\ 4 & -2 \end{pmatrix}$ matritsaning xos qiymatlar yig‘indisini toping.

A)-3 B)7 C)-10 D)1

5. A matritsaning rangi 5 ga teng bo‘lsa, A'matritsaning rangini toping.

A)10 B)2,5 C)5 D)-5

6. Agar $x^3 - mx^2 + nx^2 - 5$ ko‘phad $x^2 - 1$ ko‘phadga qoldiqsiz bo‘linsa $n+m$ ni hisoblang.

A)-6 B)4 C)3 D)-5

7. $x^5 - 7x^2 - 5x - 8$ ko‘phadni $x-2$ ko‘phadga bo‘lgandagi qoldiqni hisoblang.

A)-8 B)-5 C)-7 D)-14

8. i va k larning qanday qiymatlarida $a_{62}a_{i5}a_{33}a_{k4}a_{46}a_{21}$ ko‘paytma oltinchi tartibli determinantga minus ishora bilan kiradi?

A) $i=2, k=6$ B) $i=1, k=3$ C) $i=6, k=2$

D) $i=5, k=1$

9. $\det A = 2, \det B = 3$ bo‘lsa, $\det(A^{-1}\cdot B^2)$ ni hisoblang.

A) $\frac{11}{2}$ B)2 C)0 D) $-\frac{9}{2}$

10. $\sum_{n=1}^{2008} i^n$ hisoblang (i- mavhum birlik).

A)0 B)2008 C)1004i D)1004

11. Limitni hisoblang.

$$\lim_{n\to\infty}\frac{1}{n}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}+...+\sqrt{1+\frac{n}{n}}\right)$$

A) $\frac{2}{3}\left(2\sqrt{2}-1\right)$ B) $\frac{2}{3}\left(2\sqrt{2}-2\right)$ C) $\frac{2}{3}\left(3\sqrt{2}-1\right)$ D) $\frac{2}{3}\left(2\sqrt{2}-3\right)$

12. Hisoblang. $\frac{d}{dx}\int_0^{x^2}\sqrt{1+t^2}dt$

A) $\sqrt{1+x^4}$ B) $2x\sqrt{1+x^4}$ C) $2x+\sqrt{1+x^4}$ D) 1

13. Qator yig‘indisini toping.

$$\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+...+\frac{1}{(3n-2)\cdot(3n+1)}+...+$$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{6}$ D) 1

14. Qatorning yaqinlashish radiusini toping. $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}x^n$

A) $R = 4$ B) $R=\frac{1}{4}$ C) $R = 3$ D) $R = 0$

15. Ko‘paytmani toping. $\prod_{n=2}^{\infty}(1-\frac{1}{n^2})$

A) $\frac{1}{2}$ B) 2 C) 0 D) 1

16. Hisoblang. $\cos\frac{\pi}{5}-\cos\frac{2\pi}{5}$

A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{8}$

17. $\varphi(x) = \sin^6 x + \cos^6 x$ funksiyaning eng katta qiymatini toping.

A) 1 B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{1}{3}$

18. Tengsizlikni yeching. $\left|x^2 - 2x - 3\right| < 2x - 3$

A) (2; 4) B) (2; 5) C) (1; 6) D) (3; 4)

19. Hisoblang. $\ln tg1^0 \cdot \ln tg2^0 \cdot \ln tg3^0 \cdot ... \cdot \ln tg89^0$

A) 1 B) 0 C) $\frac{1}{2}$ D) $-\frac{1}{2}$

20. Limitni hisoblang. $\lim_{\substack{x\to 0,\\ y\to 0}}\left(x^2+y^2\right)^{x^2y^2}$

A) 1 B) 0 C) $e - 1$ D) 2

21. ABC uchburchakda AC tomonga tushirilgan balandligi 2 ga AB tomoni 5 ga, ABC uchburchakka tashqi chizilgan aylana radiusi 5 ga teng bo‘lsa, BC tomonining uzunligini toping.

A) 2 B) 5 C) 4 D) $\sqrt{21}$

22. Gipotenuzasi c ga teng bo‘lgan to‘g‘ri burchakli uchburchakning o‘tkir burchaklarining kosinuslari yig‘indisi q ga teng. Uchburchakning yuzini hisoblang.

A) $\frac{c^2(q^2-1)}{4}$ B) cq C) c^2q^2 D) $\frac{c^2(q^2+1)}{4}$

23. Uchlari A(1;1), B(2;2), C(3;1) nuqtalarda bo‘lgan uchburchakda medianalar kesishgan nuqtaning koordinatalarini toping.

A) (1; 2) B) $(2; \frac{4}{3})$ C) $(\frac{1}{3}; 3)$ D) (2;1)

24. $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$, $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3, \vec{a}$ va $\vec{b}$ orasidagi burchak 60^0 bo‘lsa, $2\vec{a}(3\vec{a}-4\vec{b})$ ni hisoblang.

A) 6 B) 24 C) -24 D) 0

25. Uchlari A(0; 0), B(2; 0), C(0; -4) nuqtalarda bo‘lgan uchburchakka tashqi chizilgan aylana markazining koordinatalarini toping.

A) (1; -3) B) (1; -2) C) (-2; 1) D) (3; 4)

26. Yon tomoni a gat eng bo‘lgan teng yonli uchburchakning asosi qanday bo‘lganda uning yuzi eng katta bo‘ladi?

A) $a\sqrt{2}$ B) $a\sqrt{3}$ C) $2a$ D) a

27. vektor $\vec{b}(1,2,3)$ vektorga kollinear bo‘lib, $\vec{a}\cdot\vec{b} = 28$ bo‘lsa, $\left|\vec{a}\right|$ ni toping.

A) $\sqrt{44}$ B) $\sqrt{56}$ C) 3 D) 14

28. $y = \frac{2}{x-3}$ chiziqning assimptotalarini toping.

A) $y = 0$ B) $x = 0$ C) $x = 1$ D) $x = 3, y = 0$

29. M(-3; -5) nuqtadan o‘tib, $7x + 4y + 3 = 0$ to‘g‘ri chiziqqa parallel bo‘lgan chiziq tenglamasini tuzing.

A) $-3x + -5y + 1 = 0$ B) $7x + 4y + 41 = 0$ C) $4x + 4y + 41 = 0$
D) $4x + 4y - 41 = 0$

30. a va b ning qanday qiymatlarida $ax - 2y - 1 = 0$ va $6x - 4y - b = 0$ to‘g‘ri chiziqlar kesishmaydi?

A) $a = 3; b = 2$ B) $a = 3; b \neq 2$ C) $a \neq 3; b \neq 2$

D) $a = 2; b = 3$

31. $x^2 + y^2 - 4x - 6 = 6$ egri chiziq bilan chegaralangan figuraning yuzini hisoblang.

A) 26π B) 10π C) 6π D) 8π

32.ABC ucburchakning B va C burchaklari ayirmasi $\frac{\pi}{2}$ ga teng. Agar b va c tomonlari yig‘indisi k ga, A uchidan tushirilgan balandliik h ga teng bo‘lsa, uchburchakning C burchagini toping.

A) $2 \arcsin 2hk$ B) $\arccos\frac{k}{h}$ C) $\frac{1}{2}\arcsin\frac{2h}{k}$

D) $\frac{1}{2}\arcsin\frac{2h(h + \sqrt{h^2 + k^2})}{k^2}$

33. ABC ucburchakning balandliklari, $AA_1 = h_a$, $BB_1 = h_b$, va C burchagining bissektrisasi $CC_1 = l$ ga teng bo‘lsa, uchburchakning C burchagini toping.

A) $\arccos\frac{lh_b}{h_a^2 + h_d^2}$ B) $arctg\frac{2lh_b}{h_a^2 + h_d^2}$ C) $2\arcsin\frac{h_a h_b}{l\left(h_a + h_d\right)}$

D) $\frac{1}{2}\arcsin\frac{h_a}{l\left(h_a + h_d\right)}$

34. ABC yucburchakning B va C burchaklari nisbati 3:1, A burchakdagi bissektrisasi uchburchakning yuzini 2:1 nisbatda bo‘lsa, uchburchak burchaklarini toping.

A) $\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3}$ B) $\frac{\pi}{2}, \frac{\pi}{2}, 0$ C) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{4}$

D) $\frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{2}$

35. *ABC* uchburchakda *AM* va *BN* – bissektrisalari *O*-nuqtada kesishadi. Agar *AO:OM*$=\sqrt{3}:1$, *BO:ON*$=1:\left(\sqrt{3}-1\right)$ bo‘lsa, uchburchakning *A,B,C* burchaklarini toping.

A) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{7\pi}{2}$ B) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ C) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{\pi}{6}$ D) $\frac{\pi}{6}, \frac{\pi}{6}, \frac{2\pi}{3}$

36. Matritsaning rangini toping. $\begin{pmatrix} 8 & 2 & 2 & -1 & 1 \\ 1 & 7 & 4 & -2 & 5 \\ -2 & 4 & 2 & -1 & 3 \end{pmatrix}$

A)1 B)2 C)3 D)4

37.Matritsaning rangini toping. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 5 & 1 & 1 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}$

A)-1 B)3 C)4 D)1

38. Ikkinchi darajali haqiqiy o‘zgaruvchili shunday $f(x)$ ko‘phad topingki, uning uchun $f(1) = 8, \ f(-1) = 2, \ f(2) = 14$

A) $x^2 + 3x + 4$ B) $x^2 - 2x + 3$ C) $x^2 + 3x - 4$ D) $x^2 - 4x - 3$

39. Sistemani tekshiring va uning umumiy yechimini λ parametrga bog‘liq ravishda aniqlang.

$$\lambda x_1 + x_2 + x_3 = 1$$
$$x_1 + \lambda x_2 + x_3 = 1$$
$$x_1 + x_2 + \lambda x_3 = 1$$

A) $\lambda = -1,3; x_1 = x_2 = x_3 = \dfrac{1}{\lambda}; \lambda = -1$ da $x_1 = 3 - x_2 - x_3, \lambda = 3$ da $x_2 = x_1 - 3x_2$

B) $\lambda = 1,-2; x_1 = x_2 = x_3 = \dfrac{1}{\lambda + 2}; \lambda = 1$ da $x_1 = 1 - x_2 - x_3, \lambda = -2$ da sistema yechimga ega emas.

C) to‘g‘ri javob yo‘q

D) $\lambda = 4,2; x_1 = x_2 = x_3 = \dfrac{4}{\lambda + 1}; \lambda = 4$ da $4x_1 = 1 - x_2 - x_3, \lambda = -2$ da sistema yechimga ega emas.

40. Quyidagi determinantni hisoblang:

$$\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$$

A) $\sin(\alpha-\beta)+\sin(\alpha-\beta)+\sin(\gamma+\beta)$ B) 1

C) $\sin(\gamma+\beta)$ D) $\sin(\beta-\gamma)+\sin(\gamma-\alpha)+\sin(\alpha-\beta)$

41. Hisoblang.

$$\lim_{n\to\infty}\left(1+\sqrt{2}+\frac{\left(\sqrt{2}\right)^2}{2!}+\frac{\left(\sqrt{2}\right)^3}{3!}+\frac{\left(\sqrt{2}\right)^4}{4!}+\frac{\left(\sqrt{2}\right)^5}{5!}+...+\frac{\left(\sqrt{2}\right)^n}{n!}\right)$$

A) e B) e^{-1} C) $e^{\frac{\sqrt{2}}{2}}$ D) $e^{\sqrt{2}}$

42. Agar $f(x)=\dfrac{x}{\sqrt{1+x^2}}$ bo‘lsa, $\underbrace{f(f(f(f...f}_{2008ta}(2008)...)))$ ni hisoblang.

A) 0 B) $\dfrac{1}{\sqrt{2009}}$ C) $\dfrac{2008}{\sqrt{1+2008^3}}$ D) $\dfrac{2008}{\sqrt{1+2008^2}}$

43. Limitni hisoblang. $\lim\limits_{x\to1}\dfrac{x^2-3x+2}{x^4-4x+3}$

A) 1 B) $\dfrac{1}{2}$ C) -2 D) $-\dfrac{1}{2}$

44. Hosilani hisoblang. $y(x)=x^x+arctg(x^2+1)$.

A) $x^x(1+2\ln x)+\dfrac{2x}{1+(x^2+1)^2}$ B) $x^x(1+\ln x)+\dfrac{x}{1+(x^2+1)^2}$

C) $x^x(1+\ln x)+\dfrac{2x}{1+(x^2+1)^2}$ D) $x^x(1+\ln x)+\dfrac{x}{2(1+(x^2+1))^2}$

45. a parametrning qanday qiymatlarida $f(x)=\begin{cases}e^x, x<0\\ a+x, x\geq 0\end{cases}$ funksiya uzluksiz bo‘ladi?

A) $a=e$ B) $a=1$ C) $a=0$ D) $a=-1$

46. Limitni hisoblang. $\lim\limits_{x\to e}\dfrac{e^{ex}-x^{e^e}}{x-e^e}$

A) $e^{e^{e+1}}(e-1)$ B) 0 C) $e^{e^e}(e-1)$ D) $e^{e^e}(e+1)$

47. $\int(2^x+3^x)^2dx$ ni hisoblang.

A) $\dfrac{6^x}{\ln 6}+2+C$ B) $\dfrac{4^x}{\ln 4}+2\dfrac{6^x}{\ln 6}+\dfrac{9^x}{\ln 9}+C$

C) $\dfrac{4^x}{\ln 5}+2\dfrac{6^x}{\ln 6}+\dfrac{9^x}{\ln 9}+C$ D) $\dfrac{4^x}{\ln 4}+\dfrac{6^x}{\ln 6}+\dfrac{9^x}{\ln 9}+C$

48. $\int tgxdx$ ni hisoblang.

A) $-\ln\left|\cos x\right|+C$ B) $\ln\left|\cos x\right|+C$

C) $\ln\left|\sin x\right|+C$ D) $-\ln\left|\sin x\right|+C$

49. $\int_0^2 \left|1-x\right| dx$ ni hisoblang.

A) e B) 2008 C) 1 D) 0

50. Limitni hisoblang. $\lim_{x\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+...+\frac{n}{n^2+n^2}\right)$

A) $\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{2}$ D) $\frac{3\pi}{4}$

2009-yil 13-14-may (UrDU)

1-tur (yozma ish)

3-variant

1. Quyidagi limitni hisoblang.

$$\lim_{n\to\infty}\left(\frac{1\cdot 3\cdot 5\cdot ...\cdot(2n-1)}{2\cdot 4\cdot 6\cdot ...\cdot 2n}\right)$$

2. $x > 0$ bo‘lganda ushbu

$$\left|\int_x^{x+1}\sin t^2 dt\right| \le \frac{1}{x}$$

tengsizlikni isbotlang.

3. Quyidagi limitni hisoblang:

$$\lim_{x\to 0}\left(\lim_{x\to 0}\frac{1}{x}(A^n-E)\right)$$

Bu yerda $A=\begin{pmatrix} 1 & \frac{x}{n} \\ -\frac{x}{n} & 1 \end{pmatrix}, E=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

4. ABC uchburchakning tomonlari a,b,c bo‘lsin. ABC nuqta uchburchak tekisligidagi ixtiyoriy nuqta bo‘lsin. Ushbu $\left|MA\right|^2+\left|MB\right|^2+\left|MC\right|^2$ ifodaning eng kichik qiymatini toping.

5. α,β,γ lar ABC uchburchakning ichki burchaklari bo‘lsa, ushbu

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma \ge \frac{3}{4}$$

tengsizlikni isbotllang.

2-tur (og‘zaki)

1-variant

Tekis uzluksizlik haqidagi Kantor teoremasi.

3-tur (ijodiy ish)

1-variant

Koshi – Bunyakovskiy, Yung, Gyolder va Minkovskiy tengsizliklari.

4-tur (test)

1-variant

1. $y = o, x^2 + y^2 = 1$ chiziqlar orasidagi yuzani toping.

A) π B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{6}$

2. $\lim\limits_{x\to\infty} \frac{\int_0^x \cos t^2 dt}{x}$ ni hisoblang.

A) to‘g‘ri javob yo‘q B) 0 C) $+\infty$ D) $\frac{\pi}{2}$

3. Agar α, β, γ $x^3 + px + q = 0$ tenglamaning ildizlari bo‘lsa,

$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ ni hisoblang.

A) 1 B) aniqlab bo‘lmaydi C) 0 D) $\alpha + \beta + \gamma$

4. $\int \frac{1}{\sqrt{x - x^2}} dx$ integralni hisoblang

A) $\frac{1}{2\sqrt{2 - x^2}} + C$ B) $\arcsin\frac{x}{2} + C$ C) $\arccos\frac{x}{2} + C$ D) $\arcsin(2x - 1) + C$

5. $x^2 + y^2 + ay = 0, (a > 0)$ aylana markazidan $y = 2(a - x)$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

A) $\frac{a\sqrt{5}}{4}$ B) $\frac{a\sqrt{3}}{2}$ C) $\frac{a\sqrt{5}}{2}$ D) $\frac{\sqrt{5}}{2a}$

6. Агар $f(x) = x^2 + 12x + 30$ бўлса $f(f(f(f(f(x))))) = 0$ tenglamani yeching

A) ildizlari ko‘p, umumiy yechimini yozib bo‘lmaydi.

B) ildizlari yo‘q C) $\pm\sqrt[32]{6}-6$ D) $\pm\sqrt[32]{6}+6$

7. Berilgan $f(x)$ funksiya uchun

$f_2(x)=f(f(x))$, $f_3(x)=f(f(f(x)))$ va hakazo $f_k(x)=f(f(f...f(x))...)$

deb belgilaymiz. Agar $f(x)=\dfrac{1}{\sqrt{1+x^2}}$ bo‘lsa, u holda $\lim\limits_{n\to\infty}(\sqrt{n}f_n(x))$ ni

hisoblang $x>0$

A) mavjud emas B) -1 C) 0 D) ∞

8. $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ ellipsga ichki chizilgan kvadrat yuzasini toping.

A) a,b B) a^2+b^2 C) $\dfrac{ab}{2(a^2+b^2)}$ D) $\dfrac{4a^2b^2}{a^2+b^2}$

9. Agar $a+b=1$ tenglik o‘rinli bo‘lsa, a^4+b^4 ifodaning eng kichik qiymatini toping.

A) $\dfrac{1}{64}$ B) $\dfrac{1}{8}$ C) $\dfrac{1}{128}$ D) $\dfrac{1}{256}$

10. Agar $|x|<1$ bo‘lsa

$$1+2x+3x^2+4x^3+...$$

yig‘indining qiymati nimaga teng.

A) $\dfrac{1}{(1-x)^2}$ B) $-\dfrac{1}{(1-x)^2}$ C) $\ln(1-x)$ D) $\ln(1+x)$

11. limitni hisoblang:

$$\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{n+n}\right)$$

A) 2 B) $\ln 2$ C) $\ln 3$ D) 0

12. Agar $f'(\sin^2 x)=1+\cos^2 x$ 2bo‘lsa $f(x)$ ni toping.

A) $2x+\dfrac{x^2}{2}$ B) $2x-\dfrac{x^2}{2}$ C) $\dfrac{x^2}{2}-2x$ D) $6x^3$

13. λ va μ larning qanday qiymatlarida $\bar{a}=(\lambda+1,1,2)$, $\bar{b}=(\mu,2,\lambda-1)$ vektorlar uchun $[\bar{a}\times\bar{b}]=\bar{0}$ bo‘ladi? Bu yerda $[.,.]$ vektor ko‘paytmani bildiradi.

A) $\lambda=-5,\ \mu=0$

B) $\lambda = 5,\ \mu = 2$

C) $\lambda = 5,\ \mu = 12$

D) $\lambda = -5,\ \mu = 12$

14. $1 + 2 * 3 + 3 * 3^2 + ... + 2010 * 3^{2009}$ yig'indini hsoblang.

A) $1003 * 2^{2010} - 1$ B) $\frac{1}{4}(4019 * 2^{2010} + 1)$ C) $\frac{1}{2}(1003 * 2^{2010} + 1)$ D) 2010

15. Agar $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i = 1$ bo'lsa $\sum_{i=1}^{n} \frac{a_i^2}{a_i + b_i}$ ning eng kichik qiymatini toping.

A) $-\infty$ B) aniqlab bo'lmaydi C) n D) 1

16. $y = 2x - 6,\ y = 0,\ x = 5$ chiziqlar bilan chegaralangan figurani OY o'qi atrofida aylanishdan hosil bo'lgan jismning hajmini toping.

A) 34π B) $34\frac{2}{3}\pi$ C) 33π D) 36π

17. $y'' + m^2 y = 0$ differensial tenglama yechimini nollari orasidagi masofani hisoblang.

A) π B) $\frac{\pi}{m}$ C) πm D) aniqlab bo'lmaydi

18. Agar $\left|x_1^2 + x_2^2\right| \le 1$ bo'lsa, $f(\psi_1,\ \psi_2) = \max(\psi_1, \psi_2)$ funksiyani toping.

A) $\left|\psi_1\right| + \left|\psi_2\right|$ B) $\sqrt{\psi_1^2 + \psi_2^2}$ C) $\psi_1^2 + \psi_2^2$ D) $\left|\psi_1\right| * \left|\psi_2\right|$

19. Agar $\left|x_1\right| \le 1,\ \left|x_2\right| \le 1$ bo'lsa $f(\psi_1, \psi_2) = \max(\psi_1 x_1 + \psi_2 x_2)$ funksiyani toping.

A) $\psi_1^2 + \psi_2^2$ B) $\left|\psi_1\right| + \left|\psi_2\right|$ C) $\left|\psi_1\right| * \left|\psi_2\right|$ D) $\sqrt{\left|\psi_1\right| + \left|\psi_2\right|}$

20. Limitni hisoblang. $\lim_{n\to\infty} \frac{\sqrt[n]{n!}}{n}$

A) 0 B) $\frac{1}{e}$ C) 1 D) e

21. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = ?$

A) $(b-a)(c-a)(c-b)$ B) $a^2b^2c^2$ C) 0

D) $b^2-a^2+c^2$

22. $\int\limits_{-1}^{1} \frac{x}{\cos^3 x} dx = ?$

A) 1 B) $\frac{2}{\cos 1}$ C) $\frac{1}{\cos 2}$ D) 0

23. $f(y) = x^2 \sin x - x^3 \cos x$ funksiya uchun $x \to 0$ da $f(x) \sim kx^m$ bo'lsa k va m ni toping.

A) $k = \frac{1}{3},\ m = 5$ B) $k = 1,\ m = 3$ C) $k = \frac{1}{3},\ m = 3$

D) $k = -\frac{1}{2},\ m = 2$

24. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ ellipsga $M(5;\, 4)$ nuqtadan o'tkazilgan urinmalarning urinish nuqtalari orasidagi masofani toping.

A) 5 B) 16 C) $\sqrt{38}$ D) $\sqrt{41}$

25. $\lim\limits_{t\to 0} \int\limits_{-1}^{1} \frac{t}{t^2 + x^2} \cos x dx = ?$

A) 1 B) 0 C) π D) $2\cos 1$

26. Uchlari $A(1;\, 2;\, 3)$ $B(5;\, 2;\, 1)$ $C(0;\, 4;\, 4)$ nuqtalarda bo'lgan uchburchakning yuzasini hisoblang.

A) $\sqrt{21}$ B) 5 C) 1 D) $\sqrt{12}$

27. $rang\left\{\left[\begin{pmatrix} 1 & 1 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{pmatrix} * \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}\right]\right\} = ?$

A) 1 B) 2 C) 3 D) 0

28. $A = \begin{pmatrix} 2001 & 2002 \\ 2003 & 2004 \end{pmatrix}$ $B = \begin{pmatrix} 2005 & 2006 \\ 2007 & 2008 \end{pmatrix}$

$\det\left(A^{-1}B^{-1}\right) = ?$

A) 0 B) -2 C) 1 D) 2009

29. $\varphi(n)$ orqali n dan kichik va n bilan o'zaro tub sonlar sonini belgilasak, $\varphi(1996)$ ni toping.

A) 1995 B) 1996 C) 996 D) 1001

30. 13^{20} ni 19 ga bo‘lganda qoldiqni toping.

A) 5 B) 12 C) 13 D) 17

31. $x^2 + 2xy + 3y^2 - 6x - 4y + 16 = 0$ egri chiziq markazini koordinatalari topilsin.

A) $(3; -1)$ B) $(1.5; -0.5)$ C) $(3,5; -0,5)$ D) $(3; 2)$

32. $f_1(x) = x^2$ *va* $f_1(x) = x - 1$ funksiyalar grafiklari orasidagi eng qisqa masofani toping.

A) 1 B) $\frac{\sqrt{3}}{4}$ C) $\frac{3\sqrt{2}}{3}$ D) $\frac{3\sqrt{2}}{8}$

33. $\lim\limits_{n\to\infty} \frac{1^3 + 2^3 + 3^3 + ... + n^3}{n^4} = ?$

A) ∞ B) 0 C) 0.25 D) $\frac{1}{3}$

34. $\lim\limits_{n\to\infty} \sqrt[n]{2n+1}$ hisoblang.

A) 1 B) 0 C) ∞ D) limit mavjud emas

35. ABCD muntazam tetraedr qirrasi 4 ga teng P, Q, M, N lar mos ravishda AB, AC BD CD qirralar o‘rtalari $\vec{AD}*\vec{AB}+\vec{PQ}*\vec{MN} = ?$

A) 0 B) 12 C) 4 D) 16

36. $\begin{cases} 2p^2 + k^2 - 2pk = 25 \\ 2pq - q^2 = 25 \end{cases}$ bo‘lsa $\frac{p+q}{k^2}$ ni toping.

A) aniqlab bo‘lmaydi B) 1 C) $0,25$ D) $\pm 0,4$

37. $A = \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$ bo‘lsa, quyidagi ko‘phadlarning qaysi birisi uchun $P(A) = 0$ bo‘ladi?

A) $P(\lambda) = \lambda^2 + 2\lambda + 9$ B) $P(\lambda) = \lambda^2 + 10\lambda + 1$

C) $P(\lambda) = \lambda^2 - 10\lambda + 1$ D) $P(\lambda) = \lambda^2 - 10\lambda + 15$

38. $u_1: \dfrac{x-2}{4}=\dfrac{y+1}{1}=\dfrac{z-1}{-1}, \quad u_2: \dfrac{x+u}{2}=\dfrac{y-2}{-2}=\dfrac{z+2}{-3}$ to‘g‘ri chiziqlarda perpendikulyar $\vec{u}$ vektorni toping.

A) $\vec{u}=(-5, 10, -10)$ B) $\vec{u}=(-2, 5, 5)$

C) $\vec{u}=(4, 1, -1)$ D) $\vec{u}=(2, -2, -3)$

39. λ ning qanday qiymatlarida $x^3+16x+\lambda=0$ kubik tenglama barcha ildizlari haqiqiy bo‘ladi?

A) $\lambda < -12$ B) ixtiyoriy qiymatlarida

C) $\lambda = 0$ D) hech bir qiymatlarida

40. Muntazam tetraedrning qirrasi 1 ga teng tetraedr ichidagi ixtiyoriy nuqtadan uning yoqlarigacha bo‘lgan masofalar yig‘indisini toping.

A) 1 B) $\dfrac{\sqrt{6}}{3}$ C) $\dfrac{\sqrt{6}}{2}$ D) $\sqrt{12}$

41. ABC uchburchakning medianalari 3, 4 va 5 ga teng. Uchburchaklarning yuzini hisoblang.

A) 5 B) 10 C) 8 D) 25

42. ABC uchburchakning yuzi 48 ga teng. Uning 10 va 12 teng medianalari. Bu uchburchakning 3 ta uchburchakka va 1 ta 4 burchakka ajratildi. Hosil bo‘lgan 4 burchakning yuzini toping.

A) 12 B) 8 C) 10 D) 16

43. $f(x_1x_2+x_2x_3+x_3x_4+x_4x_1)$ kvadrat formani kanonik ko‘rinishga keltirganda javobda nechta kvadrat qatnashadi?

A) 4 B) 3 C) 1 D) 2

44. $f(x)=\dfrac{\cos x}{1+\sin x}$ bo‘lsa $f'\left(\dfrac{\pi}{2}\right)=?$

A) 0 B) $-\dfrac{1}{2}$ C) 1 D) $\dfrac{1}{2}$

45. $f(x)=\left|x^2-3x-4\right|, \quad f'(0)=?$

A) 3 B) -3 C) mavjud emas D) 4

46. a ning qanday qiymatlarida quyidagi $f(x)$ funksiya uzluksiz bo‘ladi?

$$f(x)=\begin{cases}\dfrac{x^2\sin x+tg^3x}{x^3}, & x>0\\ e^{x+a}, & x\le 0\end{cases}$$

A) $a = 0$ B) $a = 1$ C) hech bir qiymatida uzluksiz bo‘lmaydi

D) $a = \ln 2$

47. $A=\begin{pmatrix}3 & -1 & 0\\ 6 & -3 & 2\\ 8 & -6 & 5\end{pmatrix}$ matritsa barcha xos qiymatlarining yig‘indisini toping.

A) –5 B) 1 C) –1 D) 5

48. Quyidagi parallel tekisliklar orasidagi masofani toping.

$$n_1:\ 4x+3y-5z-8=0$$
$$n_2:\ 4x+3y-5z+12=0$$

A) 20 B) 4 C) $2\sqrt{2}$ D) 10

49. $y^2=2x$ parabolaga muntazam uchburchak ichki chizilgan. Bu uchburchakning yuzini hisoblang.

A) $3\sqrt{3}$ B) $12\sqrt{3}$ C) 12 D) $6\sqrt{3}$

50. Quyidagini hisoblang: $\left(\dfrac{1+i\sqrt{3}}{1-i}\right)^{20}$

A) $2^9(1-i\sqrt{3})$ B) $2^{10}(1-i\sqrt{3})$ C) $-2^9(1-i\sqrt{3})$ D) $2^9(1+i\sqrt{3})$

2010-yil 9 – 10-may (UrDU)

1-tur (yozma ish)

2-variant

1. $\forall A,B\in C[m\times m]$ matritsalar uchun quyidagi tenglik o‘rinli ekanini isbotlang.

$$\det(I+AB)=\det(I+BA)$$

2. $f:R\to R$ akslantirishlar orasida ixtiyoriy $x,y\in R$ lar uchun $(f(x)-f(y))^2\le|x-y|^3$ shartni qanoatlantiruvchi o‘zgarmaslardan farqli funksiya mavjudmi?

3. $f(x)$ uzluksiz va irratsional qiymat qabul qilmaydi. Agar $f(\frac{1}{4})=\frac{1}{5}$ tenglik o‘rinli bo‘lsa, $f(\frac{1}{2010})$ ni toping.

4. Agar n-tartibli D_n determinantning elementlari faqat 1 va -1 lardan iborat bo‘lib, $n \geq 3$ bo‘lsa, u holda $\left|D_n\right| \leq (n-1)(n-1)!$ tengsizlikni isbotlang.

5. α, β, γ lar ABC uchburchakning ichki burchaklari bo‘lsa, quyidagi

$$\cos\alpha+\cos\beta+\cos\gamma \leq \frac{3}{2}$$

tengsizlikni isbotlang.

2-tur (og‘zaki)

2-variant

Kubik tenglamani yechish. Kordano formulasi.

3-tur (ijodiy ish)

2-variant

Chiziqli tenglamalar sistemasini Kramer usulida yechish.

4- tur (test)

3-variant

1. $y=F(x)$ funksiya $y=f(x)$ funksiya uchun boshlang‘ich funksiya bo‘lsa, u holda $y=\sqrt{\pi}f(-\sqrt{\pi}x)$ funksiyaning boshlang‘ich funksiyasini toping.

A) $-\sqrt{\pi}F(-\sqrt{\pi}x)+c$ B) $-\frac{1}{\sqrt{\pi}}F(-\sqrt{\pi}x)+c$ C) $\pi F(-\sqrt{\pi}x)+c$

D) $-F(-\sqrt{\pi}x)+c$

2. $\sqrt{x}^{x}=x^{\sqrt{x}}$ tenglama yechimlarining yig‘indisini toping.

A) 1 B) 4 C) 3 D) 5

3. $\left(x^2-x-3\right)^4$ ifoda yoyilmasida $x$ning toq darajalari oldidagi koefisentlarini yig‘indisini toping.

A) 42 B) 41 C)40 D) 43

4. $\int \frac{dx}{\sin(x+a)\sin(x+b)}$ integralni hisoblang.

A) $\frac{1}{\sin 2a}\ln\left|\frac{\sin(x-a)}{\sin(x+a)}\right|+C$ B) $\frac{2}{\sin(a-b)}\ln\left|\frac{\cos(x+a)}{\cos(x+b)}\right|+C$

C) $\frac{2}{\sin(a-b)}\ln\left|\frac{\sin(x+a)}{\cos(x+b)}\right|+C$ D) $\frac{2}{\sin(a-b)}\ln\left|\frac{\sin(x+a)}{\sin(x+b)}\right|+C$

5. $\frac{x^2}{9}+\frac{y^2}{4}=1$ ellips berilgan $(-2;1)$ nuqta orqali shu nuqtada teng ikkiga bo'linuvchi vatar o'tgazilsin.

A) $9x-8y+25=0$ B) $9x+8y+10=0$

C) $8x+9y-26=0$ D) $8x-9y+25=0$

6. Quyidagi limitni hisoblang: $\lim\limits_{n\to\infty}\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{4}+...+\frac{(-1)^{n+1}}{n}\right)$

A) 0 B) $\frac{1}{2}$ C) e^{-1} D) $\ln 2$

7. Quyidagi matrissaviy tenglama yechimining determinantini toping:

$$\begin{pmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{pmatrix} X = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 2 & 4 & 5 \end{pmatrix}$$

A) $4a$ B) $-2a$ C) $-\frac{1}{4a}$ D) $\frac{1}{a}$

8. $\begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$ matritsaga teskari matritsaning deteminantini toping.

A) 1 B) 2 C) $\sin 2\varphi$ D) 0

9. $\int\limits_{-\pi}^{\pi}\frac{1}{5+3\cos\varphi}d\varphi$ integralni hisoblang.

A) $\frac{\pi}{\sqrt{38}}$ B) $\frac{\pi}{5}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$

10. $\int\limits_{0}^{2\pi}\frac{dx}{\cos x-2}$ ni hisoblang.

A) $\frac{\pi}{\sqrt{2}}$ B) $\frac{4\pi}{\sqrt{3}}$ C) $-\frac{2\pi}{\sqrt{3}}$ D) $-\frac{4\pi}{\sqrt{3}}$

11. $\dfrac{x^2}{4}+\dfrac{y^2}{8}=1$ ellipsga $M(2;\sqrt{2})$ nuqtadan o‘tkazilgan urinmalarning urinish nuqtalari orasidagi masofani toping.

A) 4 B) 2,4 C) $2\sqrt{3}$ D) 2

12. $f(y)=x^4\cos x - x^3\sin x + x^7\sin\dfrac{3}{4}x$ funksiya $x\to 0$ da $f(x)\sim kx^m$ bo‘lsa u holda $\dfrac{k-m}{k+m}$ ni toping

A) $-\dfrac{21}{19}$ B) $\dfrac{21}{19}$ C) $-\dfrac{19}{17}$ D) $-\dfrac{9}{19}$

13. Quyidagi limitni hisoblang: $\lim\limits_{x\to 0}\left(\dfrac{m}{1-(x+1)^m}-\dfrac{n}{1-(x+1)^n}\right)$

A) 0 B) mn C) $\dfrac{m+n}{mn}$ D)1

14. Balandliklari 4, 6, 4 bo‘lgan uchburchakning yuzini toping.

A) 24 B) 12 C) $12\sqrt{3}$ D) $9\sqrt{2}$

15. 2010 songa 19 qanday daraja bilan kiradi.

A) 110 B) 101 C) aniqlab bo‘lmaydi D) 120

16. Agar $\sum\limits_{i=1}^{\infty}a_n^2$, $\sum\limits_{n=1}^{\infty}b_n^2$ qatorlar yaqinlashuvchi bo‘lsa, u holda quyidagi qatorlarning qaysilari doimo yaqinlashuvchi bo‘ladi.

1) $\sum\limits_{n=1}^{\infty}\dfrac{a_n}{n}$ 2) $\sum\limits_{n=1}^{\infty}|a_n+b_n|$ 3) $\sum\limits_{n=1}^{\infty}|a_nb_n|$

4) $\sum\limits_{n=1}^{\infty}(a_n+b_n)^2$ 5) $\sum\limits_{n=1}^{\infty}\dfrac{a_n}{b_n}$

A) 1),2),3) B) 1),3),4) C) 1),2),3),4),5) D) 4),5)

17. Agar S_n arifmetil progressiyaning birinchi n ta hadining yig‘indisi bo‘lsa, $(S_{n+3}-3S_{n+2}+3S_{n+1}-S_n)$ ni hisoblang.

A) 1 B) S_n C) n D) 0

18. Aylanaga $ABCD$ to‘rtburchak ichki chizilgan agar $AB=3$, $BC=4$, $CD=7$, $AD=5$ bo‘lsa uning diognallari ko‘paytmasini toping.

A) 47 B) 43 C) 41 D) 42

19. Quyidagi determinatni hisoblang: $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2008 & 2009 & 2010 & 2011 \\ 2008^2 & 2009^2 & 2010^2 & 2011^2 \\ 2008^3 & 2009^3 & 2010^3 & 2011^3 \end{vmatrix}$

A) 6 B) 0 C) 12 D 3

20. $x^4 + 4x^3 + \frac{5}{2}x^2 - 2x - \frac{3}{16} = 0$ tenglamaning barcha haqiqiy ildizlari joylashgan oraliqni toping.

A) (-2,1) B) (-5,5) C) $(-\frac{3}{4};\frac{1}{2})$ D) $(-\frac{3}{4};\frac{1}{4})$

21. Berilgan kvadratni teng 49 ta kvadratchalarga bo‘lindi. Hosil bo‘lgan shaklda nechta kvadrat hosil bo‘ladi?

A) 50 B) 140 C) 343 D) 344

22. Hisoblang: $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7}$

A) $\frac{1}{2}$ B) 0 C) 1 D) $\frac{1}{\sin\frac{\pi}{7}}$

23. $\varphi(n)$ orqali n dan kichik va n bilan o‘zaro tub sonlar sonini belgilasak $\varphi(2010)$ ni toping.

A) 2009 B) 528 C) 1005 D) 1001

24. $\text{rang}\left\{\begin{pmatrix} 5 & -1 & 7 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix} * \begin{pmatrix} 1 & -8 & 3 \\ 2 & 1 & 1 \\ 4 & 7 & -4 \end{pmatrix}\right\} = ?$

A) 1 B) 2 C) 3 D) 0

25. AC va BD diagonallari o‘zaro perpendikular bo‘lgan $ABCD$ to‘rtburchakka radiusi 2 ga teng bo‘lgan aylana tashqi chizilgan, agar $AB = 3$ bo‘lsa, CD ni toping

A) $\sqrt{3}$ B) 3 C) $\sqrt{5}$ D) $\sqrt{7}$

2011-yil 10-11-may (UrDU)

1-tur (yozma ish)

2-variant

1. $f(x) \in C[0,1], f(x) > 0$ bo‘lsa, $\lim_{n\to\infty}\left(\int_0^1 \sqrt[n]{f(x)}dx\right)^n$ ni hisoblang.

2. A determinanti noldan farqli kvadrat matrisa bo‘lsin.Xar bir satri noldan farqli $+1$ yoki -1 dan tuzilgan bo‘lsin.Biror natural k uchun $A^k = A^T$ ekanligini ko‘rsating.

3. $\{x_n\}$ ketma-ketlik berilgan bo‘lsin. Agar $\forall m, n \in N$ uchun $0 \le x_{m+n} \le x_m + x_n$ bo‘lsa, $\lim_{n\to\infty} \frac{x_n}{n}$ ni toping.

4. Har qanday f va g fazodagi vektorlar jufti uchun $af^2 + bfg + cg^2 \ge 0$ tengsizlik o‘rinli bo‘lishi uchun $a \ge 0, c \ge 0, 4ac \ge b^2$ bo‘lishi zarur va yetarli ekanini isbotlang.

5. $a > b > 1$ bo‘lsa, $a^{b^a} > b^{a^b}$ ekanligini isbotlang.

2-tur (og‘zaki)

3-variant

Funksiyaning tekis uzluksizligi haqidagi Kantor teoremasi.

3-tur (ijodiy ish)

2-variant

Ikkinchi tartibli egri chiziqlar va ularning klassifikatsiyasi.

4- tur (test)

3-variant

1.Qaysi biri katta? $a = \prod_{n=1}^{25}\left(1 - \frac{n}{365}\right)$ yoki $b = \frac{1}{2}$

A) $a > b$ B) $a < b$ C) $a = b$ D) $a = 2b$

2. ''Qonuniy Ma’sudiy'' asari muallifining ismi kim?

A)Beruniy B)Abu Rayhon C)Muhammad D)Ahmad

3. Yuqori limitni hisoblang $\lim\limits_{n\to\infty} \frac{2n^2}{2-n+n^2} \cdot \sin\frac{2\pi n}{3}$

A) $\frac{\sqrt{3}}{2}$ B) $\sqrt{3}$ C) $-\sqrt{3}$ D)1

4. $y = \sin(\cos^2 x)$ funksiyaning hosilasini toping.

A) $\sin(\cos^2)\cos x$ B) $\sin^2 x(\cos^2 x)$ C) $-\sin(\cos^2 x)\sin x$ D) $-\cos(\cos^2 x)\sin 2x$

5. Tenglamani yeching. $\sin\left(2x + \frac{\pi}{2}\right) = \frac{\pi}{3}$

A) $(-1)^{n+1}\frac{1}{2} - \frac{\pi}{4} + \frac{\pi n}{2},\ n \in Z$ B) $(-1)^{n+1}\arcsin\frac{\pi}{3} - \frac{\pi}{4} + \frac{\pi n}{2},\ n \in Z$

C) $(-1)^{n+1}\frac{\sqrt{3}}{2} - \frac{\pi}{4} + \frac{\pi n}{2},\ n \in Z$ D) yechimga ega emas

6. $\varphi(n)$ orqali n dan kichik va n bilan o‘zaro tub sonlar sonini belgilasak $\varphi(2010)$ ni toping.

A) 533 B)528 C)1001 D)1005

7. Aylanaga $ABCD$ to‘rtburchak ichki chizilgan. Agar AB=3 BC=4 CD=7 AD=5 bo‘lsa, uning diagonallari ko‘patmasini toping.

A) 47 B)43 C)41 D)42

8. Limitni hisoblang $\lim\limits_{k\to\infty}\sum\limits_{k=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}$

A) 0 B) $e-1$ C) $\frac{e}{2}$ D) $\frac{5}{3}$

9. $\sin x = \frac{x}{100}$ tenglama nechta yechimga ega?

A) 31 B) 32 C) 61 D) 63

10. $\begin{vmatrix} 1 & 2 & 4 & 3 \\ 4 & 5 & -1 & 6 \\ -1 & 2 & 5 & 7 \\ 7 & 8 & 1 & 9 \end{vmatrix}$ determinant a_{33} elementining algebraik to‘ldiruvchisini toping.

A) -1 B) 1 C) 0 D) 2

11. Soddalashtiring. $\left(\frac{1}{\sqrt{2}-1}\right)^{\log_6 \log_6\left(\frac{1}{\sqrt{2}-1}\right)}$

A) 1 B) $\log_6(\sqrt{2}+1)$ C) $\log_6(\sqrt{2}-1)$ D) $\frac{1}{\sqrt{2}-1}$

12. Ushbu 6-tartibli determinantning hadi yig‘indida qanday ishora bilan qatnashadi? $a_{12}a_{26}a_{41}a_{54}a_{65}$

A) “+” ishora bilan

B)”-” ishora bilan

C) 6-determinantning hadi bo‘lmaydi

D) Aniqlab bo‘lmaydi

13. $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 5 \\ 1 & 3 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{pmatrix} rangA = ?$

A) 3 B)4 C) 5 D)2

14. Tengsizlikni yeching: $\frac{4x^2}{(1-\sqrt{1+2x})^2} < 2x+9$

A) $-\frac{1}{2} < x < 5\frac{5}{8}$ B) $-\frac{1}{2} < x \le 5\frac{5}{8} \cup \{0\}$ $-\frac{1}{2} < x \le 5\frac{5}{8} \cup \{0\}$

C) $0 \le x \le 5$ D) $-\frac{1}{2} \le x < 5\frac{5}{8}$

15. $\sqrt{\log_m a + 1}$, $\sqrt{\log_n a + 1}$, $\sqrt{\log_l a + 1}$ larning nisbati, mos ravishda, 5:6:7 kabi va yig‘indisi 36 ga teng bo‘lsa, $\log_{mnl} a$ ni toping.

A)45 B) 42 C) 22,5 D)21

16. $\lim\limits_{x \to \infty} x^{\frac{1}{1-x}}$ ni hisoblang.

A) 1 B) e C) $\frac{1}{e}$ D) -1

17. i^i ni hsoblang.

A) $e^{-\frac{\pi}{2}}$ B) $e^{\frac{\pi}{2}}$ C) $\frac{\pi}{2}$ D) 1

18. ”Ziji ko‘ragoniy” asari muallifining ismi kim?

A)Muhammad Tarag‘ay B) Ulug‘bek C)Mirzo D)Ali Qushchi

19. To‘g‘ri burchakli uchburchakning yuzi $\frac{\sqrt{3}}{12}(a^2+3b^2)$ kvadrat birlikka ega. Uning burchaklarini toping.

A) $30^0, 60^0, 90^0$ B) $45^0, 45^0, 90^0$ C) $15^0, 75^0, 90^0$ D) $40^0, 50^0, 90^0$

20. Agar $f(x)=\frac{x}{\sqrt{1+x^2}}$ $bo'lsa,$ $f_n(x)=f(f(f(...f(x))))$ bo‘lsa (bu yerda n ta qavs), $f_n(x)$ ni soddalashtiring.

A) $\frac{x}{\sqrt{n(1+x^2)}}$ B) $\frac{x}{\sqrt{n+x^2}}$ C) $\frac{xn}{\sqrt{1+nx^2}}$ D) $\frac{x}{\sqrt{1+nx^2}}$

21. Tomoni 1 ga teng bo‘lgan ikkita kvadrat ustma-ust qo‘yildi. Shundan so‘ng kvadratlardan biri umumiy simmetriya markaziga nisbatan 45^0 ga burildi. Hosil bo‘lgan figura yuzini toping.

22. Muntazam to‘g‘ri burchakli prizmaning asosi yuzi Q ga teng. Uning yoqlari diagonallari nisbati 1:3 kabi. Shu prizma hajmini toping.

23. Agar $x_1^2+x_2^2\le 1$ $bo'lsa,$ $f(\psi_1,\psi_2)$ ni toping.

24. Integralni hisoblang. $\int_0^1 \ln x\ln(1-x)dx$

25. $\frac{d}{dx}\int_a^b (\sin x)^2\,dx$ ni hisoblang.

2012-yil 14 – 15-may (QDU)

1-tur (yozma ish)

1. $\{x_n\}$ ketma-ketlikning limitini toping. Bu yerda

$x_1 = \sqrt{a}$, $x_2 = \sqrt{a+\sqrt{a}}$, $x_3 = \sqrt{a+\sqrt{a+\sqrt{a}}},\dots$ $a \ge 1$.

2. $\begin{vmatrix} a & b & b & b & ..b \\ c & a & b & b & ..b \\ c & c & a & b & ..b \\ \dots & \dots & \dots & \dots & \dots \\ c & c & c & c.. & a \end{vmatrix}$ n-tartibli determinantni hisoblang.

3. $f(x)$ kvadrat uchhad $|f(x)| \le 1$, $(0 \le x \le 1)$ shartni qanoatlantiradi. U holda $f'(0)$ ning eng kichik qiymatini toping.

4. $y^2 = 2px$ parabolaga A, B, C nuqtalarda o‘tkazilgan urinmalar KLM uchburchakni hosil qiladi. $S_{KLM} = \frac{1}{2} S_{ABC}$ ekanini isbotlang.

5. Agar $\lim\limits_{n\to\infty} x_n \sum\limits_{k=1}^{n} x_k^2 = 1$, bo‘lsa $\lim\limits_{n\to\infty} \sqrt[3]{3x_n}$ limitni hisoblang.

2-tur (og‘zaki)

Uzluksiz funksiyaning chegaralanganligi haqidagi Veyershtrass teoremasi.

3-tur (ijodiy ish)

Differensial hisobning asosiy teoremalari (Ferma, Roll, Lagranj teoremalari).

4-tur (test)

C-variant

1. Agar $f\left(\frac{x}{x+1}\right) = x^2$ bo‘lsa, $f(x)$ ni toping.

A) $f(x) = \left(\frac{x}{1-x}\right)^2$ B) $f(x) = \left(-\frac{1}{1-x}\right)^2$ C) $f(x) = \left(\frac{x}{1+x}\right)^2$ D) $f(x) = x^2$

2. To‘g‘ri burchakli $(AC \perp CB)$ ABC uchburchakning A uchidan AK mediana, B uchidan BD bissektrisa tushirilgan va ular O nuqtada kesishada, BO:OD=8:5 bo‘lsa B burchagining kosinusini toping.

A) 0,5 B)0,6 C)0,8 D)0,28

3. $f(x) = x^{x^2}$, bo‘lsa $f'(x) = ?$

A) $x^{x^2}(x^x \ln(x+1) * \ln x - \frac{x^x}{x})$ B) $x^{x^2}(x^x \ln(x+1) * \ln x + \frac{x^x}{x})$

C) $x^{x^2}(x^x \ln(x-1) * \ln x + \frac{x^x}{x})$ D) $x^{x^2}(x^x \ln(x-1) * \ln x - \frac{x^x}{x})$

4. $y_1 = x$ va $y_2 = 0.5x - 1$ funktsiyalarining grafiklari orasidagi eng qisqa masofani toping.

A) $\frac{3\sqrt{3}}{8}$ B) $\frac{3\sqrt{3}}{4}$ C) $\frac{3\sqrt{5}}{8}$ D) $\frac{3\sqrt{5}}{4}$

5. $\sum_{n=0}^{\infty} \frac{\sin(x + n\frac{\pi}{2})}{n!}$, $x \in R$ qatorning yig‘indisini toping.

A) $\sin(x+1)$ B) $\sin(x-1)$ C) $\cos(x+1)$ D) $\cos(x-1)$

6. Agar $f(x) = x(x-1)(x-2)...(x-100)$ *bo'lsa,* $f'(0)$ ni toping.

A)0 B)100 C)100! D)1000!

7. Integralni hisoblang. $\int \frac{dx}{\cos(x+9)\cdot\cos(x+7)}$.

A) $\frac{2}{\sin 2}\ln\left|\frac{\cos(x+9)}{\cos(x+7)}\right|$ B) $\frac{2}{\sin 16}\ln\left|\frac{\cos(x+7)}{\cos(x+9)}\right|$

C) $\frac{2}{\sin 2}\ln\left|\frac{\cos(x+7)}{\cos(x+9)}\right|$ D) $\frac{1}{\sin 2}\ln\left|\frac{\cos(x+7)}{\cos(x+9)}\right|$

8. Limitni toping. $\lim_{n\to\infty} \sin(\pi\sqrt{n^2+1})$

A)0 B)1 C)2 D)aniqlanmagan

9. Quyidagi limitni hisoblang. $\lim_{n\to\infty}\left(\frac{2^{\frac{1}{n}}}{n+1} + \frac{2^{\frac{2}{n}}}{n+\frac{1}{2}} + ... + \frac{2^{\frac{n}{n}}}{n+\frac{1}{n}}\right)$.

A) e B) e^{-1} C) $\ln 2$ D) $\frac{1}{\ln 2}$

10. $y = \frac{2}{x-3}$ chiziqning asimptotalarini toping.

A) $y = 0$ B) $x = 0$ C) $x = 1$ D) $x = 3, y = 0$

11. $2012!$ songa 13 qanday daraja bilan kiradi.

A) 110 B) 165 C) aniqlab bo‘lmaydi D) 120

12. Determinantni hisoblang $\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$.

A) $\sin(\alpha-\beta)+\sin(\gamma+\beta)$ B) 1 C) $\sin(\alpha-\gamma)$

D) $\sin(\beta-\gamma)+\sin(\gamma-\alpha)+\sin(\alpha-\beta)$

13. Quyidagi matrissaviy tenglama yechimining determinantini toping.

$$\begin{pmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{pmatrix} X = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 2 & 4 & 5 \end{pmatrix}$$

A) $4a$ B) $-2a$ C) $-\frac{1}{4a}$ D) $\frac{1}{4a}$

14. a ning qanday qiymatlarida quyidagi $f(x)$ funksiya uzluksiz bo‘ladi?

$$f(x) = \begin{cases} \dfrac{x^2\sin x + tg^3 x}{x^3}, & x > 0 \\ e^{x+a}, & x \le 0 \end{cases}$$

A) $a=0$ B) $a=1$ C) hech bir qiymatida uzluksiz bo‘lmaydi

D) $a=\ln 2$

15. $\lim\limits_{n\to\infty} n\left(\dfrac{1}{am+a} - \int\limits_1^0 \dfrac{x^m}{a+x^n}dx\right)$ agar $a>0$ bo‘lsa, limitni hisoblang:

A) $\ln a$ B) $\ln\dfrac{a}{a+1}$ C) $\dfrac{1}{\ln a}$ D) To‘g‘ri javob yo‘q

16. Agar ABC uchburchakda $\angle A = 30^0, \angle B = 50^0$ bo‘lsa, u holda uchburchak tomonlari orasidagi munosabatni toping.

A) $a=\dfrac{c^2-b^2}{b}$ B) $a=-\dfrac{c^2-b^2}{b}$ C) $a=\dfrac{c^2-b}{b}$ D) Bog‘lanish yo‘q

17. $a_n = \sum\limits_{k=1}^{n}\dfrac{1}{C_n^k}$ bo‘lsa, limitni hisoblang $\lim\limits_{n\to\infty} a_n^n = ?$

A) $\frac{1}{e^2}$; B) e^2; C) e; D) $\frac{1}{e}$

18. $\begin{vmatrix} 1 & 2 & 4 & 3 \\ 4 & 5 & -1 & 6 \\ -1 & 2 & 5 & 7 \\ 7 & 8 & 8 & 1\ 9 \end{vmatrix}$ determinant a_{33} elementining algebraik to‘ldiruvchisini toping.

A) -1 B) 1 C) 0 D) 2

19. $\frac{x^2}{9}+\frac{y^2}{4}=1$ ellips berilgan nuqta orqali shu nuqtada teng ikkiga bo‘linuvchi vatar o‘tgazilsin.

A) $9x-8y+25=0$ B) $9x+8y+10=0$

C) $8x+9y-26=0$ D) $8x-9y+25=0$

20. $r^2=2a^2\cos 2\phi$ qaysi chiziqni ifoda etadi?

A) Arhimed spirali B)Bernulli lemniskatasi C)sikloida D) vint chizig‘i

21. $\lim\limits_{x\to\infty}\frac{e^x-e^{-x}-2x}{x-\sin x}$ ni hisoblang.

a) 2 b)0 c) $\propto$ d) mavjud emas

22. $\int\limits_0^{\pi/4}\frac{\sin x}{\sin x+\cos x}dx=?$

A) $\frac{\pi}{8}+\frac{1}{4}\ln 2$ B) $\frac{\pi}{8}+\frac{1}{2}\ln 2$ C) $\frac{\pi}{8}+\frac{1}{8}\ln 2$ D) $\frac{\pi}{4}+\frac{1}{8}\ln 2$

23. $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{4\cdot 5\cdot 6}+\frac{1}{7\cdot 8\cdot 9}+...=?$

A) $\frac{\pi\sqrt{3}}{12}+\frac{1}{2}\ln 2$ B) $\frac{\pi\sqrt{3}}{12}-\frac{1}{4}\ln 2$ C) $\frac{\pi\sqrt{3}}{12}+\frac{1}{4}\ln 2$ D) $\frac{\pi\sqrt{3}}{12}-\frac{1}{4}\ln 3$

24. $0.5[x]+10\{x\}=10$ tenglamaning barcha ildizlari yig‘indisini toping.

A) 219,5 B) 220 C) 210 D) 199,5

25. Agar $\lim\limits_{x\to\infty}\left(\sqrt[3]{1-x^3}-\lambda x-\mu\right)=0$, bo‘lsa, $\lambda+\mu$ ni toping.

A) -1 B) 1 C) 0 D) 2

2013- yil 10 – 11-may (QarDU)

1-tur (yozma ish)

3-variant

1. Tengsizlikni isbotlang. $e^{\frac{x+y}{2}} \le \frac{e^x + e^y}{2}$ $\quad x, y \in R$

2. Oshkormas ko‘rinishda berilgan

$u = f_1(x;y)$ va $v = f_2(x;y)$ funksiyalarning $(1;0;1;-2)$ nuqtasidagi xususiy hosilalarini toping.

$$\begin{cases} xu + yv - u^3 = 0 \\ x + y + u + v = 0 \end{cases}$$

3. Agar $f(x)$, $[0,1]$ da uzluksiz , diferensiallanuvchi va $f(1) - f(0) = 1$ bo‘lsa,

$$\int_0^1 (f'(x))^2 \ge 1$$

tengsizlikni isbotlang.

4. Tomonlari $a > b > c$ bo‘lgan uchburchak berilgan. Uchburchak ichidagi ixtiyoriy O nuqta orqali AO, BO, CO to‘g‘ri chiziqlar o‘tkazilgan bo‘lib, ular ABC uchburchak tomonlarini P, Q, R nuqtalarda kesadi. $OP + OQ + OR < a$ ekanligini isbotlang.

5. Limitni toping. $\lim\limits_{n\to\infty} \begin{pmatrix} 1 & \frac{\alpha}{n} \\ -\frac{\alpha}{n} & 1 \end{pmatrix}^n$

2-tur (og‘zaki)

3-variant

Differensial hisobning asosiy teoremalari

3-tur (ijodiy ish)

3-variant

1. Kvadratik formani kanonik ko‘rinishga keltirish. Lagranj usuli.

2. Ikkinchi tartibli egri chiziqlarni kanonik ko‘rinishga keltirish.

4-tur (test)

3-variant

1. Qatorning yaqinlashish radiusini toping (bunda z- kompleks son).

$$\sum_{n=0}^{\infty} 5^n z^{3n}$$

A. $\sqrt{5}$ B. $\sqrt[3]{5}$ C. $\frac{1}{\sqrt{5}}$ D. $\frac{1}{\sqrt[3]{5}}$

2. $s_n = \frac{1}{\sqrt{4n^2 - 1^2}} + \frac{1}{\sqrt{4n^2 - 2^2}} + ... + \frac{1}{\sqrt{4n^2 - n^2}}$ ketma-ketlik limitini toping.

A. $\frac{6}{\pi}$ B. $\frac{\pi}{6}$ C. 1 D. $\frac{1}{2}$

3. Agar $f(x) = x^2 + 14x + 42$, bo‘lsa $f(f(f(f(x)))) = 0$ tenglamani yeching.

A) ildizi yo‘q; B) $\pm\sqrt[16]{7} - 7;$; C) $\pm\sqrt[32]{7} + 7;$; D) $\pm\sqrt[16]{7} + 7;$

4. ABC ucburchakning B va C burchaklari ayirmasi $\frac{\pi}{2}$ ga teng. Agar b va c tomonlari yig‘indisi k ga, A uchidan tushirilgan balandliik h ga teng bo‘lsa, uchburchakning C burchagini toping.

A) $2\arcsin 2hk;$ B) $\arccos\frac{k}{h};$ C) $\frac{1}{2}\arcsin\frac{2h}{k};$ D) $\frac{1}{2}\arcsin\frac{2h(h+\sqrt{h^2+k^2})}{k^2}.$

5. Limitni hisoblang. $\lim\limits_{n\to\infty}\left(\frac{x+\sqrt{x}}{x-\sqrt{x}}\right)^x$

A) $\propto$ B) e C) $\frac{1}{e}$ D) 1

6. Limitni hisoblang. $\lim\limits_{n\to\infty}\frac{n}{\sqrt[n]{n!}}$

A) e B) 1 C) 0 D) $\frac{1}{e}$

7. $x^2 + y^2 = 4$ oshkormas funksidan y' ni toping.

A) $y' = -\frac{x}{y}$ B) $y' = \frac{y}{x}$ C) $y' = x + y$ D) topib bo‘lmaydi

8. $x = a\cos t$, $y = a\sin t$, $z = \sqrt{R^2 - a^2 t}$, $R > a > 0$, vint chizig‘ining bosh normali topilsin.

A) $\nu = -i\cos t + j\sin t$ B) $\nu = i\cos t + j\sin t + k$

C) $\nu = i\cos t - j\sin t - k$ D) $\nu = i\cos t - j + k\sin t$

9. Ketma-ketlik limitini toping. $x_1 = 0,\ \ x_{n+1} = x_n - arctgx_n + \frac{\pi}{4},\ \ n \in N$

A) 1 B) $\frac{\pi}{6}$ C) $\frac{\pi}{3}$ D) 0

10. $\sin x = \frac{x}{100}$ tenglama nechta yechimga ega?

A) 31 B) 32 C) 61 D) 63

11. $u = x\sin\frac{1}{y} + y\sin\frac{1}{x}$ bo'lsa, $\lim\limits_{x\to 0}\lim\limits_{y\to 0},\ \lim\limits_{y\to 0}\lim\limits_{x\to 0},\ \lim\limits_{\substack{x\to 0\\ y\to 0}}$ larni toping.

A) 0; 1; mavjud emas;

B) mavjud emas; mavjud emas;0;

C) 0; mavjud emas; 0;

D) 1; 0; mavjud emas;

12. Agar $a^2 + b^2 = 1$, bo'lsa, $a^6 + 3a^2b^2 + b^6$ ning qiymatini toping.

A) 0 B) 2 C) 1 D) 3

13. $x + C^2y + z - 2C = 0$ sirtlar oilasining o'ramasini toping.

A) $(x - y)z = 1$ B) $2(x + y)z = 1$

C) $xyz = 1$ D) $xy + yz = 1$

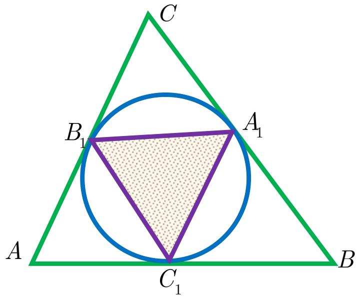
14. ABC da $AB = 7, BC = 6, CA = 5$, bo'lsa ABC uchburchakka ichki chizilgan aylana AB, BC, CA tomonlarga, mos ravishda, C_1, A_1, B_1 nuqtalarda urinsa $A_1B_1C_1$ uchburchak yuzini toping.

A) t.j.y

B) $\sqrt{6}$

C) $\frac{48\sqrt{6}}{35}$

D) $\frac{47\sqrt{6}}{35}$



15. Hisoblang $\int\limits_0^{2008} x(x-4)(x-8)...(x-2008)dx$.

A) 0 B) –1 C) 1 D) 4

16. Tomonlarining uzunligi 13, 14 va 15 bo‘lgan uchburchakka ichki va tashqi chizilgan aylanalar markazlari orasidagi masofani toping.

A) $\frac{\sqrt{65}}{8}$ B) $\frac{\sqrt{65}}{4}$ C) $\frac{\sqrt{65}}{12}$ D) $\frac{\sqrt{65}}{10}$

17. ABC uchburchakning AB va BC tomonidan $AC_1 : C_1B = 1:4$ va $BA_1 : A_1C = 1:3$ shartni qanoatlantiruvchi C_1 va A_1 nuqtalar olingan. Agar AA_1 va CC_1 kesmalar P nuqtada kesishsa, $AP_1 : PA$ *va* $CP : PC_1$ nisbatni toping.

A) 12:1 va 7:1 B) 15:1 va 4:1

C) 7:1 va 4:1 D) 16:1 va 9:1

18. $|x| + \left|\frac{x+1}{3x-1}\right| = a$ tenglama a ning qanday qiymatlarida uchta ildizga ega bo‘ladi?

A) $2; \frac{16}{9}$ B) $5; \frac{2}{11}$ C) 9 D) 2

19. $x = 3t - t^3,\ y = 3t^2,\ z = 3t + t^3$ egri chiziqning qaysi nuqtasidagi urinma $3x + y + z + 2 = 0$ tekislikka parallel bo‘ladi.

A) $A_1(-2, 3, -4)$, $A_2(2, 12, -14)$

B) $A_1(-18, 27, 36)$, $A_2(2, 12, -14)$

C) $A_1(-2, 12, 14)$, $A_2(-2, 3, -4)$

D) $A_1(-2, 12, 14)$, $A_2(-18, 27, 36)$

20. $x + y + \frac{2}{x+y} + \frac{1}{2xy}$ $(x > 0, y > 0)$ ifodaning eng katta qiymatini toping.

A) $\frac{47}{12}$ B) $\frac{7}{2}$ C) $\frac{15}{2}$ D) $\frac{\sqrt{845}}{3}$

21. $\sin 6x$ *ni* $\cos x$ *va* $\sin x$ orqali ifodalang.

A) $6\cos^5 x \sin x - 20\cos^2 \sin^3 x + 6\cos x \sin^5 x$

B) $-6\cos^5 x \sin x + 20\cos^2 \sin^3 x + 6\cos x \sin^5 x$

C) $\cos^5 x \sin x - 10\cos^3 \sin^3 x + 4\cos x \sin^5 x$

D) $\cos^6 x + 6\cos^5 \sin x + 20\cos^3 x \sin^5 x$

22. Tenglamaning haqiqiy sonlarda nechta yechimi bor?

$$(2x-1)(3x+1)(5x+1)(30x+1)=10$$

A) 1 B) 2 C) 4 D) 3

23. $\begin{vmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{vmatrix}$ determinantni hisoblang.

A) 12 B) 14 C) -12 D) 10

24. $x, y, z \in R^+$ sonlar uchun $xy+z=(x+z)(y+z)$ bo‘lsa $xyz_{\max}$ ni toping.

A) $\frac{1}{27}$ B) $\frac{3}{2}$ C) $\frac{1}{3}$ D) $\frac{9}{4}$

25. Quyidagi matrissaviy tenglama yechimining determinantini toping.

$$\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix} X = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 2 & 4 & 5 \end{vmatrix}$$

A) $4a$ B) $-2a$ C) $-\frac{1}{4a}$ D) $\frac{1}{a}$

6-§. Javoblar, yechimlar va ko'rsatmalar

2006-yil

2006.1. Biz quyidagi ifodani qaraymiz:

$$\frac{1}{b-a}\int_a^b (f(x)-1)^2\,dx = \frac{1}{b-a}\int_a^b f^2(x)\,dx - \frac{2}{b-a}\int_a^b f(x)\,dx + \frac{1}{b-a}\int_a^b dx = 1-2+1=0$$

nomanfiy funksiyaning aniq integrali ham nomanfiy ekanligidan $f(x)=1$ tenglik kelib chiqadi.

2006.2. Quyidagicha belgilash kiritamiz:

$$n\left(f\left(\frac{x^2}{n}\right)-1\right) = \frac{\left(\frac{x^4}{n^2}+1\right)e^{\frac{x^2}{n}}-1}{\frac{1}{n}} = \left|\frac{x^2}{n}=\alpha, \alpha\to 0(n\to\infty)\right| = \frac{(\alpha^2+1)e^{\alpha}-1}{\frac{\alpha}{x^2}}$$

U holda funksiyaning $x\in[0,1]$ da uzluksiz ekanligini hisobga olsak va α ga nisbatan Lopital qoidasini qo'llasak, quyidagiga ega bo'lamiz:

$$\lim_{n\to\infty} n\int_0^1 \left(f\left(\frac{x^2}{n}\right)-1\right)dx = \int_0^1 \lim_{\alpha\to 0}\frac{(\alpha^2+1)e^{\alpha}-1}{\frac{\alpha}{x^2}}dx = \int_0^1 \lim_{\alpha\to 0}\frac{(\alpha^2+2\alpha+1)e^{\alpha}}{\frac{1}{x^2}}dx = \int_0^1 x^2dx = \frac{1}{3}$$

Javob: $\frac{1}{3}$

2006.3. Agar $I+AB$ matritsaning teskarisi mavjud bo'lsa, u holda $\det(I+AB)\neq 0$. Bizdan $\det(I+BA)\neq 0$ ekanligini ko'rsatish talab etiladi. Quyidagi hollar bo'lishi mumkin:

1-hol. A-teskarilanuvchi ya'ni, $\det A\neq 0$.

Ravshanki, $I+AB = AA^{-1}+AB = A(A^{-1}+B)$ va

$I+BA = AA^{-1}+BA = (A^{-1}+B)A$

Bu tengliklardan quyidagiga ega bo'lamiz.

$\det(I+AB) = \det A(A^{-1}+B) = \det A\cdot\det(A^{-1}+B)$

$\det(I+BA) = \det(A^{-1}+B)A = \det(A^{-1}+B)\cdot\det A$

Demak, $\det(I+AB)=\det(I+BA)\neq 0$.

2-hol. $\det A = 0$ Yuqoridagi teoremaga ko'ra, $\exists\lambda_1,\lambda_2,...,\lambda_m\in C$ va $A_\lambda\in C[m\times m]$ topilib $\forall\lambda\in\mathbb{C}\setminus\{\lambda_1,\lambda_2,...\lambda_m\}$ uchun $\det A\neq 0$ va

$\lim\limits_{\lambda\to 0}\det A_\lambda = \det A$ bo'ladi. A_λ teskarilanuvchiligidan, 1-holga ko'ra, $\det\left(I + A_\lambda B\right) = \det\left(I + BA_\lambda\right)$ tenglik o'rinli.

Determinant uzluksiz funksiya ekanidan foydalanib, $\lambda \to 0$ da limitga o'tamiz va quyidagilarni hosil qilamiz.

$$\lim_{\lambda\to 0}\det\left(I + A_\lambda B\right) = \det\lim_{\lambda\to 0}\left(I + A_\lambda B\right) = \det\left(I + AB\right)$$

$$\lim_{\lambda\to 0}\det\left(I + BA_\lambda\right) = \det\lim_{\lambda\to 0}\left(I + BA_\lambda\right) = \det\left(I + BA\right)$$

Demak, $\det\left(I + BA\right) = \det\left(I + AB\right) \neq 0$

Yuqoridagi natijalardan $\det\left(I + BA\right) \neq 0$ ekanligi kelib chiqdi. Bu esa $I + BA$ matritsaning teskarilanuvchiligini bildiradi. Isbot tugadi.

2006.4. Masala shartiga ko'ra $a \le x_i \le b, \left(i = 1,2,3,...,n\right)$. U holda $0 \ge \left(x_i - a\right)\left(x_i - b\right)$ tengsizlik o'rinli. Bu tengsizlikni har bir x_i lar uchun yozib chiqamiz.

$$\begin{cases} x_1\left(a+b\right) \ge x_1^2 + ab \\ x_2\left(a+b\right) \ge x_2^2 + ab \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_n\left(a+b\right) \ge x_n^2 + ab \end{cases} \Rightarrow \begin{cases} a + b \ge x_1 + \dfrac{ab}{x_1} \\ a + b \ge x_2 + \dfrac{ab}{x_2} \\ \dots\dots\dots\dots\dots\dots \\ a + b \ge x_n + \dfrac{ab}{x_n} \end{cases} \Rightarrow$$

$$\Rightarrow n\left(a+b\right) \ge \left(x_1 + x_2 + x_n\right) + ab\left(\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n}\right) \ge$$

$$\ge 2\sqrt{ab\left(x_1 + x_2 + x_n\right)\left(\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n}\right)} \Rightarrow$$

$$\Rightarrow\left(x_1 + x_2 + x_n\right)\left(\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n}\right) \le \frac{n^2\left(a+b\right)^2}{4ab}$$

2007-yil

2007.1. $-\Delta_n = \sum\limits_{\nu=1}^{n}\left(a + \nu\cdot\dfrac{b-a}{n} - x\right)f'\left(\xi_\nu\right)dx$ bunda $a + \left(\nu - 1\right)\cdot\dfrac{b-a}{n} - x < \xi_\nu < a + \nu\cdot\dfrac{b-a}{n}$. Natijada,

$\dfrac{1}{2}\left(\dfrac{b-a}{n}\right)^2\sum\limits_{\nu=1}^{n} m_\nu \le -\Delta_n \le \dfrac{1}{2}\left(\dfrac{b-a}{n}\right)^2\sum\limits_{\nu=1}^{n} M_\nu$ $\nu - m$ yarim intervalda M_ν va

m_ν larga mos keluvchi yuqori va quyi hosilasi $f'(x)$ ga ega bo‘lamiz:

$$\lim_{n\to\infty} n\Delta_n = \frac{b-a}{2}\left[f(a)-f(b)\right]$$

2007.2. $f(x), g(x) \in C[a,b]$ va

$$\left|\int_a^b f(x)g(x)dx\right|^2 = \int_a^b |f(x)|^2 dx \cdot \int_a^b |g(x)|^2 dx$$ bo‘lsin.

$(f(x)-c\cdot g(x))^2 \geq 0$ funksiyani qaraymiz. $\forall c \in R$ son uchun $(f(x)-c\cdot g(x))^2 \geq 0$ ekanidan $\int_a^b (f(x)-c\cdot g(x))^2 dx \geq 0$ va

$\int_a^b f^2(x)dx - 2c\int_a^b f(x)g(x)dx + c^2\int_a^b g^2(x)dx \geq 0$ fiksirlangan f va g uchun bu kvadrat uchxad c ga nisbatan $(-\infty, +\infty)$ yechimga ega va

$$D = 4\left(\int_a^b f(x)g(x)dx\right)^2 - 4\int_a^b f^2(x)dx \cdot \int_a^b g^2(x)dx \leq 0$$ va

$$\left(\int_a^b f(x)g(x)dx\right)^2 \leq \int_a^b f^2(x)dx \cdot \int_a^b g^2(x)dx$$

Bu tengsizlikda tenglik faqat $f(x) - c\cdot g(x) \equiv 0$ bo‘lganda yoki f va g lardan birortasi aynan nolga teng bo‘lgan holda bajariladi.

Demak, agar

1) $f \neq 0, g \neq 0$ bo‘lganda $f(x) = c\cdot g(x)$

2) $g = 0$ bo‘lganda f ixtiyoriy uzluksiz funksiya

3) $f = 0$ bo‘lganda g ixtiyoriy uzluksiz funksiya bo‘ladi.

2007.3. $x = n \in N$ bo‘lsin,u holda $n^2+n+1 = q_1^{s_1}q_2^{s_2}...q_m^{s_m}$, $p = q_j$ bo‘lsin, u holda $n^2+n+1 = q_j\left(q_1^{s_1}q_2^{s_2}...q_j^{S_j-1}...q_m^{s_m}\right)$ tenglama uchun $x = n$ $, y = q_1^{s_1}q_2^{s_2}...q_j^{S_j-1}...q_m^{s_m}$ sonlar butun yechim bo‘ladi.

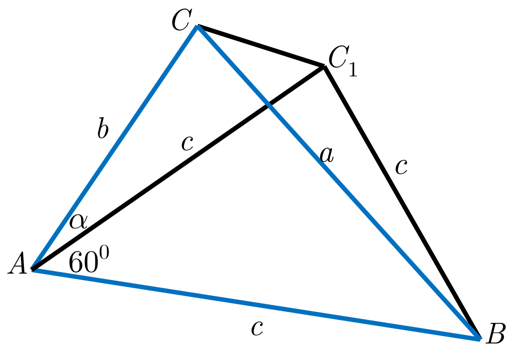
Faraz qilaylik, berilgan tenglama butun yechimlarga ega bo‘ladigan p tub sonlar soni cheklita bo‘lsin, ya`ni faqatgina $p_1, p_2, ..., p_k$ tub sonlardagina berilgan tenglama butun yechimlarga ega bo‘lsin. U holda

$x = \prod_{i=1}^{k} P_i$ $x = \prod_{i=1}^{k} P_i$ bo‘lganda $\left(\prod_{i=1}^{k} P_i\right)^2 + \left(\prod_{i=1}^{k} P_i\right) + 1$ son uchun P_i lardan birortasi ham tub bo‘luvchi bo‘la olmaydi. Demak, yuqoridagi mulohazalarga ko‘ra, shunday bir $P = P_{k+1}$ tub son topiladiki, $p_{k+1} \neq p_i,\ i = \overline{1,k}$ va $\left(\prod_{i=1}^{k} P_i\right)^2 + \left(\prod_{i=1}^{k} P_i\right) + 1 = p_{k+1}y$. Tenglik biror butun y son uchun o‘rinli bo‘ladi. Shunday qilib, biz ziddiyatga keldik.

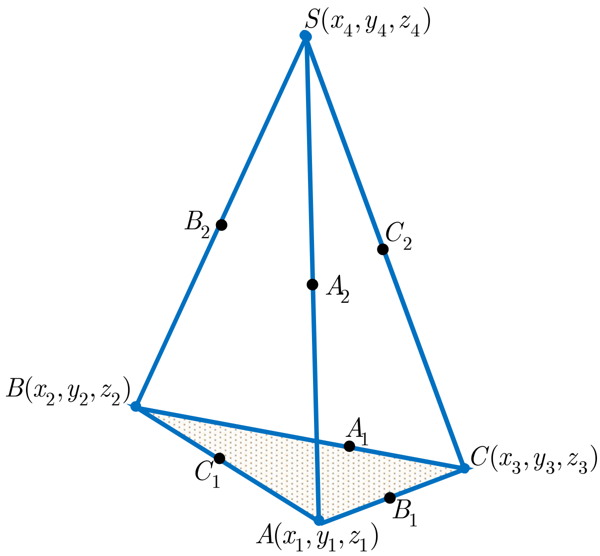
Demak, berilgan tenglama cheksiz ko‘p tub sonlarda butun yechimlarga ega bo‘ladi.

2007.4. Kosinuslar teoremasiga ko‘ra:

$$|CC_1|^2 = b^2 + c^2 - 2bc\cos\alpha = b^2 + c^2 - 2bc\cos\left(60^0 + \alpha - 60^0\right) =$$
$$= b^2 + c^2 - 2bc\left(\cos\left(60^0\right)\cos\left(60^0 + \alpha\right) + \sin\left(60^0\right)\sin\left(60^0 + \alpha\right)\right) =$$
$$= b^2 + c^2 - 2bc\left(\frac{1}{2}\cos\left(60^0 + \alpha\right) + \frac{\sqrt{3}}{2}\sin\left(60^0 + \alpha\right)\right) =$$
$$= b^2 + c^2 - \frac{2bc\cos\left(60^0 + \alpha\right)}{2} - \frac{2\sqrt{3}bc\sin\left(60^0 + \alpha\right)}{2} =$$
$$= \frac{2b^2 + 2c^2 - 2bc\cos\left(60^0 + \alpha\right)}{2} - 2\sqrt{3}S =$$
$$= \frac{b^2 + c^2 + b^2 + c^2 - 2bc\cos\left(60^0 + \alpha\right)}{2} - 2\sqrt{3}S = \frac{a^2 + b^2 + c^2}{2} - 2\sqrt{3}S$$



2007.5.



Dekart koordinatalar sistemasini kiritib olamiz. Bu sistemada $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$, $S(x_4, y_4, z_4)$ bo‘lsin. AB, BC, CA, AS, BS, CS qirralarning o‘rtalarini, mos ravishda, $C_1, A_1, B_1, A_2, B_2, C_2$ orqali belgilaymiz. A_1A_2 kesma A_2BC va A_1AS tekisliklar kesishmasiga tegishli, B_1B_2 kesma B_2AC va B_1BS tekisliklar kesishmasiga tegishli, C_1C_2 kesma C_2AB va C_1CS tekisliklar kesishmasiga tegishli. Shuning uchun A_1A_2, B_1B_2, C_1C_2 kesmalar bitta nuqtada kesishishini isbot qilish yetarli.

$$A_1\left(\frac{x_1+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right), \quad A_2\left(\frac{x_1+x_4}{2}, \frac{y_1+y_4}{2}, \frac{z_1+z_4}{2}\right)$$

bo‘lgani uchun A_1A_2 kesmaning o‘rtasi

$$K_1\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$$ bo‘ladi.

$$B_1\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right), \quad B_2\left(\frac{x_2+x_4}{2}, \frac{y_2+y_4}{2}, \frac{z_2+z_4}{2}\right)$$ bo‘lgani

uchun B_1B_2 kesmaning o‘rtasi

$$K_2\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$$

bo‘ladi. $C_1\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$, $C_2\left(\frac{x_3+x_4}{2}, \frac{y_3+y_4}{2}, \frac{z_3+z_4}{2}\right)$ bo‘lgani uchun C_1C_2 kesmaning o‘rtasi

$$K_3\left(\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right)$$

bo'ladi. Demak, K_1, K_2, K_3 nuqtalar koordinatalari bir xil va demak, A_1A_2, B_1B_2, C_1C_2 kesmalar bitta

$$K\left(\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right)$$

nuqtada kesishadi.Shunday qilib masalada aytilgan tekisliklar bitta nuqtada kesishishi isbotlandi. $\overrightarrow{SK}$ vektorni $\overrightarrow{SA}=\vec{a}, \overrightarrow{SB}=b, \overrightarrow{SC}=\vec{c}$ vektorlar orqali ifodalaymiz:

$$\overrightarrow{SK}=\left(\frac{x_1+x_2+x_3-3x_4}{4},\frac{y_1+y_2+y_3-3y_4}{4},\frac{z_1+z_2+z_3-3z_4}{4}\right)=$$

$$=\frac{1}{4}(x_1-x_4,y_1-y_4,z_1-z_4)+\frac{1}{4}(x_2-x_4,y_2-y_4,z_2-z_4)+\frac{1}{4}(x_3-x_4,y_3-y_4,z_3-z_4)=$$

$$=\frac{1}{4}\overrightarrow{SA}+\frac{1}{4}\overrightarrow{SB}+\frac{1}{4}\overrightarrow{SC}=\frac{1}{4}\left(\vec{a}+\vec{b}+\vec{c}\right)$$

Javob: $\overrightarrow{SK}=\frac{1}{4}\left(\vec{a}+\vec{b}+\vec{c}\right)$

2008-yil

2008.1. $\{a_n\}_{n=1}^{\infty}$ ketma-ketlik chegaralanganligi uchun quyidagi quyi va yuqori limitlar chekli bo'ladi:

$$\underline{\lim}_{n\to\infty}(a_n-a_{n+1})=a\,,\qquad \overline{\lim}_{n\to\infty}(a_n-a_{n+1})=b\,.$$

Quyi va yuqori limitlarning ta'rifiga ko'ra $a \le b$ bo'ladi. Biz $a=b=0$ bo'lishini isbotlaymiz. $b>0$ deb faraz qilaylik. Yuqori limitning ta'rifiga ko'ra, shunday $a_{n_k}-a_{n_k+1}$ qismiy ketma-ketlik borki, u b ga intiladi. Bu qismiy ketma-ketlikni $a_{n_k}-a_{n_k+1}>\frac{b}{2}$ tengsizlik bajariladigan qilib tanlash oson, buning uchun bu tengsizlik bajarilmaydigan cheklita hadni tashlab yuborish kifoya.

$\lim\limits_{n\to\infty}(a_n-2a_{n+1}+a_{n+2})=0$ bulgani uchun limitning ta'rifiga kura ixtiyoriy N natural son uchun shunday $n_0=n_0(N)$ nomer topiladiki, bunda $n>n_0$ bo'lganda ushbu

$$(a_n-a_{n+1})-(a_{n+1}-a_{n+2})<\frac{b}{2N}$$

tengsizlik o'rinli bo'ladi. Agar biz $n_k > n_0(N)$ deb olsak, quyidagi tengsizliklar o'rinli bo'ladi:

$$(a_{n_k+1} - a_{n_k+2}) > (a_{n_k} - a_{n_k+1}) - \frac{b}{2N} > \frac{b}{2} - \frac{b}{2N} = \frac{b(N-1)}{2N},$$

$$(a_{n_k+2} - a_{n_k+3}) > (a_{n_k+1} - a_{n_k+2}) - \frac{b}{2N} > \frac{b(N-1)}{2N} - \frac{b}{2N} = \frac{b(N-2)}{2N},$$

....

$$(a_{n_k+N-1} - a_{n_k+N}) > \frac{b}{2N}.$$

Bu tengsizliklarni qo'shib, quyidagiga ega bo'lamiz:

$$a_{n_k+1} - a_{n_k+N} > \frac{b}{2N}((N-1)+(N-2)+...+1) = \frac{b(N-1)}{4}.$$

Bunga ko'ra, N ning ixtiyoriyligi $\{a_n\}_{n=1}^{\infty}$ ning chegaralanganligiga zid keladi. Demak, farazimiz noto'gri, ya'ni $b \le 0$ ekan. $a \ge 0$ bo'lishi ham xuddi shunday isbot qilinadi. Demak, $0 \le a \le b \le 0$, ya'ni $a = b = 0$ ekan, bu esa, quyidagi limit mavjud va uning qiymati nolga teng bo'lishini bildiradi: $\lim_{n\to\infty}(a_n - a_{n+1}) = 0$.

2008.2. Koshi-Bunyakovskiy tengsizligiga ko'ra,

$$\left(\int_0^1 1\cdot\sqrt{1-f^2(x)}dx\right)^2 \le \int_0^1 (1-f^2(x))dx = 1 - \int_0^1 f^2(x)dx. \qquad (1)$$

Yanada Koshi-Bunyakovskiy tengsizligiga ko'ra,

$$\left(\int_0^1 1\cdot f(x)dx\right)^2 \le \int_0^1 f^2(x)dx \qquad (2)$$

bo'ladi. (1) va (2) tengsizliklardan esa

$$\left(\int_0^1 \sqrt{1-f^2(x)}dx\right)^2 \le 1 - \left(\int_0^1 f(x)dx\right)^2$$

kelib chiqadi. Bundan ildiz chiqarsak, talab qilingan tengsizlik hosil bo'ladi. Isbotlandi.

2008.3. $\frac{|BM|}{|MC|}=\frac{|AN|}{|ND|}=\frac{|AB|}{|CD|}=\lambda$ deb olamiz. U xolda $\overline{BM}=\lambda\overline{MC}$,

$\overline{AN}=\lambda\overline{ND}$ lardan $\overline{BM}=\frac{\lambda}{\lambda+1}\overline{BC}$ va $\overline{AN}=\frac{\lambda}{\lambda+1}\overline{AD}$ kelib chiqadi.

Bularga ko'ra

$\overline{MN}=\overline{MB}+\overline{BA}+\overline{AN}=-\frac{\lambda}{\lambda+1}\overline{BC}+\overline{BA}+\frac{\lambda}{\lambda+1}\overline{AD}=\frac{\lambda}{\lambda+1}(\overline{AD}-\overline{BC})+\overline{BA}$

Ushbu $|BA|\cdot\overline{CD}$ va $|CD|\cdot\overline{BA}$ vektorlarning uzunliklari o'zaro teng bo'lgani uchun ularning yig'indisi, ya'ni

$\overline{p}=|BA|\cdot\overline{CD}+|CD|\cdot\overline{BA}=|CD|\cdot(\lambda\overline{CD}+\overline{BA})$ vektor BA va CD tomonlar yordamida hosil qilingan burchak bissektrissasi bo'yicha yo'naladi.

$\overline{CD}=-\overline{BC}+\overline{BA}+\overline{AD}$ bulgani uchun $\overline{p}=|CD|\cdot[\lambda(\overline{AD}-\overline{BC})+(\lambda+1)\overline{BA}]=$

$$=|CD|\cdot(\lambda+1)[\frac{\lambda}{\lambda+1}(\overline{AD}-\overline{BC})+\overline{BA}]=|CD|\cdot(\lambda+1)\cdot\overline{MN}$$

Demak, $\overline{p}$ va $\overline{MN}$ o'zaro parallel ekan. Isbotlandi.

2008.4. Mumkin emas. Mumkin deb faraz qilaylik. U xolda $AB-BA=A$ tenglikdan $ABA^{-1}-B=E$ kelib chiqadi. Bu yerda E orqali n-tartibli birlik matritsa belgilangan. ABA^{-1} va B matritsalarning xarakteristik ko'phadlari ustma-ust tushadi. Bunga ko'ra ularning izlari ham bir xil bo'ladi. Demak, $tr(ABA^{-1}-B)=tr(ABA^{-1})-trB=0$ bo'ladi. Ikkinchi tomondan, esa $trE=n$ bo'ladi. Ziddiyat. Demak, farazimiz noto'g'ri ekan. Masala to'lik yechildi.

2008.5. (x,y) nuqta giperbolaning $(1,0)$ va $(-1,0)$ nuqtalardan farqli ixtiyoriy nuqtasi bo'lsin. Bitta urinma aylanaga (x_1,y_1) nuqtada urinsin. Urinish nuqtalardan o'tuvchi to'g'ri chiziq (x,y) vektorga perpendikular bo'ladi. Shuning uchun undagi ixtiyoriy nuqta quyidagi ko'rinishda bo'ladi: (x_1+ty,y_1-tx). Bu to'g'ri chiziq giperbolaga urinishini tekshirish uchun

$\max_{t\in R}\left((x_1+ty)^2-(y_1-tx)^2\right)=1$ ni isbotlaymiz:

$$\max_{t\in R}\Big(-t^2+2(x_1y+xy_1)t+(x_1^2-y_1^2)\Big)=(x_1y+xy_1)^2+(x_1^2-y_1^2)=$$
$$=(x_1y+xy_1)^2+(x_1^2-y_1^2)(x^2-y^2)=(xx_1+yy_1)^2=$$
$$=\Big((x_1^2+y_1^2)+(x-x_1)x_1+(y-y_1)y_1\Big)^2=1,$$

Chunki $x_1^2+y_1^2=1$ va (x,y), (x_1,y_1) nuqtalardan o'tuvchi urinma (x_1,y_1) vektorga ortogonal. Isbotlandi.

2009-yil

2009.1. Ushbu $a_n=\dfrac{1\cdot 3\cdot 5\cdot ...\cdot(2n-1)}{2\cdot 4\cdot 6\cdot ...2n}$ belgilashni kiritib olamiz. U holda

$$a_n^2=\frac{1^2\cdot 3^2\cdot 5^2\cdot ...\cdot(2n-1)^2}{2^2\cdot 4^2\cdot 6^2\cdot ...(2n)^2}=\frac{1\cdot 3}{2^2}\cdot\frac{3\cdot 5}{4^2}\cdot\frac{5\cdot 7}{6^2}...\frac{(2n-3)(2n-1)}{\big(2(n-1)\big)^2}\cdot\frac{(2n-1)(2n+1)}{(2n)^2}\cdot\frac{1}{2n+1}=$$

$$=\frac{2^2-1}{2^2}\cdot\frac{4^2-1}{4^2}\cdot\frac{6^2-1}{6^2}...\frac{\Big(2\big(n-1\big)^2\Big)-1}{\big(2(n-1)\big)^2}\cdot\frac{(2n)^2-1}{(2n)^2}\cdot\frac{1}{2n+1}<1\cdot 1\cdot 1\cdot ...\cdot 1\cdot 1\cdot\frac{1}{2n+1}=\frac{1}{2n+1}$$

bo'ladi. Demak, $0<a_n<\dfrac{1}{\sqrt{2n+1}}$, $n=1,2,...$ Ikkita millitsioner haqidagi teoremaga ko'ra,

$$0\le\lim_{n\to\infty}\le\lim_{n\to\infty}\frac{1}{\sqrt{2n+1}}=0$$

Javob: $\{0\}$

2009.2. Ushbu integralni qaraymiz:

$$\int_x^{x+1}t\sin t^2dt$$

Bunda aniq integral uchun o'rta qiymat haqidagi teoremani qo'llasak, shunday ξ, $x\le\xi\le x+1$ nuqta topilib, ushbu tenglik o'rinli bo'ladi

$$\int_x^{x+1}t\sin t^2dt=\xi\cdot\int_x^{x+1}\sin t^2dt$$

Natijada ushbu tenglikka kelamiz:

$$\xi\cdot\int_x^{x+1}\sin t^2dt=\int_x^{x+1}t\sin t^2dt=\frac{1}{2}[\cos x^2-\cos(x+1)^2]$$

bundan

$$\left|\xi\cdot\int\limits_x^{x+1}\sin t^2dt\right| = \left|\frac{1}{2}[\cos x^2 - \cos(x+1)^2]\right| \le 1$$

boshqa tomondan

$$x\cdot\left|\int\limits_x^{x+1}\sin t^2dt\right| \le \xi\cdot\left|\int\limits_x^{x+1}\sin t^2dt\right| = \left|\xi\cdot\int\limits_x^{x+1}\sin t^2dt\right| \le 1$$

Natijada berilgan tengsizlikka ega bo‘lamiz:

$$\left|\int\limits_x^{x+1}\sin t^2dt\right| \le \frac{1}{x}$$

Misol to‘liq yechildi.

2009.3. $\begin{vmatrix} 1-\lambda & \dfrac{x}{n} \\ -\dfrac{x}{n} & 1-\lambda \end{vmatrix} = 0 \qquad (\lambda-1)^2 + \dfrac{x^2}{n^2} = 0 \qquad \lambda - 1 = \pm i\dfrac{x}{n}$

$\lambda_1 = 1 + i\dfrac{x}{n}$, $\lambda_2 = 1 - i\dfrac{x}{n}$ xos qiymatlari $\lambda_1 = 1 + i\dfrac{x}{n}$ xos qiymatga mos keluvchi xos vektorni topamiz:

$$\begin{pmatrix} -i\dfrac{x}{n} & \dfrac{x}{n} \\ -\dfrac{x}{n} & -i\dfrac{x}{n} \end{pmatrix}\cdot\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad -i\frac{x}{n}a_1 + \frac{x}{n}a_2 = 0 \qquad a_2 = ia$$

$\overline{e}_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}$ xos vektorlar bo‘ladi. $\lambda_2 = 1 - i\dfrac{x}{n}$ xos qiymatga $\overline{e}_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$ xos vektor mos keladi. Ushbu $C = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$ matritsani tuzib olamiz.

$C^{-1} = \dfrac{1}{2}\cdot\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$ bo‘ladi.

$$C^{-1}\cdot A\cdot C = \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\cdot\begin{pmatrix} 1 & \dfrac{x}{n} \\ -\dfrac{x}{n} & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\cdot\begin{pmatrix} 1+i\dfrac{x}{n} & 1-i\dfrac{x}{n} \\ -\dfrac{x}{n}+i & -\dfrac{x}{n}-i \end{pmatrix} =$$

$$= \begin{pmatrix} 1+i\dfrac{x}{n} & 0 \\ 0 & 1-i\dfrac{x}{n} \end{pmatrix} = J$$

Demak, $A = C\cdot J\cdot C^{-1}$

$A^2 = C \cdot J \cdot C^{-1} \cdot C \cdot J \cdot C^{-1} = C \cdot J^2 \cdot C^{-1}$, $A^k = C \cdot J^k \cdot C^{-1}$ bo‘lsin deb faraz qilib, $A^{k+1} = C \cdot J^{k+1} \cdot C^{-1}$ bo‘lishini ko‘rsatamiz:

$$A^{k+1} = A^k \cdot A = C \cdot J^k \cdot C^{-1} \cdot C \cdot J \cdot C^{-1} = C \cdot J^{k+1} C^{-1}$$

Demak, ixtiyoriy n uchun

$$A^n = C \cdot J^n \cdot C^{-1} = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \cdot \begin{pmatrix} \left(1 + i\dfrac{x}{n}\right)^n & 0 \\ 0 & \left(1 - i\dfrac{x}{n}\right)^n \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$$

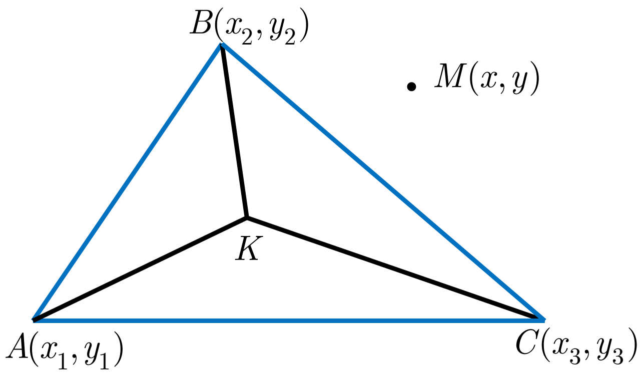
$$B = \lim_{n\to\infty} A^n = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \cdot \begin{pmatrix} e^{ix} & 0 \\ 0 & e^{-ix} \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \lim_{n\to\infty}\left(\lim_{n\to\infty}\frac{1}{x}\left(A^n - E\right)\right) =$$

$$= \lim_{n\to\infty}\frac{1}{x}\left(B - C \cdot C^{-1}\right) =$$

$$C \cdot \lim_{n\to\infty}\begin{pmatrix} \dfrac{e^{ix-1}}{x} & 0 \\ 0 & \dfrac{e^{-ix-1}}{x} \end{pmatrix} \cdot C^{-1} = C \cdot \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \cdot i^{-1} = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \cdot \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} =$$

$$\frac{1}{2}\begin{pmatrix} i & -i \\ -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$ Javob: $\left\{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right\}$

2009.4.



$$MA^2 = \left(x - x_1\right)^2 + \left(y - y_1\right)^2$$

$$MB^2 = \left(x - x_0\right)^2 + \left(y - y_0\right)^2$$

$$MC^2 = \left(x - x_3\right)^2 + \left(y - y_3\right)^2$$

$$f\left(x, y\right) = MA^2 + MB^2 + MC^2 \equiv \left(x - x_1\right)^2 + \left(y - y_1\right)^2 + \left(x - x_2\right)^2 + \left(y - y_2\right)^2 +$$
$$+\left(x - x_3\right)^2 + \left(y - y_3\right)^2$$

$$f'_x = 2\left(x - x_1\right) + 2\left(x - x_2\right) + 2\left(x - x_3\right) = 0$$

$$6x = 2x_2 + 2x_2 + 2x_3$$

$$x = \frac{x_1 + x_2 + x_3}{3}$$

$$f'_y = 2\left(y - y_1\right) + 2\left(y - y_2\right) + 2\left(y - y_3\right) = 0$$

$$y = \frac{y_1 + y_2 + y_3}{3}$$

$a_{n1} = f''_{xx} = 6 > 0$

$a_{12} = f''_{xy} = 0$ $\qquad a_{11}a_{22} - a_{12}^2 = 6 > 0$

$a_{22} = f''_{yy} = 6 > 0$ $\qquad a_{12} > 0$

Demak, $\left(x, y\right)$– minimum nuqta

$$\left(x, y\right) = \left(\frac{x_1 + x_2 + x_3}{3}, \ \frac{y_1 + y_2 + y_3}{3}\right)$$

medianalar kesishgan nuqta bo‘ladi.

$$KB = \frac{2}{3}m_b \qquad KC = \frac{2}{3}m_c \qquad KA = \frac{2}{3}m_a$$

$$KB^2 + KA^2 + KC^2 = \frac{4}{9}m_a^2 + \frac{4}{9}m_b^2 + \frac{4}{9}m_c^2 = \frac{4}{9}\left(m_a^2 + m_b^2 + m_c^2\right) = \frac{4}{9}\cdot\frac{3}{4}\left(a^2 + b^2 + c^2\right) =$$

$$= \frac{1}{3}\left(a^2 + b^2 + c^2\right)$$

2009.5.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \cos^2\gamma = 1 + \frac{\cos 2\alpha + \cos 2\beta}{2} + \cos^2\gamma =$$

$$1 + \cos\left(\alpha+\beta\right)\cos\left(\alpha-\beta\right) + \cos^2\gamma = 1 - \cos\gamma\cos\left(\alpha-\beta\right) + \cos^2\gamma = \left(\cos\gamma - \frac{1}{2}\cos\left(\alpha-\beta\right)\right)^2 +$$

$$+1 - \frac{1}{4}\cos^2\left(\alpha-\beta\right) = \left(\cos\gamma - \frac{1}{2}\cos\left(\alpha-\beta\right)\right)^2 + 1 - \frac{1}{4}\left(1 - \sin^2\left(\alpha-\beta\right)\right) = \left(\cos\gamma - \frac{1}{2}\cos\left(\alpha-\beta\right)\right)^2 +$$

$$+\frac{1}{4}\sin^2\left(\alpha-\beta\right) + \frac{3}{4} \geq 0 + 0 + \frac{3}{4} = \frac{3}{4}$$

Demak, berilgan tengsizlik isbotlandi.

Agar $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{4}$ bo‘lsa, yuqoridagi tenglikka ko‘ra

$$\left(\cos\gamma - \frac{1}{2}\cos\left(\alpha - \beta\right)\right)^2 + \frac{1}{4}\sin^2\left(\alpha - \beta\right) + \frac{3}{4} = \frac{3}{4}$$

$$\left(\cos\gamma - \frac{1}{2}\cos\left(\alpha - \beta\right)\right)^2 + \frac{1}{4}\sin^2\left(\alpha - \beta\right) = 0$$

$$\begin{cases}\sin(\alpha-\beta)=0\\ \cos\gamma-\frac{1}{2}\cos(\alpha-\beta)=0\end{cases}$$

$$\alpha-\beta=0, \quad \alpha=\beta, \quad \cos\gamma=\frac{1}{2}\Rightarrow\gamma=60^0,$$

$$\alpha+\beta+\gamma=180^0\Rightarrow 2\alpha+60^0=180^0\Rightarrow\alpha=60^0, \beta=60^0$$

Demak, tenglik faqat $\alpha=60^0$, $\beta=60^0$, $\gamma=60^0$ bo‘lgandagina bajariladi. Tengsizlik to‘la isbotlandi.

2010-yil

2010.1. Quyidagi hollarni qaraymiz.

1-hol. A-teskarilanuvchi ya’ni, $\det A\neq 0$.

Ravshanki, $I+AB=AA^{-1}+AB=A(A^{-1}+B)$ va

$I+BA=A^{-1}A+BA=(A^{-1}+B)A$

Bu tengliklardan quyidagiga ega bo‘lamiz.

$$\det(I+AB)=\det A(A^{-1}+B)=\det A\cdot\det(A^{-1}+B)$$

$$\det(I+BA)=\det(A^{-1}+B)A=\det(A^{-1}+B)\cdot\det A$$

Demak, $\det(I+AB)=\det(I+BA)$.

2-hol. $\det A=0$. Yuqoridagi teoremaga ko‘ra, $\exists\lambda_1,\lambda_2,...,\lambda_m\in C$ va $A_\lambda\in\mathbb{C}[m\times m]$ topilib $\forall\lambda\in\mathbb{C}\setminus\{\lambda_1,\lambda_2,...\lambda_m\}$ uchun $\det A\neq 0$ va $\lim\limits_{\lambda\to 0}\det A_\lambda=\det A$ bo‘ladi. A_λ teskarilanuvchiligidan, 1-holga ko‘ra, $\det(I+A_\lambda B)=\det(I+BA_\lambda)$ tenglik o‘rinli.

Determinant uzluksiz funksiya ekanidan foydalanib, $\lambda\to 0$ da limitga o‘tamiz va quyidagilarni hosil qilamiz.

$$\lim_{\lambda\to 0}\det(I+A_\lambda B)=\det\lim_{\lambda\to 0}(I+A_\lambda B)=\det(I+A\ B)$$

$$\lim_{\lambda\to 0}\det(I+BA_\lambda)=\det\lim_{\lambda\to 0}(I+BA_\lambda)=\det(I+BA\ )$$

Demak, $\det(I+AB)=\det(I+BA)$

2010.2. Misol shartiga ko‘ra: ixtiyoriy $x, y \in R$ lar uchun $(f(x)-f(y))^2 \le \left|x-y\right|^3$ bu yerdan $x \ne y \Rightarrow \dfrac{(f(x)-f(y))^2}{\left|x-y\right|^2} \le \left|x-y\right|$

$x = y + \Delta y$

$\dfrac{(f(y+\Delta y)-f(y))^2}{\left|y+\Delta y-y\right|^2} \le \left|y+\Delta y-y\right|$ bundan $\left|\dfrac{(f(y+\Delta y)-f(y))}{\Delta y}\right|^2 \le \left|\Delta y\right|$.

Endi $\Delta y \to 0$ da limitga o‘tsak, $\lim\limits_{\Delta y\to 0}\left|\dfrac{(f(y+\Delta y)-f(y))}{\Delta y}\right|^2 \le \lim\limits_{\Delta y\to 0}\left|\Delta y\right| = 0 \quad \Rightarrow \quad \left[f'(y)\right]^2 \equiv 0$. Bundan esa $f(x) = const$ ekani kelib chiqadi, chunki hosilasi nolga teng, uzluksiz funksiya faqat o‘zgarmas funksiyadir. Demak, o‘zgarmasdan farqli va misol shartini qanoatlantiruvchi funksiya mavjud emas ekan.

2010.3. $f : R \to R$ uzluksizdir, biz $f(\frac{1}{2010}) = const = \frac{1}{5}$ ekanini ko‘rsatamiz.

Faraz qilaylik, $f(\frac{1}{2010}) = x \ne \frac{1}{5}$ bo‘lsin, bundan haqiqiy sonlar to‘plami to‘liqligidan x va $\frac{1}{5}$ sonlari orasidan irratsional ξ son topiladi. Bolsano-Koshining 2-teoremasiga asosan, $f(x)$ uzluksizligidan shu irratsional ξ sonning asli mavjud. Bu esa $f(x)$ funksiya irratsional qiymat qabul qilmaydi degan shartga zid. Demak, farazimiz noto‘g‘ri. Shunday qilib, biz quyidagi xulosaga kelamiz. Agar funksiya uzluksiz bo‘lib faqat ratsional sonlar qabul qilsa, bu funksiya o‘zgarmas bo‘ladi. Demak, $f(\frac{1}{2010}) = \frac{1}{5}$.

2010.4. Matematik induksiya metodi orqali isbotlaymiz:

$$n = 3 \quad \left|D_3\right| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} \le \left|a_1\begin{vmatrix} a_5 & a_6 \\ a_8 & a_9 \end{vmatrix}\right| + \left|a_2\begin{vmatrix} a_4 & a_6 \\ a_7 & a_9 \end{vmatrix}\right| + \left|a_3\begin{vmatrix} a_4 & a_5 \\ a_7 & a_8 \end{vmatrix}\right| \qquad (1)$$

$a_i = \pm 1,\ i = \overline{1,9}$

D_3 ni 1-satr elementlari bo‘yicha yoyamiz.

A_1, A_2, A_3 algebraik to‘ldiruvchilardan kamida biri nolga teng bo‘ladi.

$$A_1=\begin{vmatrix} a_5 & a_6 \\ a_8 & a_9 \end{vmatrix},\ A_2=\begin{vmatrix} a_4 & a_6 \\ a_7 & a_9 \end{vmatrix},\quad A_3=\begin{vmatrix} a_4 & a_5 \\ a_7 & a_8 \end{vmatrix}$$

Haqiqatan ham, aksincha, $A_1 \neq 0,\ A_2 \neq 0,\ A_3 \neq 0$ desak,

$$A_1 = a_9a_5 - a_6a_8$$
$$A_2 = a_4a_9 - a_7a_6$$
$$A_3 = a_4a_8 - a_5a_7$$

A_1, A_2, A_3 nolga teng bo‘lmasligi uchun a_9a_5 va a_6a_8 ishorasi har xil bo‘lishi kerak, agar bir xil bo‘lsa, xuddi shunday a_4a_9 va a_7a_6, a_4a_8 va a_5a_7 lar ham ishorasi har xil bo‘lishi kerak. Aks holda, $a_9a_5 = a_6a_8 = \pm 1 \Rightarrow a_9a_5 - a_6a_8 = 0$ ziddiyatga kelamiz, bo‘ndan $a_9a_5a_6a_8 < 0$, $a_4a_9a_7a_6 < 0$, $a_4a_8a_5a_7 < 0$ bo‘lishi kelib chiqadi va ularni ko‘-paytirsak $\left(a_4a_5a_6a_7a_8a_9\right)^2 < 0$, ziddiyatga kelamiz. Demak, farazimiz noto‘g‘ri, A_1, A_2, A_3 lardan kamida bittasi nolga teng. Shunday qilib, determinantimizni baholaymiz:

$$\left|D_3\right| = \left|a_1A_1 + a_2A_2 + a_3A_3\right| \le \left|a_1A_1\right| + \left|a_2A_2\right| + \left|a_3A_3\right| \le \left|A_1\right| + \left|A_2\right| + \left|A_3\right|$$

Yuqorida isbotlab o‘tganimizdek qo‘shiluvchilardan kamida bittasi 0 ga teng. Masalan, $A_2 = 0$ bo‘lsin, u holda

$$D_3 \le \left|A_1\right| + \left|A_3\right| \qquad \begin{cases} \left|A_1\right| = \left|a_9a_5 - a_6a_8\right| \le 2 \\ \left|A_3\right| = \left|a_4a_8 - a_5a_7\right| \le 2 \end{cases} \qquad D_3 \le \left|A_1\right| + \left|A_3\right| \le 4$$

Demak, $n = 3$ bo‘lganda $D_3 \le (3-1)(3-1)! = 4$ tengsizlik bajarildi.

$n = k$ uchun tengsizlik to‘g‘ri deb faraz qilamiz. $D_k \le (k-1)(k-1)!$

$n = k + 1$ da to‘g‘riligini ko‘rsatamiz.

D_{k+1} ni birinchi satr elementlari bo‘yicha yoyib chiqsak,

$\left|D_{k+1}\right| = \left|a_1A_1 + ... + a_nA_n\right| \le \left|a_1A_1\right| + ... + \left|a_nA_n\right| \le \left|A_1\right| + \left|A_2\right| + ... + \left|A_n\right|$ A_i lar n-tartibli determinant bo‘lib, elementlari -1 va 1 lardan iborat, bundan $\left|A_i\right| \le (k-1)(k-1)!$

$D_{k+1} \le (k-1)(k-1)!(k+1) = \left(k^2-1\right)(k-1)! = kk!$. Tengsizlik to‘liq isbot bo‘ldi.

2010.5. Tekislikda bitta nuqtadan chiquvchi $\overline{r}_1,\ \overline{r}_2,\ \overline{r}_3$ birlik vektorlarni olamiz. Ular oralaridagi burchaklar $180^0-\alpha$, $180^0-\beta$ va $180^0-\gamma$ bo'lsin. Ushbu $(\overline{r}_1+\overline{r}_2+\overline{r}_3)^2\geq 0$ tengsizlik o'rinli bo'lishi ayon. Bunga ko'ra,

$$\overline{r}_1^2+\overline{r}_2^2+\overline{r}_3^2+2\overline{r}_1\overline{r}_2+2\overline{r}_1\overline{r}_3+2\overline{r}_2\overline{r}_3\geq 0\,,$$

$$3+2\cos(180^0-\alpha)+2\cos(180^0-\beta)+2\cos(180^0-\gamma)\geq 0\,,$$

$$3-2[\cos\alpha+\cos\beta+\cos\gamma]\geq 0\,.$$

Oxirgi tengsizlikdan $\cos\alpha+\cos\beta+\cos\gamma\leq\dfrac{3}{2}$ kelib chiqadi.

2011-yil

2011.1. $f(x)\in C[0,1], f(x)>0$ bo'lsa, $\lim\limits_{n\to\infty}\left(\int\limits_0^1\sqrt[n]{f(x)}dx\right)^n$ ni hisoblang.

Yechilishi: $x_n=\left(\int\limits_0^1\sqrt[n]{f(x)}dx\right)^n$ bo'lsin. $f(x)>0$ bo'lgani uchun $x_n>0$ ekani kelib chiqadi. Shuning uchun quyidagi ketma-ketlikni qarashga haqlimiz.

$t_\alpha=\ln x_\alpha$, bu yerda $\alpha=\dfrac{1}{n}$. Ravshanki, $t_\alpha=\dfrac{\int\limits_0^1 f^\alpha(x)dx}{\alpha}$ va $n\to\infty\Leftrightarrow\alpha\to 0$. Lopital qoidasidan foydalansak,

$$\lim_{\alpha\to 0}t_\alpha=\lim_{\alpha\to 0}\frac{\int\limits_0^1 f^\alpha(x)dx}{\alpha}=\lim_{\alpha\to 0}\frac{\int\limits_0^1 f^\alpha(x)\ln f(x)dx}{1}=\int\limits_0^1\ln f(x)dx\,\cdot$$

Demak, $\{\ln x_n\}$ -ketma-ketlik yaqinlashuvchi va $\lim\limits_{n\to\infty}\ln x_n=\int\limits_0^1\ln f(x)dx$. Bundan $\lim\limits_{n\to\infty}x_n=e^{\int_0^1\ln f(x)dx}$ ekani kelib chiqadi.

Javob : $\lim\limits_{n\to\infty}\left(\int\limits_0^1\sqrt[n]{f(x)}dx\right)^n=e^{\int_0^1\ln f(x)dx}$.

2011.2. M masala shartini qanoatlantiradigan matrisalar to'plami bo'lsin.

1-lemma: Agar $A,B\in M$ matrisalar uchun $AB\in M$ bo'ladi.

Isboti. A,B-ning noldan farqli elementlari a_{1i_1}, a_{2i_2}, b, A ning noldan farqli elementlari $a_{1\alpha_2}, a_{2\alpha_2}, \ldots\ldots, a_{n\alpha_n}$ bo‘lsin. U holda

$$\begin{pmatrix} 0\ 0\ 0 \ldots a_{1\alpha_1} 0 \\ 0\ 0\ 0 \ldots a_{2\alpha_2} 00 \end{pmatrix}$$ kabi bo‘ladi.

A ni elementi a_{ik} B ni elementi b_{ik} bo‘lsa, $A\ B$ matrisani $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$ ko‘rinishda bo‘ladi.

$A \cdot B$ ni elementi c_{ik} bo‘lsa u holda $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

$$x_{ik} = \begin{cases} a \quad agar \ \ k \neq b_{\alpha i}\, bo'lsa \\ 1 \ yoki \ -1 \ \ agar \ k = b_{\alpha i} \ bo'adi \end{cases}$$

Bu yerda $b_{\alpha i}$ α_i yo‘lning noldan farqli elementi.

Isboti. $x_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk} = a_{i1} b_{jk} + \ldots . + a_{in} b_{jk}$ xar bir satr bittadan noldan farqli element bo‘lgani uchun ya’ni i satrda x_i element noldan farqli bo‘lsin α_i ustunda $B_{\alpha i}$ noldan farqli bo‘lsin

$$a_{ij} a_{jk} = \begin{cases} 1 \ yoki \ -1 \ agar \ k = B\alpha_i \ \ i = \alpha_i \\ 0, \quad k = \beta_i \end{cases} \Rightarrow x_{ik} = \begin{cases} \pm 1 \ agar \ k = \beta_{\alpha i} \\ 0 \ agar \ k \neq \beta_{ai} \end{cases} \Rightarrow$$

$\Rightarrow$ A^2 ning ham har satrida n faqat bitta ± 1 bor boshqa barcha elementi $0 \Rightarrow A^2 \in M$. Isbotlandi.

2-lemma. Har bir satrida noldan farqli bitta elimenti bo‘lsa uni diterminanti noldan farqli.

Isboti. $(\alpha_1', \ldots, \alpha_n')$ $\quad \det A = \sum(-1) a_{1\alpha_1}, \ldots, a_{n\alpha_n} = \pm 1$

$\alpha_, \alpha_2, \ldots, \alpha_n$ Isbotlandi.

3-lemma. M to‘plamni quvvati chekli.

Isboti. $(-1, 0, +1)$ sonlaridan tuzilgan nxn matritsalar soni chekli bo‘lgani uchun kelib chiqadi.

Isbotlandi.

4-lemma. $\exists$ α son $A^{\alpha} = E$ bo‘ladi

Isbot. $\begin{cases} A \in M \\ A \in M \end{cases} \Rightarrow \quad A^2 \in M$ lemma 1 dan va hakazo $A^k \in M$

$k \in N$ uchun

M to‘plom quvvati chekli bo‘lgani uchun uni quvvatini m bilan belgilaymiz. $A, A^2, ..., A^{m+1}$ elementlarni olamiz. Bular soni $(m+1)$ ta ta $\Rightarrow$ $A^k = A^l \Rightarrow \det A \neq 0 \Rightarrow \exists A^{-1} \Rightarrow A^{k-l} = E \Rightarrow \alpha = k - l$ deb olsak $\exists \alpha$ bo‘ladi.

5-lemma. $A^T A = E$ bo‘ladi.

Isboti. A ni noldan farqli elementini $a_{1\alpha_1}, ..., a_{n\alpha_n}$ bo‘lsa u holda $A^T A$ ni elementi b_{ij} bo‘lsin. $b_{ij} = \begin{cases} 1, \ agar \ i = j \\ 0, \ agar \ i \neq j \end{cases}$ chunki

$a_{ij} a_{kj} = \begin{cases} 1, \ agar \ k = i \ j = \alpha \\ o, \ agar \ k \neq i \end{cases}$ bo‘lsa $\Rightarrow b_{ii} = \sum_{j=1}^{n} a_{ij} \cdot a_{kj} = 1 \quad \Rightarrow i \neq j$ bo‘lsa,

$b_{ij} = 0$.,

Demak, $b_{ij} = \begin{cases} 1, \ agar \ i = j \\ 0, agar \ i \neq j \end{cases}$ $A^T A = E$ chunki diaganallarda 1 boshqa barcha elementi nol $\Rightarrow A^T A = E$

Lemma 4 ga ko‘ra, $\exists \ \alpha \exists \begin{cases} A^\alpha = E \\ A^T A = E \end{cases} \Rightarrow$ lemma 5 ga ko‘ra, $\Rightarrow A^t = A^\alpha$

$\det A \neq 0 \Rightarrow \exists A^{-1} \Rightarrow A^T = A^{\alpha-1} \Rightarrow \exists k = \alpha - 1$ ekan isbotlandi.

2011.3. $\forall m, n \in N$ uchun $x_{m+n} \leq x_m + x_n$ bo‘lsin. U holda $x_m = x_{1+1+...+1} \leq x_1 + x_1 + ...x_1 = mx_1 \Rightarrow 0 \leq \frac{x_m}{m} \leq x_1$. Demak, $\{\frac{x_m}{m}\}$- ketma-ketlik chegaralangan. Aytaylik, $\inf\{\frac{x_n}{n}\} = a$ bo‘lsin. Ketma-ketlik infinumi ta’rifiga ko‘ra, $\forall \varepsilon > 0$ son uchun $\exists k_0 \in N$ topilib, $\forall k > k_0$ sonlar uchun $a \leq \frac{x_k}{k} < a + \frac{\varepsilon}{2}$ tengsizlik bajariladi. $\forall n \in N$ sonni $n = q \cdot k + r$ ko‘rinishda yozish mumkin, bu yerda $r < k, q \geq 0, q \in N$.

$$a \leq \frac{x_n}{n} = \frac{x_{qk+r}}{qk+r} \leq \frac{q \cdot x_k + x_r}{qk+r} = \frac{q \cdot x_k}{qk+r} + \frac{x_r}{n} \leq \frac{x_k}{k} + \frac{x_r}{n}.$$

x_r ni fiksirlaymiz. $n \to \infty$ da $\frac{x_r}{n}$ yetarlcha kichik bo'ladi. Demak, $\forall \varepsilon > 0$ soni uchun biror $\exists n_0 \in N$ nomer topilib, bu nomerdan boshlab , $\frac{x_r}{n} < \varepsilon$ bo'ladi.U holda $a \le \frac{x_n}{n} \le \frac{x_k}{k} + \frac{x_r}{n} < a + \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = a + \varepsilon$.

Demak, $\forall \varepsilon > 0$ soni uchun $\exists n_0(\varepsilon) \in N$ topilib, $\forall n > n_0$ sonlar uchun

$$\left|x_k - a\right| < \varepsilon \text{ bo'ladi, ya'ni } \lim_{n\to\infty} \frac{x_n}{n} = a = \inf\{\frac{x_n}{n}\}.$$

2011.4. f va g vektorlar to'g'ri chiziqda deb qaraymiz. Bunda f va g lar nuqtalardan iborat bo'ladi. Ularni $f(x)$ va $g(y)$ yani x va y nuqtalar deb olaylik.

U holda

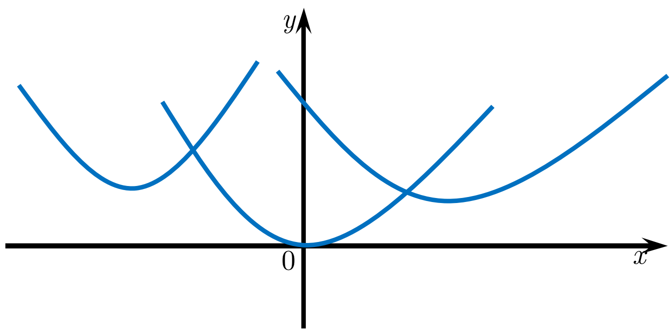
$$af^2 + bfg + cg^2 \ge 0 \qquad (1)$$

tengsizligimizga o'zimiz tanlab olgan x va y nuqtalarimizni qo'ysak,

$$ax^2 + bxg + cy^2 \ge 0 \qquad (2)$$

tengsizlikka ega bo'lamiz. Endi (2) tengsizligimizni avval x ga nisbatan tengsizlik deb olamiz. Demak, $ax^2 + bxg + cy^2 \ge 0$

Biz bilgan kvadrat tengsizlik hosil bo'ladi. Bilamizki, bu tengsizlik kvadrat tengsizlik bo'lgani uchun xassalardan bizga ma'lumki, y nomanfiy qiymatlarni qabul qilishi uchun avval x^2 oldidagi koeffitseiynt nomanfiy bo'lishi u $(a \ge 0)$ va diskriminant musbat bo'lmasligi kerak. Ya'ni grafikda parabolaning shohchalari yuqoriga qaragan va ko'pi bilan ox o'qiga uringan bo'ladi yoki, umuman, urinmasligi ham mumkin. Bu grafikni chizmada ifodalasak, $x_0 = -\frac{by}{2a}$



Demak, $a \geq 0$ va $ax^2 + bxy + cy^2 \geq 0$ dan $D \leq 0$ yani $b^2y^2 - 4acy^2 \leq 0$

$$y^2(b^2 - 4ac) \leq 0 \qquad y^2 > 0$$

$$b^2 - 4ac \leq 0 \qquad b^2 \leq 4ac$$

Endi bajaradigan ishimiz, $c \geq 0$ ekanligini ko‘rsatish. Buning uchun (2) tengsizligimizni y ga nisbatan tengsizlik deb olishimiz kifoya.

$$cy^2 + bxy + ax^2 \geq 0$$

Bu holda ham huddi yuqoridagiday $c \geq 0$ va $b^2 \leq 4ac$ kelib chiqadi. Biz yuqorida fazo deb to‘g‘ri chizqda qaradik. Endi, fazo sifatida 2-o‘lchovli Evklid tekisligini olaylik. $f(x_1, x_2)$ $g(y_1, y_2)$ deb qaraylik, u holda (1) tengsizligimiz;

$$a(x_1^2 + x_2^2) + b(x_1y_1 + x_2y_2) + c(y_1^2 + y_2^2) \geq 0$$

ko‘rinishni oladi.

Fazoni 3 o‘lchovli deb olsak va $f = f(x_1, x_2, x_3)$, $g = g(y_1, y_2, y_3)$ deb kiritsak, (1) tengsizligimiz

$$a(x_1^2 + x_2^2 + x_3^2) + b(x_1y_1 + x_2y_2 + x_3y_3) + c(y_1^2 + y_2^2 + y_3^2) \geq 0$$

ko‘rinishni oladi.

Jarayonni umumlashtirib fazoni n o‘lchovli deb olsak va

$f = f(x_1, x_2, \ldots, x_n), g = g(y_1, y_2, \ldots, y_n)$ deb kiritsak, (1) tengsizligimiz

$$a(x_1^2 + x_2^2 + \ldots + x_n^2) + b(x_1y_1 + x_2y_2 + \ldots + x_ny_n) + c(y_1^2 + y_2^2 + \ldots + y_n^2) \geq 0$$

ko‘rinishni oladi.

Bu tengsizliklarni isbotlashda

$$x_1^2 + x_2^2 + \ldots + x_n^2 = \alpha \qquad x_1y_1 + x_2y_2 + \ldots + x_ny_n = \beta$$

$$y_1^2 + y_2^2 + \ldots + y_n^2 = \gamma$$

deb belgilash kiritsak tengsizlik $a\alpha^2 + b\beta + c\gamma^2 \geq 0$ ko‘rinishga keladi va (2) tengsizlik kabi shart isbotlanadi, ya’ni tengsizlik α va β larga nisbatan olinadi.

2011.5. $a > b > 1$ bo‘lsin. $f(x) = \dfrac{\ln x}{x}$ funksiyani qaraymiz ($x \geq 1$ da)

$$f'(x) = \frac{(\ln x)'x - x'\ln x}{x^2} = \frac{1 - \ln x}{x^2} \qquad (x \geq 1)$$

$$f''(x) = \frac{-\frac{1}{x}x^2 - 2x(1-\ln x)}{x^4} = \frac{-1-2+2\ln x}{x^3} = \frac{2\ln x - 3}{x^3}$$

$f'(x) = \dfrac{1-\ln x}{x^2} \le 0 = f'(1) \Rightarrow$

Demak, $f(x)$ o‘suvch ekani kelib chiqadi. Demak, $f(x) = \dfrac{\ln x}{x}$ ($x \ge 1$ da) o‘suvchi. $a > b \Rightarrow$ f(a) > f(b) $\Rightarrow \dfrac{\ln a}{a} > \dfrac{\ln b}{b} \Rightarrow b \ln a > a \ln b \Rightarrow a^b > b^a$.

Shu holni yana bir marta qo‘llab, $a^{b^a} > b^{a^b}$ ni hosil qilamiz. Bu yerda $a > b > 1$.

2012-yil

2012.1. Ushbu $x^2 = a + x$ kvadrat tenglama yagona musbat $c = \frac{1}{2} + \sqrt{\frac{1}{4} + a}$ ildizga ega. Biz, $\lim\limits_{n\to\infty} x_n = c$ ekanini isbotlaymiz.

Buning uchun ketma-ketlikning monoton o‘suvchi va yuqoridan chegaralanganligini ko‘rsatishimiz kerak. Matematik induksiya usuli bilan $x_n < c$ ekanini ko‘rsatamiz: $n = 1$ da $c^2 = c + a > a = x_1^2 \Rightarrow x_1 < c$ to‘g‘ri. Bu tasdiqni n uchun to‘g‘ri deb faraz qilsak, u holda $x_{n+1} = \sqrt{a + x_n} < \sqrt{a + c} = c$ ekani kelib chiqadi. Demak, $\forall n \in N$ uchun $x_n < c$ o‘rinli ekan. U holda ketma-ketlik chekli limitga ega (Monoton ketma-ketliklar haqidagi teoremaga asosan). $A = \lim\limits_{n\to\infty} x_n$ desak, $x_{n+1}^2 = x_n + a \Rightarrow d^2 = d + a \Rightarrow d = c$ ekani kelib chiqadi. $\lim\limits_{n\to\infty} = \frac{1}{2} + \sqrt{\frac{1}{4} + a}$.

2012.2. Biz
$$W(x) = \begin{vmatrix} a+x & b+x & b+x & b+x \ldots b+x \\ c+x & a+x & b+x & b+x \ldots b+x \\ c+x & c+x & a+x & b+.x \ldots b+x \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ c+x & c+x & c+x & c+x \ldots a+x \end{vmatrix}, x \in R$$

funksiyani qaraymiz. Uning 1-satr elementlarini -1 ga ko‘paytirib, 2-,3-,...n-satr elementlariga qo‘shib chiqilsa, quyidagicha determinant hosil bo‘ladi:

$$W(x)=\begin{vmatrix} a+x & b+x & b+x & b+x ... b+x \\ c-a & a-b & 0 & 0\ ...\ 0 \\ c-a & c-b & a-b & 0...\ 0 \\ .... \\ c-a & c-b & c-b & c-b ... a-b \end{vmatrix}$$ bu determinant hisoblansa, biror $W(x)=Ax+B$ x ga nisbatan chiziqli funksiya hosil bo'ladi. Chunki uning 1-satridan boshqa satr elementlarida x ishtirok etmagan. Bu yerdagi A biror n-tartibli determinant, B esa $x=0$ bo'lganda hosil bo'ladigan biz izlayotgan determinant. Endi $x=-b$ va $x=-c$ bo'lgan hollarni qaraymiz: $\begin{matrix} W(-b)=-Ab+B=(a-b)^n \\ W(-c)=-Ac+B=(a-c)^n \end{matrix}$ birinchi tenglikning ikkala tomonini c ga, ikkin-chi tenglikni esa $-b$ ga ko'paytirib, ikkalasini qo'shamiz:

$$B(c-b)=c(a-b)^n-b(a-c)^n \Rightarrow B=\frac{c(a-b)^n-b(a-c)^n}{(c-b)}$$ hosil bo'ladi.

Bunda quyidagi hollar bo'lishi mumkin:

1-hol: $a=b=c$ u holda $B=0$;

2-hol: $a=b\neq c$ u holda $B=b(b-c)^{n-1}$;

3-hol: $a=c\neq b$ u holda $B=c(c-b)^{n-1}$;

4-hol: $a\neq b\neq c$ u holda $B=\dfrac{c(a-b)^n-b(a-c)^n}{(c-b)}$.

2012.3. $f(x)=ax^2+bx+c$ ko'rinishda bo'lsin. Masala shartiga ko'ra.

$$|f(0)|=|c|\le 1,\ \left|f\left(\frac{1}{2}\right)\right|=\left|\frac{a}{4}+\frac{b}{2}+c\right|\le 1,\ |f(1)|=|a+b+c|\le 1 \Rightarrow -3c\le 3,$$
$$4\left(\frac{a}{4}+\frac{b}{2}+c\right)\le 4,\ -(a+b+c)\le 1 \Rightarrow -3c+4\left(\frac{a}{4}+\frac{b}{2}+c\right)-(a+b+c)\le$$
$$\le 3+4+1=8 \Rightarrow b\le 8$$

Ammo ikkinchi tomondan $f'(0)=b\le 8$. Demak, $f'(0)$ ning eng kichik qiymati 8 ga teng ekan. Bu kvadrat uchhadni qurish oson. $f(x)=-8x^2+8x-1$. Bunda $f'(0)=8$ tenglik bajariladi.

2012.5. $S_n = x_n \sum_{k=1}^{n} x_k^2 = 1$ deb belgilab olamiz. U holda $x_n S_n$ ko‘paytma 1 ga yaqinlashadi. $\lim_{n\to\infty} S_n = \lim_{n\to\infty} x_n \sum_{k=1}^{n} x_k^2 = +\infty$ va $\lim_{n\to\infty} x_n = 0$ bo‘ladi. Quyidagi ifodalarni qaraymiz:

$$S_n^3 - S_{n-1}^3 = x_n^2\left(S_n^2 + S_n S_{n-1} + S_{n-1}^2\right) \to 3, \left(n \to \infty\right)$$

$$\lim_{n\to\infty} \frac{S_n^3}{n} = \lim_{n\to\infty} \frac{S_n^3 - S_{n-1}^3}{n - \left(n-1\right)} = 3 \Rightarrow \frac{\sqrt[3]{3n}}{S_n} \to 1 \Rightarrow \sqrt[3]{3}x_n = \frac{\sqrt[3]{3n}}{S_n} x_n S_n \to 1\left(n \to \infty\right)$$

2013-yil

2013.1. $\forall x, y \in R$ lar uchun $\left(e^{\frac{x}{2}} - e^{\frac{y}{2}}\right)^2 \geq 0$ ekani ma’lum. Bundan $e^x - 2e^{\frac{x+y}{2}} + e^y \geq 0 \Rightarrow e^x + e^y \geq 2e^{\frac{x+y}{2}} \Rightarrow \frac{e^x + e^y}{2} \geq e^{\frac{x+y}{2}}$. Tenglik belgisi $x = y$ bo‘lganda bajariladi.

2013.2. Sistemaning ikkala tenglamasidan ham x va y o‘zgaruvchilar bo‘yicha hosila olamiz:

$$\begin{cases} u + xu'_x + yv'_x - 3u^2u'_x = 0 \\ u'_y + v + yv'_y - 3u^2u'_y = 0 \\ 1 + u'_x + v'_x = 0 \\ 1 + u'_y + v'_y = 0 \end{cases} \Rightarrow u'_x = \frac{u - y}{3u^2 + y - x} = \frac{1 - 0}{3 \cdot 1^2 + 0 - 1} = \frac{1}{2},$$

$$u'_y = \frac{v - y}{3u^2 + y - 1} = \frac{-2 - 0}{3 \cdot 1^2 + 0 - 1} = -1$$

$v'_x = -1 - u'_x = -\frac{3}{2}, v'_y = -1 - u'_y = 0$.

Javob: $u'_x = \frac{1}{2}, u'_y = -1, v'_x = -\frac{3}{2}, v'_y = 0$

2013.3. Koshi-Bunyakovskiy tengsizligini qo‘llaymiz:

$\int_0^1 \left(f'\left(x\right)\right)^2 dx \geq \left(\int_0^0 1 \cdot f'\left(x\right) dx\right)^2 = \left(f\left(x\right)\Big|_0^1\right)^2 = \left(f\left(1\right) - f\left(0\right)\right)^2 = 1$. Isbot tugadi.

2013.5.

$$\begin{vmatrix} 1-\lambda & \dfrac{\alpha}{n} \\ -\dfrac{\alpha}{n} & 1-\lambda \end{vmatrix} = 0 \qquad (\lambda-1)^2+\frac{\alpha^2}{n^2}=0 \qquad \lambda-1=\pm i\frac{\alpha}{n}$$

$\lambda_1 = 1+i\dfrac{\alpha}{n}$, $\lambda_2 = 1-i\dfrac{\alpha}{n}$ xos qiymatlari $\lambda_1 = 1+i\dfrac{\alpha}{n}$ xos qiymatga mos keluvchi xos vektorni topamiz:

$$\begin{pmatrix} -i\dfrac{\alpha}{n} & \dfrac{\alpha}{n} \\ -\dfrac{\alpha}{n} & -i\dfrac{\alpha}{n} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad -i\frac{\alpha}{n}a_1+\frac{\alpha}{n}a_2=0 \qquad a_2 = ia$$

$\overline{e}_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}$ xos vektorlar bo‘ladi.

$\lambda_2 = 1-i\dfrac{\alpha}{n}$ xos qiymatga $\overline{e}_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$ xos vektor mos keladi. Ushbu

$C = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$ matritsani tuzib olamiz. $C^{-1} = \dfrac{1}{2}\cdot\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$ bo‘ladi.

$$C^{-1}\cdot A\cdot C = \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\cdot\begin{pmatrix} 1 & \dfrac{\alpha}{n} \\ -\dfrac{\alpha}{n} & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\cdot\begin{pmatrix} 1+i\dfrac{\alpha}{n} & 1-i\dfrac{\alpha}{n} \\ -\dfrac{\alpha}{n}+i & -\dfrac{\alpha}{n}-i \end{pmatrix} =$$

$$= \begin{pmatrix} 1+i\dfrac{\alpha}{n} & 0 \\ 0 & 1-i\dfrac{\alpha}{n} \end{pmatrix} = J$$

Demak, $A = C\cdot J\cdot C^{-1}$

$$A^2 = C\cdot J\cdot C^{-1}\cdot C\cdot J\cdot C^{-1} = C\cdot J^2\cdot C^{-1},$$

$A^k = C\cdot J^k\cdot C^{-1}$ bo‘lsin deb faraz qilib, $A^{k+1} = C\cdot J^{k+1}\cdot C^{-1}$ bo‘lishini ko‘rsatamiz:

$$A^{k+1} = A^k\cdot A = C\cdot J^k\cdot C^{-1}\cdot C\cdot J\cdot C^{-1} = C\cdot J^{k+1}C^{-1}$$

Demak, ixtiyoriy n uchun

$$A^n = C\cdot J^n\cdot C^{-1} = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\cdot\begin{pmatrix} \left(1+i\dfrac{\alpha}{n}\right)^n & 0 \\ 0 & \left(1-i\dfrac{\alpha}{n}\right)^n \end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$$

$$\lim_{n\to\infty} A^n = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\cdot\begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

TEST KALITLARI

2006-YIL

	0	1	2	3	4	5	6	7	8	9
0		A	C	A	C	B	B	D	A	B
1	D	A	E	B	C	A	E	A	B	E
2	B									

2007-YIL

	0	1	2	3	4	5	6	7	8	9
0		C	B	A	C	A	C	D	C	B
1	C	D	B	B	C	A	D	A	C	D
2	A									

2008-YIL

	0	1	2	3	4	5	6	7	8	9
0		C	A	B	A	C	A	D	D	D
1	A	A	B	B	A	A	B	A	B	B
2	A	C	A	B	D	B	A	B	D	B
3	B	A	D	C	D	C	B	C	A	B
4	D	D	C	B	C	B	A	B	A	C
5	A									

2009-YIL

	0	1	2	3	4	5	6	7	8	9
0		C	A	C	D	C	C	D	D	B
1	A	B	B	C	B	A	B	B	B	B
2	B	A	D	A	D	C	A	B	B	C
3	D	C	D	C	A	B	D	C	A	A
4	B	C	D	D	B	A	D	D	C	B
5	A									

2010-YIL

	0	1	2	3	4	5	6	7	8	9
0		D	D	C	D	C	D	C	A	D
1	D	D	A	D	D	A	B	D	C	C
2	A	B	A	B	C	B				

2011-YIL

	0	1	2	3	4	5	6	7	8	9
0		B	C	B	D	D	B	C	D	D
1	C	B	A	B	D	A	C	A	A	A
2	C	-	-	-	-	-				

2012-YIL

	0	1	2	3	4	5	6	7	8	9
0		A	B	B	C	A	C	D	A	D
1	D	B	D	C	D	D	A	C	C	D
2	B	A	B	D	D	A				

2013-YIL

	0	1	2	3	4	5	6	7	8	9
0		D	B	B	D	A	A	A	A	A
1	D	B	C	A	C	A	A	D	A	C
2	B	A	B	A	A	C				

Olimpiada g'oliblari

2006-yil 15 – 16-may kunlari Urganch davlat universitetida matematika mutaxassisligi fanidan o'tkazilgan talabalar respublika fan olimpiadasining II bosqich natijalari

Qatnashgan talabalarning F.I.Sh	**OTM nomi**	**Yakuniy ballari**	**O'rni**
Mamayusupov Xudoyor Sultonovich	Qarshi DU	88,65	1
Xolmatov Shoxrux Yusupovich	SamDU	86,35	2
Ataev Frunza Kamiljonovich	UrDU	83,10	3

2007-yil 16 – 17-may kunlari Urganch davlat universitetida matematika mutaxassisligi fanidan o'tkazilgan talabalar respublika fan olimpiadasining II bosqich natijalari

Qatnashgan talabalarning F.I.Sh	**OTM nomi**	**Yakuniy ballari**	**O'rni**
Xolmatov Shoxrux Yusupovich	SamDU	97,25	1
Mamayusupov Xudoyor Sultonovich	Qarshi DU	81,25	2
Jumabayev Zafar	O'zMU	77, 75	3

2008-yil 16 – 17-may kunlari Urganch davlat universitetida matematika mutaxassisligi fanidan o'tkazilgan talabalar respublika fan olimpiadasining II bosqich natijalari

Qatnashgan talabalarning F.I.Sh	**OTM nomi**	**Yakuniy ballari**	**O'rni**
Ataev Fruzna Kamiljonovich	UrDU	85.75	1
Ibragimov Orif Olimovich	SamDU	76	2
Курьяков Александр Юрьевеч	O'zMU	75.5	3

2009-yil 13 – 14-may kunlari Urganch davlat universitetida matematika mutaxassisligi fanidan o‘tkazilgan talabalar respublika fan olimpiadasining II bosqichi natijalari

Qatnashgan talabalarning F.I.Sh	**OTM nomi**	**Yakuniy ballari**	**O‘rni**
Ibragimov Orif Olimovich	SamDU	89.25	1
Abdullayev Jonibek Shokirovich	UrDU	85.75	2
Turdibayev Rustam Mirzaliyevich	O‘zMU	65	3

2010-yil 13 – 14-may kunlari Urganch davlat universitetida matematika mutaxassisligi fanidan o‘tkazilgan talabalar respublika fan olimpiadasining II bosqichi natijalari

Qatnashgan talabalarning F.I.Sh	**OTM nomi**	**Yakuniy ballari**	**O‘rni**
Abdullayev Jonibek Shokirovich	UrDU	86.75	1
Pirnapasov Abror Tolipovich	O‘zMU	81	2
Rahimov Karim Hoshimovich	Navoiy DPI	76.25	3

2011-yil 10 – 11-may kunlari Urganch davlat universitetida matematika mutaxassisligi fanidan o‘tkazilgan talabalar respublika fan olimpiadasining II bosqichi natijalari

Qatnashgan talabalarning F.I.Sh	**OTM nomi**	**Yakuniy ballari**	**O‘rni**
Pirnapasov Abror Tolipovich	O‘zMU	84.75	1
Kamalov Ne’matjon Bahodirovich	UrDU	83.75	2
Alladustov Shuhrat Ulug‘muratovich	SamDU	81.5	3

2012-yil 14 – 15-may kunlari Qoraqalpoq davlat universitetida matematika mutaxassisligi fanidan o‘tkazilgan talabalar respublika fan olimpiadasining II bosqichi natijalari

Qatnashgan talabalarning F.I.Sh	**OTM nomi**	**Yakuniy ballari**	**O‘rni**
Pirnapasov Abror Tolipovich	O‘zMU	75	1
Alladustov Shuhrat Ulug‘muratovich	SamDU	74.25	2
Kamalov Ne’matjon Bahodirovich	UrDU	74	3

2013-yil 10 – 11-may kunlari Qarshi davlat universitetida matematika mutaxassisligi fanidan o‘tkazilgan talabalar respublika fan olimpiadasining II bosqichi natijalari

Qatnashgan talabalarning F.I.Sh	**OTM nomi**	**Yakuniy ballari**	**O‘rni**
Pirnapasov Abror Tolipovich	O‘zMU	87.5	1
Kamalov Ne’matjon Bahodirovich	UrDU	81.5	2
Musurmonova Shahlo	Qarshi DU	75.5	3

Foydalanilgan adabiyotlar

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Qaydlar uchun

Ro‘zimboy Madrahimov, Ne’matjon Kamalov

Baxtiyor Yusupov, Sadoqat Bekmetova

TALABALAR MATEMATIKA OLIMPIADASI MASALALARI

(Uslubiy qo‘llanma)

Muharrir **Ro‘zimboy Yo‘ldoshev**

Texnik muharrir **Sherali Yo‘ldoshev**

Musahhih **Tamara Turumova**

UrDU noshirlik bo‘limi O‘zbekiston matbuot va axborot agentligining 2009-yil 19-avgustdagi №148 raqamli buyrug‘i bilan qayta ro‘yxatdan o‘tkazilgan.

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UrDU noshirlik bo‘limida tayyorlandi.

Manzil: 220110. Urganch shahri,

H. Olimjon ko‘chasi, 14-uy.

Telefon: (8-362)-224-27-58.

UrDU bosmaxonasida chop etildi.

Manzil: 220110. Urganch shahri,

H. Olimjon ko‘chasi, 14-uy.

Telefon: (8-362)-224-27-58.