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SAMARQAND DAVLAT UNIVERSITETI

SONLI QATORLAR, FUNKSIONAL QATORLAR, XOSMAS INTEGRALLAR VA PARAMETRGA BOG'LIQ XOS VA XOSMAS INTEGRALLAR

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MUNDARIJA

SONLI QATORLAR

Kirish……………………………………………………………………………..5
§1.1. Asosiy tushunchalar.Sonli qator ta'rifi, yaqinlashuvchiligi, sodda xossalari….6
§1.2. Musbat hadli sonli qatorlarning yaqinlashuvchiligi………………………….9
§1.3. Koshi va Dalamber alomatlari …………………………………………………11
§1.4. Raabe, Gauss va Koshining integral alomatlari……………………………......13
§1.5. Musbat hadli sonli qatorlar uchun taqqoslash alomatlari..…………………......15
§1.6. Ixtiyoriy ishorali sonli qatorlar 17
§1.7. Ixtiyoriy ishorali qatorlar uchun Leybnis, Abel va Dirixle alomatlari..............19
§1.8. Absolyut va shartli yaqinlashuvchi qatorlar……………………………..........20

FUNKSIONAL KETMA - KETLIKLAR VA QATORLAR

§2.1. Funksional ketma – ketliklar va qatorlar...22
§2.2. Funksional ketma- ketlikning tekis yaqinlashishi 23
§2.3. Funksional qatorlarning yaqinlashishi, absolyut yaqinlashishi va yaqinlashish sohasi…...26
§2.4. Darajali qatorlar. Darajali qatorlarning yaqinlashish radiusi va intervali. Koshi-Adamar formulasi...27

XOSMAS INTEGRALLAR

§3.1.Cheksiz oraliq bo'yicha xosmas integrallar va ularning yaqinlashuvchiligi…...31
§3.2.Yaqinlashuvchi integrallarning xossalari……………………….....................34
§3.3.Xosmas integral yaqinlashuvchiligi haqidagi teoremalar. Integralning absolyut yaqinlashuvchiligi…………………………………………………………37
§3.4. Chegaralanmagan funksiyaning xosmas integrallari va ularning yaqin-lashuvchiligi va ularning yaqinlashuvchiligi tushunchalari…………………...40
§3.5.Yaqinlashuvchi integrallarning xossalari………………………………………42
§3.6.Xosmas integrallarning yaqinlashuvchiligi haqidagi teoremalar. Integralning absolyut yaqinlashuvchiligi…………………………………………………44

PARAMETRGA BOG'LIQ INTEGRALLAR

§4.1. Parametrga bog'liq integral tushunchasi………………………………………48
§4.2. Limit funksiya.Tekis yaqinlashish. Limit funksiyaning uzluksizligi…………49
§4.3. Parametrga bog'liq integrallarning funksional xossalari………………………52
§4.4. Parametrga bog'liq integrallar (umumiy hol)………………………………….56
§4.5. Parametrga bog'liq xosmas integral tushunchasi ……………………………..61
§4.6. Parametrga bog'liq xosmas integrallarning funksional xossalari......................67
§4.7. Eyler integrallari ………………………………….................................70
Adabiyotlar……………………………………………………………………74

KIRISH

Sonli qatorlar, funksional ketma-ketliklar, funksional qatorlar, xosmas integrallar va parametrga bog’liq xosmas integrallar bo’limi matematik analiz fanining asosiy bo’limlaridan bo’lib hisoblanadi. Bu bo’limlarni chuqur o’rganish matematikaning ko’pgina sohalarini o’zlashtirishni va amaliy masalalarni yechishni osonlashtiradi.

Sonli qatorlar bo’limida sonli qatorlarga oid ta’rif va tushunchalar, xossalari, yaqinlashish alomatlari keltirilgan, hamda bu xossalar misol va mashqlar yordamida batafsil tahlil qilingan.

Funksional ketma-ketliklar va qatorlar, xosmas integrallar va parametrga bog’liq integrallar bo’limida ham ular haqidagi ta’rif va tushunchalar, va ularning xossalari keltirilgan. Misol va nazariy mashqlarni yechish yo’llari ko’rsatilgan holda ko’pgina tushuncha va ta’riflar yoritilib o’tilgan. Mustaqil yechish uchun ham yetarli darajada misol va nazariy mashqlar keltirilgan.

Ushbu metodik ko’rsatma (qo’llanma) 2-kurs talabalariga va bu bo’limlarni mustaqil o’zlashtiruvchilar uchun mo’ljallangan bo’lib, talabalarga sonli qatorlar, funksional ketma-ketliklar, funksional qatorlar, xosmas integrallar, parametrga bog’liq integrallarni o’rganishning asosiy metodlarini, hamda funksional ketma-ketlik va qatorlarning, xosmas integrallar va parametrga bog’liq xosmas integrallarning yaqinlashishga tekshirish texnikasini yaxshiroq o’zlashtirib olishga yordam beradi. Shuningdek, funksiyalarni qatorlar yordamida tekshirish, xosmas integrallarning qo’llanishiga doir amaliy ahamiyatga ega bo’lgan malaka va ko’nikmalarni egallashda ham asosiy manbalardan biri bo’lishi mumkin.

SONLI QATORLAR

§1.1. Asosiy tushunchalar. Sonli qator ta'rifi, yaqinlashuvchiligi, sodda xossalari

Ta'rif. Ushbu $\{a_n\}_{n=1}^{\infty}$: $a_1, a_2, a_3, ..., a_n, ...$ sonli ketma-ketlik hadlaridan tuzilgan

$$a_1 + a_2 + a_3 + ... + a_n + ... \qquad (1)$$

ifodaga sonli qator deyiladi va qisqacha $\sum_{n=1}^{\infty} a_n$ kabi belgilanadi, ya'ni:

$$a_1 + a_2 + a_3 + ... + a_n + ... = \sum_{n=1}^{\infty} a_n\ , \qquad (2)$$

(2) – sonli qator hadlaridan quyidagi ketma-ketlikni tuzamiz:

$$A_1 = a_1; \quad A_2 = a_1 + a_2; ...; \ A_n = a_1 + a_2 + a_3 + ... + a_n; ...$$

bu $\{A_n\}_{n=1}^{\infty}$ ketma-ketlik (2) – sonli qatorning qismiy yig'indilar ketma-ketligi deyiladi.

Ta'rif. Agar $\{A_n\}_{n=1}^{\infty}$ ketma-ketlikning limiti chekli, ya'ni $\lim_{n\to\infty} A_n = A$ (A-chekli) bo'lsa, qator yaqinlashuvchi (yoki (1) – ifodaning qiymati deyiladi) va A soni qator yig'indisi deyiladi $\sum_{n=1}^{\infty} a_n = A$ kabi belgilanadi.

Agar $\{A_n\}_{n=1}^{\infty}$ ketma-ketlik chekli limitga ega bo'lmasa, u holda (2) – sonli qator uzoqlashuvchi deyiladi.

$$a_{m+1} + a_{m+2} + ... + a_{m+k} + ... = \sum_{n=m+1}^{\infty} a_n\ , \qquad (3)$$

yig'indiga (2) – sonli qatorning qoldiq qatori deyiladi.

1-teorema. (2) – sonli qatorning yaqinlashuvchi bo'lishi uchun (3) – qatorning yaqinlashuvchi bo'lishi zarur va yetarli.

2-teorema. Agar (2) – qator yaqinlashuvchi va yig'indisi A soniga teng bo'lsa, u holda ixtiyoriy chekli o'zgarmas c soni uchun $ca_1 + ca_2 + ca_3 + ... + ca_n + ... = \sum_{n=1}^{\infty} ca_n$ qator ham yaqinlashuvchi va yig'indisi $c \cdot A$ ga teng bo'ladi.

3-teorema. Ikkita yaqinlashuvchi $\sum_{n=1}^{\infty} a_n$ va $\sum_{n=1}^{\infty} b_n$ sonli qatorlar yig'indilari mos ravishda A va B sonlarga teng bo'lsa, u holda bu qatorlar hadlarining algebraik yig'indisidan tuzilgan

$$(a_1 \pm b_1) + (a_2 \pm b_2) + ... + (a_n \pm b_n) + ... = \sum_{n=1}^{\infty} (a_n \pm b_n)$$

qator ham yaqinlashuvchi va yig'indisi $A \pm B$ ga teng bo'ladi.

4-teorema. Agar (2) – sonli qator yaqinlashuvchi bo'lsa, u holda $\lim_{n\to\infty} a_n = 0$ bo'ladi (agar $\lim_{n\to\infty} a_n \neq 0$ bo'lsa, u holda (2) – sonli qator uzoqlashuvchi bo'ladi).

Masalan: $\frac{2}{3}+\frac{3}{5}+\frac{4}{7}+\frac{5}{9}+...$ qatorda

$a_n = \frac{n+1}{2n+1}$ bo'lib, $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n+1}{2n+1} = \lim_{n\to\infty}\frac{1+\frac{1}{n}}{2+\frac{1}{n}} = \frac{1}{2} \neq 0$ bo'lganligi uchun bu qator uzoqlashuvchi.

1-misol. $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+...+\frac{1}{(2n-1)(2n+1)}+...$ qatorning yaqinlashuvchiligini ko'rsating va yig'indisini toping.

Yechish: Bu qator uchun avval $\{A_n\}_{n=1}^{\infty}$ qismiy yig'indilar ketma-ketligini tuzamiz va limitini qaraymiz:

$A_n = \frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+...+\frac{1}{(2n-1)(2n+1)}$ buning limitini hisoblash uchun bu yig'indini quyidagicha tasvirlaymiz

$$A_n = \frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+...+\frac{1}{(2n-1)(2n+1)} =$$

$$= \frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+...+\frac{1}{2n-3}-\frac{1}{2n-1}+\frac{1}{2n-1}-\frac{1}{2n+1}\right) = \frac{1}{2}\left(1-\frac{1}{2n+1}\right)$$

bundan

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty}\frac{1}{2}\left(1-\frac{1}{2n+1}\right) = \frac{1}{2}$$

ga ega bo'lamiz. Demak, qator yaqinlashuvchi va yig'indisi $\frac{1}{2}$ ga teng bo'ladi.

2-misol. $1+\frac{1}{3}+\frac{1}{3^2}+...+\frac{1}{3^{n+1}}+...$ qatorning yaqinlashuvchiligini ko'rsating va yig'indisini toping.

Yechish. Bu holda $\{A_n\}_{n=1}^{\infty}$ ketma-ketlik quyidagicha:

$A_n = 1+\frac{1}{3}+\frac{1}{3^2}+...+\frac{1}{3^{n-1}} = \frac{1-\frac{1}{3^n}}{1-\frac{1}{3}} = \frac{3}{2}-\frac{1}{2\cdot 3^{n-1}}$ bo'lib,

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty}\left(\frac{3}{2}-\frac{1}{2\cdot 3^{n-1}}\right) = \frac{3}{2}$$

ega bo'lamiz. Demak, qator yaqinlashuvchiligi ta'rifiga ko'ra, berilgan qator yaqinlashuvchi va yig'indisi $\frac{3}{2}$ ga teng bo'ladi.

1.1 uchun misollar.

Qator yaqinlashishining zaruriy sharti bajariladimi?

1. $\frac{1^2}{1\cdot 3}+\frac{2^2}{3\cdot 5}+\frac{3^2}{5\cdot 7}+...;$

2. $\sin 1+\sin 2+\sin 3+...+\sin n+...;$

3. $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+...\ ;$

4. $\frac{3}{2}+\frac{4}{\sqrt{2}\cdot 3}+\frac{5}{\sqrt{3}\cdot 4}+\frac{6}{\sqrt{4}\cdot 5}+...;$

5. $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+...\ ;$

6. $\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^n;$

7. $\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}+...;$

8. $\sum_{n=1}^{\infty}\sqrt{\frac{3n+4}{5n+1}};$

9. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+...\ ;$

10. $-1+\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-...;$

11. $\frac{1}{1\cdot 3}+\frac{2}{3\cdot 5}+\frac{3}{5\cdot 7}+...;$

12. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n}}{\ln^2(n+1)};$

13. $\frac{1^2}{1+1^2}+\frac{2^2}{1+2^2}+\frac{3^2}{1+3^2}+...;$

14. $\sum_{n=1}^{\infty}\left(n^2+2\right)\ln\frac{n^2+1}{n^2};$

15. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+...;$

16. $\sum_{n=1}^{\infty}(n+1)arctg\,\frac{1}{n+2};$

17. $1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+...;$

18. $\sum_{n=1}^{\infty}\left(\frac{3n^3-2}{3n^3+4}\right)^{n^3};$

19. $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+...;$

20. $\sum_{n=1}^{\infty}\frac{n^3+1}{n+3}\arcsin\frac{1}{n^2+2};$

21. $\sum_{n=1}^{\infty}\sqrt[n]{0{,}01};$

22. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[n]{\ln(n+1)}};$

23. $-\frac{1}{3}+\left(\frac{5}{9}\right)^4-\left(\frac{15}{19}\right)^9+...+(-1)^n\left(\frac{2n^2-3}{2n^2+1}\right)^{n^2}+...;$

24. $\sum_{n=1}^{\infty}\frac{n^{n+\frac{1}{n}}}{\left(n+\frac{1}{n}\right)^n}.$

Quyidagi sonli qatorlarning yaqinlashuvchiligini ko'rsating va yig'indisini toping:

1. $\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+...+\frac{2n-1}{2^n}+...;$

2. $\sum_{n=1}^{\infty}\frac{1}{(n+2)(n+5)};$

3. $\frac{1}{2\cdot 5}+\frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\frac{1}{(3n-1)(3n+2)}+...;$

4. $\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+...;$

5. $1+q\sin\frac{\pi}{5}+q^2\sin\frac{2\pi}{5}+...+q^n\sin\frac{n\pi}{5}+...;$

6. $\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+...;$

7. $1+\frac{\cos 1}{2}+\frac{\cos 2}{2^2}+...+\frac{\cos n}{2^n}+...;$

8. $\frac{0}{1!}+\frac{1}{2!}+\frac{2}{3!}+...;$

9. $\left(1+\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{1}{2^2}\right)+\left(\frac{1}{3^2}+\frac{1}{2^3}\right)+...+\left(\frac{1}{3^{n-1}}+\frac{1}{2^n}\right)+...;$

10. $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+...+\frac{(-1)^{n-1}}{2^{n-1}}+...;$

11. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+...;$

12. $\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\ldots+\left(\frac{1}{2^n}-\frac{1}{3^n}\right)+\ldots;$

13. $1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\ldots;$

14. $\sum_{n=1}^{\infty}\left(\sqrt{n+1}-2\sqrt{n}+\sqrt{n-1}\right);$

15. $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\ldots;$

16. $\sum_{n=1}^{\infty}\left(\sqrt{n+3}-2\sqrt{n+2}+\sqrt{n+1}\right);$

17. $-1+\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\ldots;$

18. $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\ldots;$

19. $1-\frac{1}{2^3}+\frac{1}{2^6}-\frac{1}{2^9}+\ldots;$

20. $\frac{1}{1\cdot 3\cdot 5}+\frac{2}{3\cdot 5\cdot 7}+\frac{3}{5\cdot 7\cdot 9}+\ldots;$

21. $1+\frac{1}{2}-\frac{1}{4}+\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\ldots;$

22. $\frac{1}{1\cdot 3\cdot 5}+\frac{1}{3\cdot 5\cdot 7}+\frac{1}{5\cdot 7\cdot 9}+\ldots;$

23. $\sum_{n=1}^{\infty}\frac{1}{n(n+3)}.$

§1.2. Musbat hadli sonli qatorlarning yaqinlashuvchiligi

$$\sum_{n=1}^{\infty}a_n,\ \ (a_n\ge 0)\ ,\ \ n=1,2,3,\ldots \qquad (4)$$

qator musbat hadli sonli qator deyiladi. $A_{n+1}=A_n+a_{n+1}\ge A_n$ ekanligidan $\{A_n\}_{n=1}^{\infty}$ qismiy yig'indilari ketma-ketligining o'suvchiligi kelib chiqadi.

1-teorema. (4) – sonli qatorning yaqinlashuvchi bo'lishi uchun, $\{A_n\}_{n=1}^{\infty}$ ketma-ketlikning yuqoridan chegaralangan bo'lishi zarur va yetarli.

2-teorema. Agar (4) – sonli qatorda $a_n\ge a_{n+1},\ \ n=1,2,3,\ldots$ bo'lsa, u holda (4) – qator bilan bir vaqtda

$$\sum_{n=1}^{\infty}2^n\cdot a_{2^n}, \qquad (5)$$

sonli qator yaqinlashuvchi yoki uzoqlashuvchi bo'ladi.

1-misol. $\frac{1}{1!}+\frac{1}{2!}+\ldots+\frac{1}{n!}+\ldots$ qatorni yaqinlashishga tekshiring.

Yechish. $a_n=\frac{1}{n!}>0$, ya'ni berilgan qator musbat hadli, $A_n=\frac{1}{1!}+\frac{1}{2!}+\ldots+\frac{1}{n!}$ yig'indining yuqoridan chegaralanganligini tekshiramiz,

$$A_n=\frac{1}{1!}+\frac{1}{2!}+\ldots+\frac{1}{n!}<1+\frac{1}{2}+\frac{1}{2^n}+\ldots+\frac{1}{2^{n-1}}=\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}=2-\frac{1}{2^{n-1}}<2$$

Demak, $\{A_n\}$ ketma-ketlik yaqinlashuvchi, ya'ni berilgan qator yaqinlashuvchi bo'ladi.

2-misol. $\sum_{n=1}^{\infty}\frac{1}{n^2}$ qatorni yaqinlashishga tekshiring.

Yechish. $a_n=\frac{1}{n^2}>0$ bo'lib, bu qator musbat hadli, $a_{n+1}=\frac{1}{(n+1)^2}<\frac{1}{n^2}=a_n$ ekanligidan qator hadlarining monoton kamayuvchiligi kelib chiqadi, 2-teoremaga ko'ra, $\sum_{n=1}^{\infty}2^n\cdot a_{2^n}=\sum_{n=1}^{\infty}2^n\cdot\frac{1}{2^{2n}}=\sum_{n=1}^{\infty}\frac{1}{2^n}$ qatorning yaqinlashuvchiligidan berilgan qatorning yaqinlashuvchiligi kelib chiqadi.

1.2 uchun misollar.

Quyidagi musbat hadli sonli qatorlarni yaqinlashishga tekshiring:

1. $1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots+\frac{1}{(2n-1)^2}+\ldots;$
2. $\sum_{n=2}^{\infty}\frac{1}{\ln n};$
3. $\frac{1}{\sqrt{1}\cdot\sqrt{2}}+\frac{1}{\sqrt{2}\cdot\sqrt{3}}+\frac{1}{\sqrt{n}\cdot\sqrt{n+1}}+\ldots;$
4. $\sum_{n=1}^{\infty}\frac{1}{n^2+3};$
5. $1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-1}+\ldots;$
6. $\sum_{n=1}^{\infty}\frac{\ln n}{n};$
7. $\frac{1}{11}+\frac{1}{21}+\ldots+\frac{1}{10n+1}+\ldots;$
8. $\sum_{n=1}^{\infty}\frac{n+1}{n^2+1};$
9. $\frac{1}{\sqrt{1\cdot 3}}+\frac{1}{\sqrt{3\cdot 5}}+\ldots+\frac{1}{\sqrt{(2n-1)(2n+1)}}+\ldots;$
10. $\sum_{n=1}^{\infty}\frac{n-1}{n!};$
11. $\frac{1}{\sqrt{1}\cdot 2}+\frac{1}{\sqrt{2}\cdot 3}+\ldots+\frac{1}{\sqrt{n}\cdot(n+1)}+\ldots;$
12. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)(n+2)};$
13. Agar $\sum_{n=1}^{\infty}a_n$ va $\sum_{n=1}^{\infty}b_n$, $(a_n\geq 0;\quad b_n\geq 0)$

qatorlar uzoqlashuvchi bo'lsa, u holda $\sum_{n=1}^{\infty}\min(a_n;\ b_n)$ va $\sum_{n=1}^{\infty}\max(a_n;\ b_n)$ qatorlarni yaqinlashishga tekshiring.

14. $\sum_{n=1}^{\infty}\frac{1}{n+5};$
15. $\sum_{n=1}^{\infty}\frac{1}{25n^2+5n-6};$
16. $\sum_{n=1}^{\infty}\frac{1}{n^2+8};$
17. $\sum_{n=1}^{\infty}\frac{1}{4n^2+4n-3};$
18. $\sum_{n=1}^{\infty}\frac{1}{n!(n+2)};$
19. $\sum_{n=1}^{\infty}\frac{1}{36n^2+12n-35};$
20. $\sum_{n=1}^{\infty}\frac{n}{(2n-1)^2(2n+1)^2};$
21. $\sum_{n=2}^{\infty}\frac{1}{n^2-1};$
22. $\sum_{n=1}^{\infty}\frac{1}{16n^2-8n-3};$
23. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n+3)(2n+5)};$
24. $\sum_{n=1}^{\infty}\frac{1}{49n^2+7n-12}.$

§ 1.3. Koshi va Dalamber alomatlari

$$\sum_{n=1}^{\infty} a_n , \qquad a_n > 0 \quad n = 1,2,3,... \qquad (6)$$

qator berilgan bo'lsin.

1-teorema. (Dalamber alomati)

Agar $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$ limit mavjud bo'lib, q ga teng, ya'ni $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = q$ bo'lib, $q<1$ bo'lsa (6) – qator yaqinlashuvchi, $q>1$ bo'lsa (6) – qator uzoqlashuvchi, $q = 1$ bo'lsa (6) – qator yaqinlashuvchi bo'lishi ham bo'lmasligi ham mumkin.

2-teorema. Agar $\overline{\lim\limits_{n\to\infty}} \frac{a_{n+1}}{a_n} = q$ bo'lib, $q<1$ bo'lsa (6) – qator yaqinlashuvchi bo'ladi.

3-teorema. (Koshi alomati)

Agar $\lim\limits_{n\to\infty} \sqrt[n]{a_n}$ limit mavjud bo'lib, k ga teng bo'lsa, ya'ni $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = k$ bo'lib, $k<1$ bo'lsa (6) – qator yaqinlashuvchi, $k>1$ bo'lsa (6) – qator uzoqlashuvchi, $k = 1$ bo'lsa (6) – qator yaqinlashuvchi bo'lishi ham bo'lmasligi ham mumkin.

4-teorema. (Umumlashgan Koshi alomati).

Agar $\overline{\lim\limits_{n\to\infty}} \sqrt[n]{a_n} = k$ bo'lib, $k<1$ bo'lsa (6) – qator yaqinlashuvchi, $k>1$ bo'lsa (6) – qator uzoqlashuvchi, $k = 1$ bo'lsa (6) – qator yaqinlashuvchi bo'lishi ham bo'lmasligi ham mumkin.

5-teorema. Agar $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = q$ bo'lsa, u holda $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = q$ bo'ladi.

1-misol. $\frac{3}{1!} + \frac{3^2}{2!} + ... + \frac{3^n}{n!} + ...$ qatorni yaqinlashishiga tekshiring.

Yechish. $a_n = \frac{3^n}{n!}$ va $a_{n+1} = \frac{3^{n+1}}{(n+1)!}$ bo'lib, $\frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} = \frac{3}{n+1}$

$\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{3}{n+1} = 0 < 1,$ demak, berilgan qator Dalamber alomatiga ko'ra yaqinlashuvchi bo'ladi.

2-misol. $\sum\limits_{n=1}^{\infty} \frac{n}{\left(3 + \frac{1}{n}\right)^n}$ qatorni yaqinlashishgaga tekshiring.

Yechish. $a_n = \frac{n}{\left(3+\frac{1}{n}\right)^n}$; $\sqrt[n]{a_n} = \frac{\sqrt[n]{n}}{3+\frac{1}{n}}$ bo'lib, $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\frac{\sqrt[n]{n}}{3+\frac{1}{n}} = \frac{1}{3} < 1$, ekanligidan Koshi alomatiga ko'ra, berilgan qator yaqinlashuvchi bo'ladi.

3-misol. $\sum_{n=1}^{\infty}\frac{3+(-1)^n}{n^2}$ qatorni yaqinlashishga tekshiring.

Yechish. $a_n = \frac{3+(-1)^n}{n^2}$ bo'lib, berilgan qator musbat hadli qator yaqinlashish kriteriyasiga ko'ra yaqinlashuvchi, lekin

$$\overline{\lim_{n\to\infty}}\frac{a_{n+1}}{a_n} = \overline{\lim_{n\to\infty}}\frac{3-(-1)^n}{3+(-1)^n}\cdot\left(\frac{n}{n+1}\right)^2 = 2 > 1.$$

1.3 uchun misollar.

Quyidagi sonli qatorlarni Koshi yoki Dalamber alomatlari yordamida yaqinlashishga tekshiring:

1. $\frac{(1!)^2}{3!}+\frac{(2!)^2}{5!}+\ldots+\frac{(n!)^2}{(2n+1)!}+\ldots$;

2. $\sum_{n=1}^{\infty}\frac{n^2\left(\sqrt{3}+(-1)^n\right)^n}{2^n}$;

3. $\frac{3\cdot 1!}{1}+\frac{3^2\cdot 2!}{4^2}+\frac{3^3\cdot 3!}{4^3}+\ldots+\frac{3^n\cdot n!}{4^n}+\ldots$;

4. $\sum_{n=1}^{\infty}\left(\frac{1+\sin n}{2+\sin n}\right)^{2n+\ln n}$;

5. $\frac{1!}{1}+\frac{2!}{2^2}+\frac{3!}{3^3}+\ldots+\frac{n!}{n^n}+\ldots$;

6. $\sum_{n=1}^{\infty}\frac{4+(-1)^n}{2^{n+3}}$;

7. $\frac{(1!)^2}{2}+\frac{(2!)^2}{2^4}+\frac{(3!)^2}{2^9}+\ldots+\frac{(n!)^2}{2^{n^2}}+\ldots$;

8. $\sum_{n=1}^{\infty}\frac{\cos^2\frac{n\pi}{4}}{2^n}$;

9. $\frac{10}{1}+\frac{10\cdot 11}{1\cdot 3}+\frac{10\cdot 11\cdot 12}{1\cdot 3\cdot 5}+\ldots$;

10. $\frac{1}{2}+\frac{3!}{2\cdot 4}+\frac{5!}{2\cdot 4\cdot 6}+\ldots$;

11. $\frac{1}{\sqrt{3}}+\frac{5}{\sqrt{2\cdot 3^2}}+\frac{9}{\sqrt{3\cdot 3^3}}+\frac{13}{\sqrt{4\cdot 3^4}}+\ldots$;

12. $\frac{1}{3}+\frac{1}{4}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots$;

13. $\sum_{n=1}^{\infty}a_n$, $a_n = \begin{cases}\frac{1}{n} & агар \quad n = m^2\\ \frac{1}{n^2} & агар \quad n \neq m^2\end{cases}$ $m\in N$;

14. $\sum_{n=1}^{\infty}\frac{n}{2^n}$;

15. $\sum_{n=1}^{\infty}\frac{n^{n-1}}{\left(3n^3+2n+1\right)^{\frac{n+1}{3}}}$;

16. $\sum_{n=1}^{\infty}\frac{3^n}{n!}$;

17. $1+\frac{1\cdot 2}{1\cdot 3}+\frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5}+\ldots$;

18. $\sum_{n=1}^{\infty}\frac{n}{\left(2-\frac{1}{n}\right)^n}$;

19. $\sum_{n=1}^{\infty}\frac{n^3}{3^n+4^n}$; 20. $\sum_{n=1}^{\infty}\frac{\left(\sqrt{2}-(-1)^n\right)^n}{3^{n+1}}$;

21. $\sum_{n=1}^{\infty}\left(\frac{n+1}{n+2}\right)^{n(n+1)}$; 22. $\frac{2}{1}+\frac{4}{3!}+\frac{6}{5!}+\frac{8}{7!}+...$;

23. $\sum_{n=1}^{\infty}\frac{3+(-1)^n}{3^n}$; 24. $1+\frac{2}{2!}+\frac{4}{3!}+\frac{8}{4!}+...$;

25. $\sum_{n=1}^{\infty}\left(\sqrt{3}-\sqrt[3]{3}\right)\left(\sqrt{3}-\sqrt[5]{3}\right)...\left(\sqrt{3}-\sqrt[2n+1]{3}\right)$.

§1.4. Raabe, Gauss va Koshining integral alomatlari

$$\sum_{n=1}^{\infty}a_n, \qquad a_n>0 \quad n=1,2,3,... \tag{6}$$

qator berilgan bo'lsin.

1-teorema. (Raabe alomati). Agar $\lim_{n\to\infty}n\left(1-\frac{a_{n+1}}{a_n}\right)$ limit mavjud bo'lib, *p* ga teng bo'lsa, ya'ni $\lim_{n\to\infty}n\left(1-\frac{a_{n+1}}{a_n}\right)=p$ bo'lsa, u holda *p>1* bo'lsa qator yaqinlashuvchi, *p<1* bo'lsa (6) – qator uzoqlashuvchi, *p=1* bo'lsa (6) – qator yaqinlashuvchi bo'lishi ham bo'lmasligi ham mumkin.

2-teorema. (Gauss alomati).

Agar $\frac{a_n}{a_{n+1}}=\lambda+\frac{\mu}{n}+\frac{\theta_n}{n^{1+\varepsilon}}$ bo'lsa (bunda $|\theta_n|<c$ chegaralangan va $\varepsilon>0$), u holda $\lambda>1$ bo'lganda (6) – qator yaqinlashuvchi, $\lambda<1$ bo'lganda (6) – qator uzoqlashuvchi, $\lambda=1$ bo'lganda $\mu>1$ bo'lsa (6) – qator yaqinlashuvchi, $\mu\le1$ bo'lsa qator uzoqlashuvchi bo'ladi.

3-teorema. (Koshining integral alomati).

Agar $f(x)$ $(x\ge1)$ funksiya manfiymas, o'smaydigan, uzluksiz funksiya va $a_n=f(n)$ bo'lsin.

$$\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}f(n), \tag{7}$$

qator bilan birga

$$\lim_{A\to+\infty}\int_1^A f(x)dx, \tag{8}$$

limitni qaraymiz.

Agar (8) limit chekli bo’lsa (7) – qator yaqinlashuvchi, agar (8) limit chekli bo’lmasa, u holda (7) – qator uzoqlashuvchi bo’ladi.

1-misol. $\frac{1}{1+1^3}+\frac{1}{1+2^3}+\frac{1}{1+3^3}+...$ qatorni Raabe alomatiga ko’ra yaqinlashishga tekshiring.

Yechish. $a_n=\frac{1}{1+n^3}$; $a_{n+1}=\frac{1}{1+(n+1)^3}$ bo’lib $p_n=n\cdot\left(1-\frac{a_{n+1}}{a_n}\right)=n\cdot\left(1-\frac{1+n^3}{1+(n+1)^3}\right)=\frac{3n^3+3n^2+n}{n^3+3n^2+3n+2}\to 3>1$, berilgan qator Raabe alomatiga ko’ra yaqinlashuvchi bo’ladi.

2-misol. $1+\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{7}}+\frac{1}{\sqrt{10}}+...$ qatorni Koshining integral alomatiga ko’ra yaqinlashishga teshirng.

Yechish. $a_n=\frac{1}{\sqrt{3n-2}}=f(n)$ bo’lib, $f(x)=\frac{1}{\sqrt{3x-2}}\geq 0 \quad (x\geq 1)$ manfiymas $1\leq x_2\leq x_1$ uchun

$$f(x_1)=\frac{1}{\sqrt{3x_1-2}}\leq\frac{1}{\sqrt{3x_2-2}}=f(x_2)$$

o’smaydi. $\lim\limits_{A\to+\infty}\int\limits_1^A f(x)dx=\lim\limits_{A\to+\infty}\int\limits_1^A\frac{dx}{\sqrt{3x-2}}=\lim\limits_{A\to\infty}\left(\frac{2}{3}\sqrt{3x-2}\right)\Big|_1^A=\lim\limits_{A\to+\infty}\left(\frac{2}{3}\sqrt{3A-2}-\frac{2}{3}\right)=+\infty$, demak, berilgan qator Koshining integral alomatiga ko’ra uzoqlashuvchi bo’ladi.

3-misol. $1+\frac{1}{2^3}+\frac{1}{3^3}+...$ qatorni Gauss alomatiga ko’ra yaqinlashishga tekshiring.

Yechish. $a_n=\frac{1}{n^3}$; $a_{n+1}=\frac{1}{(n+1)^3}$ bo’lib,

$$\frac{a_n}{a_{n+1}}=\frac{(n+1)^3}{n^3}=1+\frac{3}{n}+\frac{3+\frac{1}{n}}{n^2}$$

ekanligidan $\lambda=1$; $\mu=3$; $\theta_n=3+\frac{1}{n}$, demak, Gauss alomatiga ko’ra qator yaqinlashuvchi bo’ladi..

1.4. uchun misollar.

Quyidagi sonli qatorlarni Raabe, Gauss yoki Koshining integral alomatlari yordamida yaqinlashishga tekshiring:

1. $1+\frac{1}{3^3}+\frac{1}{5^3}+\frac{1}{7^3}+...$;
2. $\frac{1}{1+1^2}+\frac{1}{1+2^2}+\frac{1}{1+3^2}+...$;
3. $1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+...$;
4. $1+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{7}}+...$;

5. $\frac{1}{1+1^4}+\frac{2}{1+2^4}+\frac{3}{1+3^4}+\dots;$

6. $\frac{1}{1+1^2}+\frac{2}{1+2^2}+\frac{3}{1+3^2}+\dots;$

7. $1+\frac{1}{4^2}+\frac{1}{7^2}+\frac{1}{10^2}+\dots;$

8. $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\dots;$

9. $\frac{1}{1+1^3}+\frac{2}{1+2^3}+\frac{3}{1+3^3}+\frac{4}{1+4^3}+\dots;$

10. $\frac{1}{2\ln^2 2}+\frac{1}{3\ln^2 3}+\frac{1}{4\ln^2 4}+\dots;$

11. $\frac{a}{b}+\frac{a(a+d)}{b(b+d)}+\frac{a(a+d)(a+2d)}{b(b+d)(b+2d)}+\dots,$; $(a,b,d>0)$

12. $\frac{1}{2\ln 2}+\frac{1}{3\ln 3}+\frac{1}{4\ln 4}+\dots;$

13. $\sum_{n=1}^{\infty}\frac{\sqrt{n!}}{(3+\sqrt{1})(3+\sqrt{2})\dots(3+\sqrt{n})};$

14. $\frac{1}{3\ln^3 3}+\frac{1}{5\ln^3 5}+\frac{1}{7\ln^3 7}+\dots;$

15. $\left(\frac{1}{2}\right)^3+\left(\frac{1\cdot 3}{2\cdot 4}\right)^3+\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^3+\dots;$

16. $\frac{1}{2\ln 2}+\frac{1}{4\ln 4}+\frac{1}{6\ln 6}+\dots;$

17. $\frac{1^2}{1\cdot 3}+\frac{2^2}{3\cdot 5}+\dots+\frac{n^2}{(2n-1)(2n+1)}+\dots;$

18. $\frac{1}{3\ln 3}+\frac{1}{5\ln 5}+\frac{1}{7\ln 7}+\dots;$

19. $1+\left(\frac{1\cdot 3}{1\cdot 4}\right)^2+\left(\frac{1\cdot 3\cdot 5}{1\cdot 4\cdot 7}\right)^2+\dots;$

20. $\sum_{n=1}^{\infty}\frac{(2n-1)!!}{(2n)!!};$

21. $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\dots;$

22. $\sum_{n=1}^{\infty}\left(\frac{(a+1)(a+2)\dots(a+n)}{(b+1)(b+2)\dots(b+n)}\right)^2;$

23. $\frac{1}{2^3}+\frac{2}{3^3}+\frac{3}{4^3}+\dots;$

24. $\sum_{n=1}^{\infty}\left(\frac{(2n+1)!!}{(2n+2)!!}\right)^{\alpha}\cdot\frac{1}{n^{\beta}}.$

§1.5. Musbat hadli qatorlar uchun taqqoslash alomatlari

$$\sum_{n=1}^{\infty}a_n, \qquad (a_n\ge 0, \qquad n=1,2,3,\dots) \tag{9}$$

$$\sum_{n=1}^{\infty}b_n, \qquad (b_n\ge 0 \qquad n=1,2,3,\dots) \tag{10}$$

qatorlar berilgan bo’lsin.

1-teorema. Agar $\exists n_0\in N$ bo’lib, $\forall n\ge n_0$ uchun $a_n\le b_n$ bo’lsa, va (10) – qator yaqinlashuvchi bo’lsa, u holda (9) – qator ham yaqinlashuvchi bo’ladi, agar (9) – qator uzoqlashuvchi bo’lsa, u holda (10) qator ham uzoqlashuvchi bo’ladi.
Xususan, $n\to\infty$ da $a_n\sim b_n$ bo’lsa, u holda (9) va (10) – qatorlar bir vaqtda yaqinlashuvchi yoki bir vaqtda uzoqlashuvchi bo’ladi.

2-teorema. Agar $n\to\infty$ da $a_n=O(b_n)$ bo’lsa, u holda (9) va (10) – qatorlar bir vaqtda yaqinlashuvchi yoki bir vaqtda uzoqlashuvchi bo’ladi.

3-teorema. Agar $n \to \infty$ da $a_n = O\left(\frac{1}{n^\alpha}\right)$ bo'lib, $\alpha > 1$ bo'lsa (9) – qator yaqinlashuvchi, $\alpha \le 1$ bo'lsa (9) – qator uzoqlashuvchi bo'ladi.

1-misol. $1 + \frac{1}{2 \cdot 5} + \frac{1}{3 \cdot 5^2} + \frac{1}{4 \cdot 5^3} + \ldots$ qatorni taqqoslash alomatlaridan foydalanib yaqinlashishga tekshiring.

Yechish. $a_n = \frac{1}{n \cdot 5^{n-1}} \le \frac{1}{5^{n-1}} = b_n$ ekanligidan va $\sum_{n=1}^{\infty} \frac{1}{5^{n-1}}$ qatorning yaqinlashuvchiligidan, 1- teoremaga ko'ra berilgan qator yaqinlashuvchi bo'ladi.

2-misol. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n^2}$ qatorni yaqinlashishga tekshiring.

Yechish. $a_n = \frac{\sqrt{n+1} - \sqrt{n}}{n^2} = \frac{1}{n^{\frac{5}{2}}\left(\sqrt{1+\frac{1}{n}} + 1\right)} = O\left(\frac{1}{n^{\frac{5}{2}}}\right), (n \to \infty)$ bo'lib, $\frac{5}{2} > 1$ bo'lganligi uchun, berilgan qator 2-teoremaga ko'ra yaqinlashuvchi bo'ladi.

1.5. uchun misollar.

Quyidagi sonli qatorlarni taqqoslash alomatlari yordamida yaqinlashishga tekshiring:

1. $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} + \ldots;$

2. $\sum_{n=1}^{\infty} \frac{\cos\frac{\pi}{4n}}{\sqrt[5]{2n^5 - 1}};$

3. $\frac{1}{\ln 2} + \frac{1}{\ln 3} + \frac{1}{\ln 4} + \ldots;$

4. $\sum_{n=1}^{\infty} \frac{arctgn}{n^2 + 1};$

5. $\sum_{n=1}^{\infty} \left(\sqrt{n+a} - \sqrt[4]{n^2 + n + 1}\right);$

6. $\sum_{n=1}^{\infty} \frac{arctg(n^2 + 2n)}{3^n + n^2};$

7. $\sum_{n=1}^{\infty} \frac{\ln(n!)}{n^3};$

8. $\sum_{n=1}^{\infty} \frac{4 + 3 \cdot (-1)^{n+1}}{2^n};$

9. $\sum_{n=1}^{\infty} \left(tg\frac{n\pi}{4n+1} - \sin\frac{n\pi}{2n-1}\right);$

10. $\sum_{n=1}^{\infty} n^2 \cdot e^{-n};$

11. $\sum_{n=1}^{\infty} n \cdot \bar{e}^{\sqrt{n}};$

12. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} arctg\frac{1}{2\sqrt{n}};$

13. $\sum_{n=1}^{\infty} e^{\frac{a\ln n + d}{b\ln n + c}};$

14. $\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n\sqrt{n}}\right);$

15. $\sum_{n=1}^{\infty} e^{-\sqrt{n}};$

16. $\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - 1\right)\sin\frac{1}{\sqrt{n+1}};$

17. $\sum_{n=1}^{\infty} \frac{1}{\ln^2\left(\sin^2\frac{1}{n}\right)};$

18. $\sum_{n=1}^{\infty} \sin\frac{2n+1}{n^3 + 5n + 3};$

19. $\sum_{n=1}^{\infty}\left(a^{\frac{1}{n}}-\frac{b^{\frac{1}{n}}+c^{\frac{1}{n}}}{2}\right)$, $(a>0;\ b>0;\ c>0)$

20. $\sum_{n=1}^{\infty}\ln\frac{n+3}{n^2+4}$;

21. $\sum_{n=1}^{\infty}\frac{\sin^2 3n}{n\sqrt{n}}$;

22. $\sum_{n=1}^{\infty}\ln\frac{n^2+4}{n^2+3}$;

23. $\sum_{n=1}^{\infty}\sin\frac{3+(-1)^n}{n^2}$.

§1.6. Ixtiyoriy ishorali sonli qatorlar

Teorema. (Koshi kriteriyasi)

$$\sum_{n=1}^{\infty} a_n\ , \qquad (11)$$

sonli qator yaqinlashuvchi bo'lishi uchun $\{A_n\}_{n=1}^{\infty}$ ketma-ketlik fundamental, ya'ni **Ошибка! Объект не может быть создан из кодов полей редактирования.** uchun $\exists n_0 \in N$ bo'lib $\forall n \geq n_0$ va $\forall p \in N$ uchun

$$\left|A_{n+p}-A_n\right| = \left|a_{n+1}+a_{n+2}+\ldots+a_{n+p}\right| < \varepsilon$$

bo'lishi zarur va yetarli.

1-misol. $b_0+\frac{b_1}{10}+\ldots+\frac{b_n}{10^n}+\ldots \quad \left(\left|b_n\right|\leq 9\right)$ qatorning yaqinlashuvchiligini Koshi kriteriyasidan foydalanib isbotlang.

Isbot. $A_n = b_0+\frac{b_1}{10}+\ldots+\frac{b_n}{10^n}$, $A_{n+p} = b_0+\frac{b_1}{10}+\ldots+\frac{b_n}{10^n}+\frac{b_{n+1}}{10^{n+1}}+\frac{b_{n+2}}{10^{n+2}}+\ldots+\frac{b_{n+p}}{10^{n+p}}$ bo'lib,

$$\left|A_{n+p}-A_n\right| = \left|\frac{b_{n+1}}{10^{n+1}}+\frac{b_{n+2}}{10^{n+2}}+\ldots+\frac{b_{n+p}}{10^{n+p}}\right| \leq \frac{9}{10^{n+1}}\left(1+\frac{1}{10}+\ldots+\frac{1}{10^{p-1}}\right) \leq$$

$$\leq \frac{9}{10^{n+1}}\left(1+\frac{1}{10}+\ldots+\frac{1}{10^{p-1}}+\frac{1}{10^p}+\ldots\right) = \frac{9}{10^{n+1}}\cdot\frac{1}{1-\frac{1}{10}} = \frac{1}{10^n} < \varepsilon,$$

$n > \lg\frac{1}{\varepsilon} \geq \left[\lg\frac{1}{\varepsilon}\right] = n_0$ deb tanlasak, $\forall p \in N$ va $\forall n \geq n_0 = \left[\lg\frac{1}{\varepsilon}\right]$ uchun $\{A_n\}_{n=1}^{\infty}$ ketma-ketlik fundamental, ya'ni berilgan qator yaqinlashuvchi bo'ladi.

2-misol. $\frac{1}{2}+\frac{1}{5}+\frac{1}{8}+\ldots+\frac{1}{3n-1}+\ldots.$ qatorni Koshi kriteriyasidan foydalanib uzoqlashuvchi ekanligini ko'rsating.

Isbot. $A_n = \frac{1}{2}+\frac{1}{5}+\frac{1}{8}+\ldots+\frac{1}{3n-1}$,

$$A_{n+p}=\frac{1}{2}+\frac{1}{5}+\frac{1}{8}+...+\frac{1}{3n-1}+\frac{1}{3n+2}+...+\frac{1}{3n+3p-1},$$

$$\left|A_{n+p}-A_n\right|=\frac{1}{3n+2}+\frac{1}{3n+5}+...+\frac{1}{3n+3p-1}\geq\frac{p}{3n+3p}=\frac{p}{6p}=\frac{1}{6}$$ bo'ladi, bundan $n=p$ desak, $\{A_n\}_{n1}^{\infty}$ ketma-ketlikning fundamental emasligi kelib chiqadi, ya'ni qator uzoqlashuvchi bo'ladi.

1.6. uchun misollar.

Quyidagi sonli qatorlarni Koshi kriteriyasidan foydalanib yaqinlashuvchiligini ko'rsating:

1. $\frac{\sin 1}{2}+\frac{\sin 2}{2^2}+...+\frac{\sin n}{2^n}+...;$
2. $\sum_{n=1}^{\infty}\frac{\cos nx}{2^n};$
3. $\frac{\cos 3-\cos 6}{1}+\frac{\cos 6-\cos 9}{2}+...;$
4. $\sum_{n=1}^{\infty}\frac{\sin n\alpha}{n(n+1)};$
5. $\frac{\cos 1^2}{1^2}+\frac{\cos 2^2}{2^2}+\frac{\cos 3^2}{3^2}+...;$
6. $\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+...+\frac{1}{(n+2)(n+3)}+...;$
7. $\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+\frac{1}{(4n-3)(4n+1)}+...;$
8. $\sum_{n=1}^{\infty}\frac{\sin^3 nx}{(n+1)(n+3)};$
9. $\frac{1}{1\cdot 6}+\frac{1}{6\cdot 11}+...+\frac{1}{(5n-4)(5n+1)}+...;$
10. $\sum_{n=1}^{\infty}\frac{x_n}{3^n}\quad (|x_n|<3);$
11. $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...;$
12. $\sum_{n=1}^{\infty}\frac{\cos x^n}{n^2}.$

Quyidagi sonli qatorlarni Koshi kriteriyasidan foydalanib uzoqlashuvchiligini ko'rsating:

1. $1+\frac{1}{\sqrt[3]{2}}+\frac{1}{\sqrt[3]{3}}+\frac{1}{\sqrt[3]{4}}+...;$
2. $\sum_{n=1}^{\infty}\frac{1}{2n+1};$
3. $1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+...;$
4. $\sum_{n=1}^{\infty}\frac{n+1}{n^2+4};$
5. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...;$
6. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right);$
7. $\frac{1}{\sqrt{2\cdot 3}}+\frac{1}{\sqrt{3\cdot 4}}+\frac{1}{\sqrt{4\cdot 5}}+...;$
8. $\sum_{n=2}^{\infty}\frac{1}{\sqrt{(n-1)n}};$
9. $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+...;$
10. $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+...;$
11. $1+\frac{1}{4}+\frac{1}{7}+\frac{1}{10}+...;$
12. $\sum_{n=1}^{\infty}\frac{n}{n^2+1}.$

§1.7. Ixtiyoriy ishorali qatorlar uchun Leybnis, Abel va Dirixle alomatlari

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n,\ \ (b_n \geq 0,\ \ n=1,2,3,...) \qquad (12)$$

sonli qator ishorasi navbat bilan almashinuvchi sonli qator deyiladi.

1-teorema. (Leybnis alomati). (12) – qatordagi $\{b_n\}_{n=1}^{\infty}$ ketma-ketlik monoton bo'lib nolga intilsa, u holda bu qator yaqinlashuvchi bo'ladi.

$$\sum_{n=1}^{\infty} a_n \cdot b_n \qquad (13)$$

qator berilgan bo'lsin.

2-teorema. (Abel alomati).

Agar $\sum_{n=1}^{\infty} a_n$ qator yaqinlashuvchi va $\{b_n\}_{n=1}^{\infty}$ ketma-ketlik monoton va chegaralangan bo'lsa, u holda (13) – qator yaqinlashuvchi bo'ladi.

3-teorema. (Dirixle alomati).

Agar $\{A_n\}_{n=1}^{\infty}=\{a_1+a_2+...+a_n\}_{n=1}^{\infty}$ qismiy yig'indilar ketma-ketligi chegaralangan, $\{b_n\}_{n=1}^{\infty}$ ketma-ketlik monoton bo'lib nolga intilsa, u holda (13) – qator yaqinlashuvchi bo'ladi.

1-misol. $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+...$ qatorni yaqinlashishga tekshiring.

Yechish. $b_n=\frac{1}{\sqrt{n}}$ bo'lib, bu monoton kamayib nolga intiladi. Leybnis alomatiga ko'ra qator yaqinlashuvchi bo'ladi.

2-misol. $\sum_{n=1}^{\infty}\frac{(-1)^{\frac{n(n-1)}{2}}}{2^n}$ qatorni yaqinlashishga tekshiring.

Yechish. Dirixle alomatini qo'llaymiz, unga ko'ra: $A_n=\sum_{k=1}^{n}(-1)^{\frac{k(k-1)}{2}}$ bo'lib, $|A_n|\leq 2$ chegaralangan, $b_n=\frac{1}{2^n}$ kamayuvchi va $b_n=\frac{1}{2^n}\to 0, n\to\infty$. Demak, berilgan qator yaqinlashuvchi bo'ladi.

3-misol. $\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{n^2}$ qatorni Abel alomatiga ko'ra yaqinlashishga tekshiring.

Yechish. $b_n=\frac{\ln n}{n}$ va $a_n=\frac{(-1)^n}{n}$ desak, u holda $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ qatorning Leybnis alomatiga ko'ra yaqinlashuvchiligi va $\{b_n\}=\left\{\frac{\ln n}{n}\right\}$ ketma-ketlikning monoton va chegaralanganligidan, berilgan qatorning Abel alomatiga ko'ra yaqinlashuvchiligi kelib chiqadi.

1.7. uchun misollar.

Quyidagi ixtiyoriy ishorali sonli qatorlarni yaqinlashishga tekshiring:

1. $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n+10}{2n+1}\right)^n$;

2. $\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2+(-1)^n}$;

3. $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt[3]{n}}{n+1}$;

4. $\sum_{n=1}^{\infty}\frac{(-1)^{\frac{n(n+1)}{2}}}{n}$;

5. $\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt[4]{n}+(-1)^n}$;

6. $\frac{1}{2\ln 2}-\frac{1}{3\ln 3}+\frac{1}{4\ln 4}-\ldots$;

7. $\sum_{n=1}^{\infty}\sin\left(\pi\sqrt{n^2+1}\right)$;

8. $1-\frac{1}{\sqrt[100]{2}}+\frac{1}{\sqrt[100]{3}}-\frac{1}{\sqrt[100]{4}}+\ldots$;

9. $\sum_{n=1}^{\infty}\frac{(-1)^{[\sqrt{n}]}}{n}$;

10. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n+1}}$;

11. $\sum_{n=1}^{\infty}\frac{\ln^2 n}{n}\cdot\sin\frac{n\pi}{4}$;

12. $\sum_{n=1}^{\infty}\cos\left(\frac{\pi}{4}+\pi n\right)\sin\frac{1}{n}$;

13. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{\cos^2 n}{n}$;

14. $\sum_{n=1}^{\infty}(-1)^n\left(1-\cos\frac{\pi}{\sqrt{n}}\right)$;

15. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[n]{n!}}$;

16. $\sum_{n=1}^{\infty}\frac{\sin n}{\sqrt[5]{n}}$;

17. $\sum_{n=1}^{\infty}\frac{\ln^3 n}{n}\cos\frac{n\pi}{4}$;

18. $\sum_{n=1}^{\infty}(-1)^n\frac{\cos^2 2n}{\sqrt{n}}$;

19. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[n]{n^2}}$;

20. $\sum_{n=1}^{\infty}\frac{(-1)^n\cdot\sin^2\left(\frac{n}{2}\right)}{\sqrt[5]{n+1}}$;

21. $\sum_{n=2}^{\infty}\frac{1}{\ln^2 n}\cdot\cos\frac{n^2\pi}{n+1}$;

22. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{\sqrt[n]{n^2+1}}$;

23. $\sum_{n=2}^{\infty}\frac{1}{\ln^2 n}\sin\frac{\pi n^2}{n+1}$;

24. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot\ln n}{\sqrt{n}}$.

§1.8. Absolyut va shartli yaqinlashuvchi sonli qatorlar

$$\sum_{n=1}^{\infty}a_n=a_1+a_2+a_3+\ldots+a_n+\ldots, \qquad (14)$$

$$\sum_{n=1}^{\infty}|a_n| = |a_1| + |a_2| + |a_3| + \ldots + |a_n| + \ldots, \qquad (15)$$

sonli qatorlar berilgan bo'lsin.

1-ta'rif. Agar (15) – qator yaqinlashuvchi bo'lsa, u holda (14) – qator absolyut yaqinlashuvchi qator deyiladi.

2-ta'rif. Agar (14) – qator yaqinlashuvchi bo'lib, (15) – qator uzoqlashuvchi bo'lsa, u holda (14) – qator shartli yaqinlashuvchi sonli qator deyiladi.

Sonli qatorni absolyut yaqinlashishga tekshirish uchun (15) – qatorga (u musbat hadli bo'lganligi uchun) yuqoridagi alomatlarning birortasini qo'llab tekshiriladi.

1-misol. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+4}$ qatorni absolyut va shartli yaqinlashishga tekshiring.

Yechish. Berilgan sonli qator hadlarining modullaridan tuzilgan $\sum_{n=1}^{\infty}\frac{1}{n^2+4}$ qator yaqinlashuvchi (Koshining integral alomati yoki 1-taqqoslash alomatiga ko'ra), demak, berilgan qator absolyut yaqinlashuvchi bo'ladi.

2-misol. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[4]{n}}$ qatorni absolyut va shartli yaqinlashishga tekshiring.

Yechish. Berilgan qator hadlarining modullaridan tuzilgan $\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n}}$ qator uzoqlashuvchi, lekin berilgan qatorda $b_n = \frac{1}{\sqrt[4]{n}}$ desak, bu monoton va nolga intiladi, ya'ni berilgan qator Leybnis alomatiga ko'ra yaqinlashuvchi. Demak, bu qator shartli yaqinlashuvchi bo'ladi.

1.8. uchun misollar.

Quyidagi sonli qatorlarni absolyut va shartli yaqinlashishga tekshiring:

1. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{p+\frac{1}{n}}}$;
2. $\sum_{n=1}^{\infty}\frac{(-n)^n}{(2n)!}$;
3. $\sum_{n=2}^{\infty}\ln\left(1+\frac{(-1)^n}{n^2}\right)$;
4. **Ошибка! Объект не может быть создан из кодов полей редактирования.**;
5. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^2}$;
6. $\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi}{4}}{n^p+\sin\frac{n\pi}{4}}$;

7. $1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+...;$

8. $\sum_{n=2}^{\infty}\frac{\sin\frac{n\pi}{12}}{\ln n};$

9. $\sum_{n=1}^{\infty}(-1)^n\frac{n-1}{n+1}\cdot\frac{1}{n^2};$

10. $\sum_{n=1}^{\infty}(-1)^{n-1}\cdot\frac{2^n\sin^{2n}x}{n};$

11. $\sum_{n=1}^{\infty}(-1)^{\frac{n(n-1)}{2}}\cdot\frac{n^{100}}{2^n};$

12. $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdot6\cdots(2n)}\right)^p;$

13. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt[n]{n^3}};$

14. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n}{n+1}\cdot\frac{1}{\sqrt[5]{n}};$

15. $\sum_{n=2}^{\infty}\frac{(-1)^n}{\left(n+(-1)^n\right)^2};$

16. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n^2\cdot2^n}{3^n+1};$

17. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n-1}{n+1}\cdot\frac{1}{\sqrt{n}};$

18. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n}}\cdot\cos\frac{\pi(n^2+1)}{n};$

19. $\sum_{n=1}^{\infty}\frac{(-1)^{[\ln n]}}{n};$

20. $\sum_{n=1}^{\infty}\frac{\cos n\cdot\sin n^2}{n};$

21. $\sum_{n=1}^{\infty}\frac{\sin\left(2n+\frac{\pi}{4}\right)}{n\cdot\sqrt[3]{n+2}};$

22. $\sum_{n=1}^{\infty}\frac{\sin nx}{n\cdot\ln^{\alpha}(n+1)};\quad 0<x<\pi$

23. $\sum_{n=1}^{\infty}n^3\cdot\sin n\cdot e^{-\sqrt{n}};$

24. $\sum_{n=1}^{\infty}\frac{\sin 2n\cdot\ln^2 n}{n^{\alpha}}.$

FUNKSIONAL KETMA - KETLIKLAR VA QATORLAR

§2.1. Funksional ketma - ketliklar

$f_n(x),\ n\in N$ funksiyalar E $(E\subset R)$ to’plamda aniqlangan bo’lsin.

$$f_1(x), f_2(x),..., f_n(x),... \qquad (1)$$

E to’plamda berilgan funksional ketma-ketlik deyiladi va u $\{f_n(x)\}$ kabi belgilanadi. (1) to’plamning elementlari $f_n(x),\ n\in N$ funksiyalar uning indeksi n orqali ham farqlanadi. $x_0\in E$ bo’lsin. Agar $\{f_n(x_0)\}$ sonli ketma-ketlik yaqinlashuvchi bo’lsa, u holda $\{f_n(x)\}$ funksional ketma-ketlik x_0 nuqtada yaqinlashuvchi deyiladi.

Agar $\{f_n(x)\}$ funksional ketma-ketlik E to’plamning har bir nuqtasida yaqinlashuvchi bo’lsa, u holda $\{f_n(x)\}$ funksional ketma-ketlik E to’plamda yaqinlashuvchi deyiladi. Bu holda E to’plamda shunday $f(x)$ funksiya aniqlangan bo'ladiki, bu funksiyaning $x_0 \in E$ nuqtadagi qiymati $\{f_n(x_0)\}$ sonli ketma-ketlikning limitiga teng bo'ladi. Bu $f(x)$ funksiyani $\{f_n(x)\}$ funksional ketma-ketlikning limitik funksiyasi deyiladi va

$$\lim_{n\to\infty} f_n(x) = f(x), \quad x \in E \tag{2}$$

yoki

$$f_n(x) \to f(x), \quad x \in E$$

kabi belgilanadi.

Limit ta'rifiga ko'ra (2) yozuv

$$\forall \varepsilon > 0, \quad \exists N = N_\varepsilon(x), \quad \forall n > N \to \left| f_n(x) - f(x) \right| < \varepsilon \tag{3}$$

munosabatning bajarilishini bildiradi.

1-misol. $f_n(x) = n\sin\dfrac{1}{nx}, \quad x \in E = (0; +\infty)$ funksional ketma-ketlikning limitik funksiyasini toping.

Yechish: $t \to 0$ da $\sin t \sim t$ munosabatdan foydalanib,

$$n\sin\frac{1}{nx} \sim \frac{1}{nx}, \ (n \to \infty, x \neq 0)$$

ekanligini olamiz. Demak, $\lim\limits_{n\to\infty} n\sin\dfrac{1}{nx} = \lim\limits_{n\to\infty}\dfrac{n\sin\frac{1}{nx}}{n\frac{1}{nx}x} = \dfrac{1}{x}$, ya'ni $f(x) = \dfrac{1}{x}$ ekan.

§2.2. Funksional ketma-ketliklarning tekis yaqinlashishi

Ta'rif. Agar

$$\forall \varepsilon > 0, \quad \exists N_\varepsilon, \quad \forall n > N_\varepsilon, \quad \forall x \in E \to \left| f_n(x) - f(x) \right| < \varepsilon \tag{4}$$

munosabat bajarilsa, u holda $\{f_n(x)\}$ funksional ketma-ketlik E to'plamda $f(x)$ funksiyaga tekis yaqinlashadi deyiladi va

$$f_n(x) \rightrightarrows f(x), x \in E \quad \text{yoki} \quad f_n \rightrightarrows f, x \in E$$

kabi yoziladi. Bu ta'rifda N_ε nomer x ga bog'liq emasligini takidlash muhim.

$\{f_n(x)\}$ funksional ketma-ketlik uchun (4) munosabatni qanoatlantiruvchi $f(x)$ funksiya mavjud bo'lsa, u holda bu funksional ketma-ketlik E to'plamda tekis yaqinlashuvchi deyiladi.

Shunday $\{a_n\}$ sonli ketma-ketlik va shunday n_0 nomer mavjud bo'lib

$$\forall n > n_0, \quad \forall x \in E \to \left| f_n(x) - f(x) \right| \leq a_n$$

munosabat bajarilib $\lim\limits_{n\to\infty} a_n = 0$ bo'lsa, u holda

$f_n(x) \rightrightarrows f(x), x \in E$ bo'ladi.

2-misol. $f_n(x)=\frac{n+1}{n+x^2}$, $x\in E=[-1,1]$ funksional ketma-ketlikning E to'plamda tekis yaqinlashishini va uning limitik funksiyasini toping.

Yechish: $f_n(x)=\frac{1+\frac{1}{n}}{1+\frac{x^2}{n}}$ bo'lganligi uchun $f(x)=1$ bo'ladi. $|x|\le 1$ bo'lganligi uchun $|f_n(x)-f(x)|=\frac{1-x^2}{n+x^2}\le\frac{1}{n}$ bo'ladi. $\frac{1}{n}\to 0,(n\to\infty)$ ekanligidan yuqoridagi mulohazaga asosan,

$\frac{n+1}{n+x^2}\rightrightarrows 1,\ x\in[-1;1]$ ekanligini olamiz.

1-teorema. $\{f_n(x)\}$ funksional ketma-ketlik E to'plamda $f(x)$ funksiyaga tekis yaqinlashishi uchun

$$\lim_{x\to\infty}\sup_{x\in E}|f_n(x)-f(x)|=0 \qquad (5)$$

tenglikning bajarilishi zarur va yetarli.

3-misol. $f_n(x)=nx^2e^{-nx}$, $E=[2,+\infty]$ funksional ketma-ketlikning E to'plamda tekis yaqinlashishini isbotlang.

Yechish: $x>0$ bo'lganligi uchun $\lim_{x\to\infty}\frac{nx^2}{e^{nx}}=0$ ekanligini ko'rish qiyin emas. Demak, $f(x)=0,\ \ x\in E$. Bunda $f_n'(x)=nxe^{-nx}(2-xn)$ tenglik o'rinli. $nxe^{-nx}(2-xn)<0$ tengsizlik $x>\frac{2}{n}$ bo'lganda o'rinli. Bu yerdan $f_n(x)$ funksiyaning $\left[\frac{2}{n},+\infty\right)$ oraliqda kamayuvchi ekanligi kelib chiqadi. Shuning uchun

$\sup_{x\in E}f_n(x)\le f_n\left(\frac{2}{n}\right)=\frac{4}{n}e^{-2}\to 0,\quad n\to\infty$ bo'ladi. Demak, $\lim_{n\to\infty}\sup_{x\in E}|f_n(x)-f(x)|=0$ tenglik o'rinli ekan va 1-teoremaga asosan qaralayotgan funksional ketma-ketlikning E to'plamda tekis yaqinlashuvchiligi kelib chiqadi.

2-teorema. (funksional ketma-ketliklarning tekis yaqinlashish haqidagi Koshi kriteriyasi)
$\{f_n(x)\}$ funksional ketma-ketlikning E to'plamda tekis yaqinlashishi uchun

$$\forall\varepsilon>0,\ \exists N_\varepsilon,\ \forall n>N_\varepsilon,\ \forall p\in E,\ \forall x\in E\to|f_{n+p}(x)-f_n(x)|<\varepsilon \qquad (6)$$

munosabatning bajarilishi zarur va yetarli.

Agar (6) munosabat bajarilmasa, ya'ni

$$\exists\varepsilon_0>0,\ \forall k\in N,\ \exists n>k,\ \exists p\in N,\ \exists\tilde{x}\in E\to|f_{n+p}(\tilde{x})-f_n(\tilde{x})|\ge\varepsilon_0 \qquad (7)$$

munosabat bajarilsa, u holda $\{f_n(x)\}$ funksional ketma-ketlik E to'plamda tekis yaqinlashuvchi bo'lmaydi.

4-misol. $f_n(x)=\frac{\ln nx}{\sqrt{nx}}$ ketma-ketlikning $E=(0,1)$ to'plamda tekis yaqinlashuvchi emasligini isbotlang.

Yechish: $\forall k\in N$ uchun $n=k,\ \ p=k=n,\ \ \tilde{x}=\frac{1}{k}=\frac{1}{n}$ deb olsak, u holda

$$\left|f_{n+p}(\tilde{x})-f_n(\tilde{x})\right|=\left|f_{2n}\left(\frac{1}{n}\right)-f_n\left(\frac{1}{n}\right)\right|=\left|\frac{\ln 2}{\sqrt{2}}-\ln 1\right|=\frac{\ln 2}{\sqrt{2}}\geq\varepsilon_0$$

bo'ladi. Bu esa (7) munosabatning bajarilishini bildiradi. Demak, qaralayotgan ketma-ketlik $E=(0,1)$ to'plamda tekis yaqinlashuvchi emas ekan.

Agar $\{f_n(x)\}$ funksional ketma-ketlikning E to'plamda limitik funksiyasi $f(x)$ bo'lsa va (3) munosabat bajarilmasa, ya'ni

$$\exists\varepsilon_0>0,\ \forall k\in N,\ \ \exists n\geq k,\ \ \exists\tilde{x}\in N\rightarrow|f_n(\tilde{x})-f(\tilde{x})|\geq\varepsilon_0 \qquad (8)$$

munosabat bajarilsa, u holda $\{f_n(x)\}$ funksional ketma-ketlik E to'plamda $f(x)$ funksiyaga notekis yaqinlashadi deyiladi.

2.1, 2.2 uchun misollar.

Quyidagi funksional ketma-ketliklarning limitik funksiyasini toping va tekis yaqinlashishga tekshiring:

1. $f_n(x)=\ln\left(3+\frac{n^2e^x}{n^4+e^{2x}}\right),\quad E=[0;\ +\infty)$

2. $f_n(x)=\frac{nx}{1+n^2x^2},\quad E=R$

3. $f_n(x)=\frac{n+1}{n+x^2},\quad E=[-1;\ 1]$

4. $f_n(x)=\sqrt{x^2+\frac{1}{n}},\quad E=R$

5. $f_n(x)=n\sin\frac{1}{nx},\quad E=[1;\ +\infty)$

6. $f_n(x)=\frac{2n^2x}{1+n^\alpha x^2},\quad \alpha>4,\ \ E=R$

7. $f_n(x)=nx^2e^{-nx},\quad E=[2,\ +\infty)$

8. $f_n(x)=\frac{arctgnx}{\sqrt{n+x}},\quad E=[0;\ +\infty)$

9. $f_n(x)=\frac{\ln nx}{nx^2},\ E=[1;\ +\infty)$

10. $f_n(x)=xe^{-nx}\ln^2 n,\quad E=[0;+\infty)$

11. $f_n(x)=x^{2n},\quad E=[0,\ q],\ \ 0<q<1$

12. $f_n(x)=e^{-(x-n)^2},\quad E=[-1;\ 1]$

13. $f_n(x)=\frac{\sin\sqrt{x}}{\ln(n+1)},\quad E=[0;\ +\infty)$

14. $f_n(x)=\sqrt{n}\cdot\sin\frac{x}{n\sqrt{n}},\ E=R$

15. $f_n(x)=\frac{2n^2x}{1+n^6x^2},\quad E=R$

16. $f_n(x)=\frac{nx^2}{n+x},\quad E=[1;+\infty)$

17. $f_n(x)=\frac{n}{x^2+n^2}arctg\sqrt{nx}, E=[0;\ +\infty)$

18. $f_n(x)=x^n-x^{n+2},\ E=[0;1]$

19. $f_n(x)=\sin\frac{x}{n^\alpha},\quad \alpha>0,\ \ E=R$

20. $f_n(x)=x^n+x^{2n}-2x^{3n},\ E=[0;\ 1]$

§2.3. Funksional qatorlarning yaqinlashishi, absolyut yaqinlashishi va yaqinlashish sohasi

$u_n(x),\ n\in N$ funksiyalar E to'plamda aniqlangan bo'lsin.

$$u_1(x)+u_2(x)+u_3(x)+...+u_n(x)+... \qquad (9)$$

ifodaga funksional qator deyiladi. (9) ifoda

$$\sum_{n=1}^{\infty}u_n(x), \qquad (10)$$

ko'rinishda ham yoziladi. $x_0\in E$ bo'lsin. Agar

$$\sum_{n=1}^{\infty}u_n(x_0)$$

sonli qator yaqinlashuvchi bo'lsa, u holda (10) funksional qator $x_0\in E$ nuqtada yaqinlashuvchi deyiladi. Agar

$$\sum_{n=1}^{\infty}|u_n(x_0)| \qquad (11)$$

sonli qator yaqinlashuvchi bo'lsa, u holda (10) funksional qator $x_0\in E$ nuqtada absolyut yaqinlashuvchi deyiladi.

Agar (10) funksional qator E to'plamning har bir nuqtasida yaqinlashuvchi bo'lsa, u holda bu funksional qatorni E to'plamda yaqinlashuvchi qator deyiladi. Agar (10) funksional qator E to'plamning har bir nuqtasida absolyut yaqinlashuvchi bo'lsa, u holda bu funksional qator E to'plamda absolyut yaqinlashuvchi deyiladi.

$$S_n(x)=\sum_{k=1}^{n}u_k(x) \qquad (12)$$

yigindi (10) funksional qatorning n – qismiy yigindisi deyiladi. Agar $S_n(x)$ funksional ketma-ketlik E to'plamda yaqinlashuvchi bo'lsa, u holda bu ketma-ketlikning limiti $S(x)$ funksiyaga (10) qatorning yigindisi deyiladi.

(10) qator yaqinlashuvchi bo'lgan x ning barcha qiymatlari to'plamiga bu qatorning yaqinlashish sohasi deyiladi.

$$\sum_{n=1}^{\infty}|u_n(x)|, \qquad (13)$$

qator yaqinlashuvchi bo'lgan x ning barcha qiymatlari to'plami (10) qatorning absolyut yaqinlashish sohasi deyiladi.

5-misol. $\sum_{n=1}^{\infty}\frac{\ln^n x}{n}$ qatorning yaqinlashish va absolyut yaqinlashish sohasini toping.

Yechish: $|\ln x|<1$ bo'lganda, ya'ni $e^{-1}<x<e$ bo'lganda bu qator absolyut yaqinlashuvchi bo'ladi. $|\ln x|>1$ bo'lganda bu qator uzoqlashuvchi bo'ladi, chunki bu holda $\lim_{n\to\infty}\frac{\ln^n x}{n}=\infty$ bo'ladi. Agar $\ln x=1$ bo'lsa, ya'ni $x=e$ bo'lsa, bu qator $\sum_{n=1}^{\infty}\frac{1}{n}$

ko'rinishni oladi. Bizga ma'lumki, bu qator uzoqlashuvchi. $\ln x = -1$ bo'lsa, ya'ni $x = e^{-1}$ bo'lsa, qaralayotgan qator $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ko'rinishda bo'ladi. Bu qator absolyut uzoqlashuvchi, lekin Leybnis teoremasiga ko'ra shartli yaqinlashuvchi bo'ladi. Demak, qaralayotgan funksional qatorning absolyut yaqinlashish sohasi (e, e^{-1}) oraliqdan, yaqinlashish sohasi $(e,\ e^{-1}]$ oraliqdan iborat bo'lar ekan.

2.3. uchun misollar.

Quyidagi funksional qatorlarning yaqinlashish va absolyut yaqinlashish sohalarini toping:

1. $\sum_{n=1}^{\infty} \frac{1}{x^n}$; 2. $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{\pi x}{n}$; 3. $\sum_{n=1}^{\infty} \frac{1}{n(x+2)^n}$; 4. $\sum_{n=1}^{\infty} \frac{\ln^n x}{n^2}$;

5. $\sum_{n=1}^{\infty} (3 - x^2)^n$; 6. $\sum_{n=1}^{\infty} n^2 e^{-nx^2}$; 7. $\sum_{n=1}^{\infty} \frac{tg^n x}{n^2+4}$; 8. $\sum_{n=1}^{\infty} \frac{\cos \pi nx}{n \ln^2 (n+1)}$;

9. $\sum_{n=1}^{\infty} n^2 \left(\frac{2x-3}{4} \right)^n$; 10. $\sum_{n=1}^{\infty} \frac{\cos nx}{\sqrt[3]{n}}$; 11. $\sum_{n=1}^{\infty} e^{-nx} \sin nx$; 12. $\sum \frac{(-1)^n}{x^2 + \sqrt{n}}$;

13. $\sum_{n=1}^{\infty} \frac{\cos nx}{n}$; 14. $\sum_{n=1}^{\infty} \frac{\sin nx}{n^{\alpha}}$ $\alpha > 0$ 15. $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$; 16. $\sum_{n=1}^{\infty} \frac{\cos nx}{n^{\alpha}}$;

17. $\sum_{n=1}^{\infty} \frac{\cos nx}{\sqrt{n}}$; 18. $\sum_{n=1}^{\infty} \frac{\sin nx}{\sqrt[3]{n}}$; 19. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n + x^2}$; 20. $\sum_{n=1}^{\infty} \left(1 + \frac{x}{n} \right) n^{-x}$.

§2.4. Funksional qatorlarning tekis yaqinlashishi

Agar (10) funksional qatorning (12) qismiy yigindilar ketma-ketligi $\{S_n(x)\}$ E to'plamda tekis yaqinlashuvchi bo'lsa, u holda (10) funksional qator E to'plamda tekis yaqinlashuvchi deyiladi.

$r_n(x) = S(x) - S_n(x)$ belgilashni kiritamiz. $r_n(x)$ funksiya (10) funksional qatorning n-qoldig'i deyiladi. (10) funksional qator E to'plamda tekis yaqinlashuvchi bo'lishi uchun $r_n(x) \underset{\to}{\to} 0, x \in E$ munosabat bajarilishi kerak, ya'ni

$$\forall \varepsilon > 0, \quad \exists N_{\varepsilon}, \quad \forall n > N_{\varepsilon}, \quad \forall x \in E \to |r_n(x)| < \varepsilon \tag{14}$$

munosabat bajarilsa (10) funksional qator E to'plamda tekis yaqinlashuvchi deyiladi. 1-teoremaga asosan (10) qator E to'plamda tekis yaqinlashuvchi bo'lishi uchun

$$\lim_{n\to\infty} \sup_{x \in E} |r_n(x)| = 0 \tag{15}$$

munosabatning bajarilishi zarur va yetarli.

Agar (10) funksional qator E to'plamda yaqinlashuvchi bo'lib, (14) shart yoki unga teng kuchli (15) shart bajarilmasa, (10) funksional qator E to'plamda notekis yaqinlashadi deyiladi. Demak, agar

$$\exists\varepsilon_0 > 0,\ \forall k \in N,\ \ \exists n > k,\ \ \exists\tilde{x} \in E \to |r_n(\tilde{x})| \geq \varepsilon_0 \qquad (16)$$

yoki

$$\lim_{n\to\infty}\sup_{x\in E}|r_n(x)| \neq 0 \qquad (17)$$

shartlar bajarilsa, u holda (10) E to'plamda notekis yaqinlashadi deyiladi.

6-misol. $\sum\limits_{n=1}^{\infty} x^{n-1},\quad x \in E = (-1, 1)$ qatorni yaqinlashishga va tekis yaqinlashishga tekshiring.

Yechish: $S_n(x) = \sum\limits_{k=1}^{n} x^{k-1} = \frac{1-x^n}{1-x}$ va $S_n(x) \underset{n\to\infty}{\to} \frac{1}{1-x},\quad \forall x \in E$ bo'lganligi uchun $r_n(x) = \frac{x^n}{1-x}$ tenglik o'rinli $\tilde{x} = 1 - \frac{1}{n}$ deb olsak, ixtiyoriy $n \in N$ uchun $\tilde{x} \in E$ va $r_n(\tilde{x}) = n\left(1 - \frac{1}{n}\right)^n \underset{n\to\infty}{\to} \infty$ bo'ladi. Bu yerdan (17) shartning bajarilishi kelib chiqadi. Demak, qaralayotgan qator E to'plamda notekis yaqinlashar ekan.

3-teorema. (Qator yaqinlashishi uchun Koshi kriteriyasi)

(10) funksional qator E to'plamda tekis yaqinlashuvchi bo'lishi uchun

$$\forall\varepsilon > 0,\ \exists N_\varepsilon,\ \forall n > N_\varepsilon,\ \forall x \in E \to \left|\sum_{k=n+1}^{n+p} u_k(x)\right| < \varepsilon \qquad (18)$$

munosabatning bajarilishi zarur va yetarli.

Agar (18) shart bajarilmasa, ya'ni

$$\exists\varepsilon_0 > 0,\ \forall m \in N,\ \exists n > m,\ \ \exists p \in N,\ \exists\tilde{x} \in E \to \left|\sum_{k=n+1}^{n+p} u_k(\tilde{x})\right| \geq \varepsilon_0$$

shart bajarilsa, u holda (10) qator E to'plamda tekis yaqinlashuvchi bo'lmaydi.

2.4. uchun misollar.

Quyidagi funksional qatorlarni tekis yaqinlashuvchilikka tekshiring:

1. $\sum\limits_{n=1}^{\infty} 2^{-n}\cos\pi nx,\quad x \in E = (-\infty; +\infty)$
2. $\sum\limits_{n=1}^{\infty} \frac{arctgnx}{x^4 + n\sqrt[3]{n}},\quad x \in E = (-\infty; +\infty)$
3. $\sum\limits_{n=1}^{\infty} \frac{\sin^2 2nx}{\sqrt[3]{n^4 + x^2}},\quad x \in E = (-\infty; +\infty)$
4. $\sum\limits_{n=1}^{\infty} \frac{x^2}{+n^{\frac{3}{2}}x^2},\quad x \in E = (-\infty; +\infty)$
5. $\sum\limits_{n=1}^{\infty} \frac{1}{(n+x)^2},\quad x \in [0; +\infty)$
6. $\sum\limits_{n=1}^{\infty} \frac{x^n}{n^2},\quad -1 \leq x \leq 1$
7. $\sum\limits_{n=2}^{\infty} \sin\frac{x^2}{n\ln^2 n},\quad x \in [-c, c],\ c > 0$
8. $\sum\limits_{n=2}^{\infty} \ln\frac{n^2 + x^2}{n^2},\quad x \in R$
9. $\sum\limits_{n=1}^{\infty} \frac{nx^2}{n^3 + x^3},\quad x \in [0,\ c],\ c > 0$
10. $\sum\limits_{n=0}^{\infty} \frac{x\sqrt[3]{n}}{1 + n^3x^2},\quad x \in R$

11. $\sum_{n=1}^{\infty} x^2\left(1-x^2\right)^{n-1}, \quad x\in[-1,1]$

12. $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, x\in[-c, c], c>0$

13. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+x^2}, \quad x\in R$

14. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n+x^2}+x^2}, \quad x\in R$

15. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}+\cos x}, \quad x\in R$

16. $\sum_{n=1}^{\infty} \sin\frac{x}{2^n}, \quad x\in R$

17. $\sum_{n=1}^{\infty} \frac{2^n+x^n}{1+3^n x^n}, \quad x\in R$

18. $\sum_{n=1}^{\infty} \frac{x^n}{1+x^n}, \quad x\in[0,1]$

19. $\sum_{n=1}^{\infty} 3^n\cdot\sin\frac{1}{4^n x}, \quad x\in R$

20. $\sum_{n=1}^{\infty} (1-x)x^n, \quad x\in[0,1]$

21. $\sum_{n=0}^{\infty} \left(1-x^2\right)x^n, \quad x\in[0,1]$

22. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^{\frac{4}{2}}, \quad x\in[0,1]$

§2.5. Darajali qatorlar. Darajali qatorning yaqinlashish radiusi va intervali. Koshi – Adamar formulasi

Funksional qatorlarning xususiy holi bo'lgan quyidagi qatorlarni qaraymiz:

$$\sum_{n=1}^{\infty} a_n x^n , \tag{19}$$

yoki

$$\sum_{n=1}^{\infty} a_n (x-x_0)^n , \tag{20}$$

Bunda $a_0, a_1, a_2, ..., a_n, ...$ o'zgarmas haqiqiy sonlar, darajali qatorning koeffisentlari deyiladi. (19) va (20) qatorlarga darajali qator deyiladi.

Funksional qatorda $u_n(x)=a_n x^n$ yoki $u_n(x)=a_n(x-x_0)^n$ desak, u holda (19) yoki (20) shaklidagi qatorlar hosil bo'ladi. $x-x_0=t$ deb belgilash bilan (20) qator (19) darajali qator shakliga keltiriladi. Shuning uchun (19) shaklidagi darajali qatorni o'rganish kifoyadir.

Darajali qatorning yaqinlashish sohasini topishda Abel teoremasi muhim rol o'ynaydi.

4-teorema.(Abel) Agar (19) darajali qator x ning $x=x_0$ qiymatida yaqinlashuvchi bo'lsa**,** u holda x ning $|x|<|x_0|$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida (19) darajali qator absolyut yaqinlashuvchi bo'ladi.

Natija. Agar (19) darajali qator x ning $x = x_0$ qiymatida uzoqlashuvchi bo'lsa**,** u holda u x ning $|x| > |x_0|$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida uzoqlashuvchi bo'ladi.

5-teorema. Har qanday (19) darajali qator uchun $\exists R$ son mavjud bo'lib,

a) agar $R \neq 0$ va $R \neq \infty$ bo'lsa, u holda (19) qator $K = \{x : |x| < R\}$ intervalda absolyut yaqinlashuvchi bo'ladi va K intervalning tashqarisida uzoqlashuvchi bo'ladi, bu intervalni (19) darajali qatorning yaqinlashish intervali deyiladi;

b) agar $R = 0$ bo'lsa, u holda (19) darajali qator faqat $x = 0$ nuqtada yaqinlashuvchi bo'ladi;

v) agar $R = \infty$ bo'lsa, u holda qator sonlar o'qining hamma nuqtalarida yaqinlashuvchi bo'ladi.

6-teorema. (Koshi-Adamar) Agar 1) chekli yoki cheksiz $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|}$ mavjud bo'lsa, u holda (19) qatorning yaqinlashish radiusi R uchun

$$\frac{1}{R} = \lim_{n\to\infty} \sqrt[n]{|a_n|}, \qquad (21)$$

bo'ladi;

2) chekli yoki cheksiz

$$\lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

mavjud bo'lsa, u holda $R = \lim\limits_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$ bo'ladi.

7- misol. $\sum\limits_{n=0}^{\infty} \frac{2^n}{n!} x^n$ darajali qatorning yaqinlashish radiusini va yaqinlashish intervalini toping.

Yechish: $\lim\limits_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim\limits_{n\to\infty} \frac{2^n (n+1)!}{n!\, 2^{n+1}} = \lim\limits_{n\to\infty} \frac{(n+1)}{2} = +\infty$. Demak, 6- teoremaga ko'ra, $R = +\infty$. Berilgan qator $(-\infty, +\infty)$ intervalda yaqinlashuvchi bo'ladi.

8- misol. $\sum\limits_{n=0}^{\infty} \frac{4^n}{n} (x-1)^n$ darajali qatorning yaqinlashish radiusini va yaqinlashish sohasini toping.

Yechish: $\frac{1}{R} = \lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{4^n}{n}} = \lim\limits_{n\to\infty} \frac{4}{\sqrt[n]{n}} = 4$. Demak, 6- teoremaga ko'ra, darajali qatorning yaqinlashish intervali $(\frac{3}{4}; \frac{5}{4})$ bo'ladi.

$x = \frac{3}{4}$ bo'lsa, qator $\sum\limits_{n=0}^{\infty} \frac{(-1)^n}{n}$ ko'rinishni oladi, ma'lumki, bu qator Leybnis teoremasiga ko'ra yaqinlashuvchi.

$x=\frac{5}{4}$ bo'lsa, qator $\sum_{n=0}^{\infty}\frac{1}{n}$ ko'rinishni oladi, ma'lumki, bu qator uzoqlashuvchi.
Demak, qatorning yaqinlashish sohasi $\left[\frac{3}{4};\frac{5}{4}\right)$ ekan.

2.5. uchun misollar.

Quyidagi darajali qatorlarning yaqinlashish radiusini va yaqinlashish sohasini toping:

1. $\sum_{n=0}^{\infty}3^n(x+1)^n$;	2. $\sum_{n=1}^{\infty}\frac{(n+1)}{4^{n+2}}(x-2)^n$;	3. $\sum_{n=1}^{\infty}\left(\sin\frac{n^2}{2^n}\right)(x-3)^n$;
4. $\sum_{n=1}^{\infty}\frac{2^n\cdot n}{n^n}(x-1)^n$;	5. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{3n+2}\right)^n(x+2)^n$;	6. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}\left(\frac{x-1}{3}\right)^n$;
7. $\sum_{n=0}^{\infty}n!x^n$;	8. $\sum_{n=0}^{\infty}\frac{x^n}{n!}$;	9. $\sum_{n=1}^{\infty}\frac{x^n}{n}$;
10. $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$;	11. $\sum_{n=1}^{\infty}\left(2+(-1)^n\right)^n\cdot\frac{x^n}{n^2}$;	12. $\sum_{n=0}^{\infty}\frac{x^n}{\sqrt{n!}}$;
13. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{(-1)^n n^2}x^n$;	14. $\sum_{n=1}^{\infty}n^n x^n$;	15. $\sum_{n=0}^{\infty}nx^n$;
16. $\sum_{n=0}^{\infty}\frac{x^n}{\sqrt{n!}}$;	17. $\sum_{n=1}^{\infty}\frac{n^n}{n!}x^n$;	18. $\sum_{n=1}^{\infty}5^n x^n$;
19. $\sum_{n=1}^{\infty}2n^2x^n$;	20. $\sum_{n=1}^{\infty}\frac{x^n}{n^3}$;	21. $\sum_{n=1}^{\infty}\frac{1}{2^n}x^n$;
22. $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cdot x^n$.		

XOSMAS INTEGRALLAR

§3.1. Cheksiz oraliq bo'yicha xosmas integrallar va ularning yaqinlashuvchiligi

f(x) funksiya [a,+∞] oraliqda aniqlangan bo'lib, bu oraliqning istalgan [a,A] qismida integrallanuvchi bo'lsin.

1-ta'rif. Agar

$$\lim_{A\to+\infty}\int_a^A f(x)dx \qquad (1)$$

mavjud bo'lsa, bu limitga $f(x)$ funksiyaning [a,+∞] oraliq bo'yicha xosmas integrali deyiladi va $\int_a^{\infty} f(x)dx$ kabi belgilanadi.

2-ta'rif. Agar (1) limit mavjud bo'lib, u chekli bo'lsa, $\int_a^{+\infty} f(x)dx$ xosmas integral yaqinlashuvchi deyiladi, *f(x)* funksiya esa [a,+∞] oraliqda xosmas ma'noda integrallanuvchi deyiladi.

Agar (1) limit cheksiz bo'lsa yoki mavjud bo'lmasa, $\int_a^{+\infty} f(x)dx$ xosmas integral uzoqlashuvchi deyiladi.

f(x) funksiyaning (-∞,a] va (-∞,+∞) oraliqlar bo'yicha xosmas integrallarini analogik ravishda yuqoridagi kabi ta'riflash mumkin, ya'ni

$$\int_{-\infty}^{a} f(x)dx = \lim_{A'\to-\infty}\int_{A'}^{a} f(x)dx$$

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{\substack{A\to+\infty \\ A'\to-\infty}}\int_{A'}^{A} f(x)dx$$

Agar $\int_{-\infty}^{a} f(x)dx$ va $\int_a^{+\infty} f(x)dx$ integrallar mavjud bo'lsa, $\int_{-\infty}^{\infty} f(x)dx$ integralni quyidagi tenglik bo'yicha aniqlasa ham bo'ladi:

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_a^{\infty} f(x)dx .$$

1-misol. $\int_0^{+\infty}\frac{dx}{x^2+4}$ xosmas integralning yaqinlashuvchiligini ko'rsating va qiymatini toping.

Yechish. $f(x)=\frac{1}{x^2+4}$ funksiya har bir chekli [0,A] (A>0) oraliqda integrallanuvchi.1-ta'rifga ko'ra,

$$\int_0^{+\infty}\frac{dx}{x^2+4} = \lim_{A\to+\infty}\int_0^{A}\frac{dx}{x^2+4} = \lim_{A\to+\infty}\left(\frac{1}{2}arctg\frac{A}{2}-\frac{1}{2}arctg\frac{0}{2}\right) = \lim_{A\to+\infty}\left(\frac{1}{2}arctg\frac{A}{2}\right) = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4} .$$

Demak, berilgan xosmas integral yaqinlashuvchi va $\int_0^{+\infty}\frac{dx}{x^2+4} = \frac{\pi}{4}$.

2-misol. $\int_{-\infty}^{+\infty}\frac{dx}{x^2+4x+9}$ xosmas integralning yaqinlashuvchi ekanligini ko'rsating va qiymatini toping.

Yechish. Ta'rifga ko'ra,

$$\int_{-\infty}^{+\infty}\frac{dx}{x^2+4x+9}=\int_{-\infty}^{0}\frac{dx}{x^2+4x+9}+\int_{0}^{+\infty}\frac{dx}{x^2+4x+9}=$$

$$=\lim_{A\to-\infty}\int_{A}^{0}\frac{dx}{x^2+4x+9}+\lim_{A'\to-\infty}\int_{0}^{A'}\frac{dx}{x^2+4x+9}=$$

$$=\lim_{A\to-\infty}\left(\frac{1}{\sqrt{5}}arctg\,\frac{x+2}{\sqrt{5}}\Bigg|_{A}^{0}\right)+\lim_{A'\to+\infty}\left(\frac{1}{\sqrt{5}}arctg\,\frac{x+2}{\sqrt{5}}\Bigg|_{0}^{A'}\right)=$$

$$=\frac{1}{\sqrt{5}}arctg\,\frac{2}{\sqrt{5}}-\frac{1}{\sqrt{5}}\lim_{A\to-\infty}arctg\,\frac{A+2}{\sqrt{5}}+\frac{1}{\sqrt{5}}\lim_{A'\to+\infty}arctg\,\frac{A'+2}{\sqrt{5}}-\frac{1}{5}arctg\,\frac{2}{\sqrt{5}}=$$

$$=-\frac{1}{\sqrt{5}}\cdot\left(-\frac{\pi}{2}\right)+\frac{1}{\sqrt{5}}\cdot\frac{\pi}{2}=\frac{\pi}{\sqrt{5}}.$$

Demak, berilgan xosmas integral yaqinlashuvchi va $\int_{0}^{+\infty}\frac{dx}{x^2+4x+9}=\frac{\pi}{\sqrt{5}}$.

3-misol. $\int_{0}^{\infty}\cos 4xdx$ xosmas integralning uzoqlashuvchi ekanligini ko’rsating.

Yechish. Ta’rifga ko’ra,

$$\int_{0}^{+\infty}\cos 4xdx=\lim_{A\to+\infty}\int_{0}^{A}\cos 4xdx=\lim_{A\to+\infty}\left(\frac{1}{4}\sin 4x\Bigg|_{0}^{A}\right)=\lim_{A\to+\infty}\frac{1}{4}\sin 4A,$$

bu limit mavjud bo’lmaganligi uchun berilgan xosmas integral uzoqlashuvchi bo’ladi.

3.1 uchun misollar.

Quyidagi xosmas integrallarni hisoblang yoki uzoqlashuvchi ekanligini ko’rsating:

1. $\int_{2}^{+\infty}\frac{dx}{x^2}$;
2. $\int_{1}^{+\infty}\frac{dx}{2\sqrt{x}}$;
3. $\int_{2}^{+\infty}\frac{dx}{x^2+x-2}$;
4. $\int_{1}^{+\infty}\frac{dx}{\sqrt{4+x^2}}$;
5. $\int_{0}^{+\infty}e^{-ax}\cos bxdx,\quad (a>0)$;
6. $\int_{-\infty}^{0}xe^{x}dx$;
7. $\int_{1}^{+\infty}e^{-3x}dx$;
8. $\int_{-\infty}^{0}\frac{x+1}{x^2+1}dx$;
9. $\int_{-\infty}^{0}\frac{dx}{x+1}$;
10. $\int_{-\infty}^{\infty}\frac{dx}{2x^2-5x+7}$;
11. $\int_{e}^{+\infty}\frac{dx}{x\ln x}$;
12. $\int_{3}^{+\infty}\frac{2x+5}{x^2+3x-10}dx$;
13. $\int_{2}^{+\infty}x\cdot 2^{-x}dx$;
14. $\int_{2}^{+\infty}\frac{dx}{x^2-1}$;

15. $\int_{1}^{+\infty} \frac{arctgx}{1+x^2} dx$;

16. $\int_{e}^{+\infty} \frac{dx}{x \ln^2 x}$;

17. $\int_{0}^{+\infty} \frac{dx}{x\sqrt{x}}$;

18. $\int_{1}^{+\infty} x \cdot \cos x dx$;

19. $\int_{0}^{+\infty} \sin 2x dx$;

20. $\int_{1}^{+\infty} \frac{dx}{x\sqrt{x^2+x+1}}$;

21. $\int_{\frac{2}{\pi}}^{+\infty} \frac{1}{x^2} \sin\frac{1}{x} dx$;

22. $\int_{-\infty}^{0} e^{5x} dx$;

23. $\int_{0}^{+\infty} e^{-ax} \sin bx dx, \quad (a>0)$;

24. $\int_{-\infty}^{\infty} \frac{dx}{x^2+9}$.

§3.2. Yaqinlashuvchi integrallarning xossalari

1. Agar $\int_{a}^{+\infty} f(x)dx$ va $\int_{a}^{+\infty} g(x)dx$ xosmas integrallar yaqinlashuvchi bo'lsa, u holda ixtiyoriy α va β o'zgarmas sonlar uchun

$$\int_{a}^{+\infty} \left(\alpha f(x) \pm \beta g(x)\right)dx \qquad (1)$$

xosmas integral ham yaqinlashuvchi bo'lib, quyidagi tenglik o'rinli bo'ladi

$$\int_{a}^{+\infty} \left(\alpha f(x) \pm \beta g(x)\right)dx = \alpha \int_{a}^{+\infty} f(x)dx \pm \beta \int_{a}^{+\infty} g(x)dx \qquad (2)$$

2. Nyuton – Leybnis formulasi.

$f(x)$ funksiya $[a, +\infty)$ oraliqda uzluksiz va $F(x)$ esa uning shu oraliqdagi boshlang'ich funksiyasi bo'lsin, u holda

$$\int_{a}^{+\infty} f(x)dx = F(x)\Big|_{a}^{+\infty} = F(+\infty) - F(a) \qquad (3)$$

bo'ladi, bu yerda $F(+\infty) = \lim_{x\to+\infty} F(x)$.

3. O'zgaruvchini almashtirish formulasi.

$f(x)$funksiya $[a, +\infty)$ oraliqda uzluksiz, $\varphi(t)$ funksiya $t \in [\alpha, \beta)$ da uzluksiz differensiallanuvchi va $a = \varphi(\alpha) \le \varphi(t) < \lim_{t\to\beta-0} \varphi(t) = +\infty$ bo'lsa, u holda

$$\int_{a}^{+\infty} f(x)dx = \int_{\alpha}^{\beta} f(\varphi(t))\varphi'(t)dt \qquad (4)$$

bo'ladi.

4. Bo'laklab integrallash formulasi.

Agar $u(x)$ va $v(x)$ funksiyalar $[a,+\infty)$ oraliqda uzluksiz differensiallanuvchi bo'lib, $\lim_{x\to+\infty}(u\cdot v)$ mavjud bo'lsa, u holda

$$\int_a^{+\infty} u\,dv = u\cdot v\Big|_a^{+\infty} - \int_a^{+\infty} v\,du \qquad (5)$$

bo'ladi, bu yerda

$$(u\cdot v)\Big|_a^{+\infty} = \lim_{x\to+\infty}(u\cdot v) - u(a)\cdot v(a).$$

5. Agar $\forall x\in[a,+\infty)$ uchun $f(x)\le g(x)$ bo'lib, $\int_a^{+\infty} f(x)dx$ va $\int_a^{+\infty} g(x)dx$ integrallar yaqinlashuvchi bo'lsa, u holda

$$\int_a^{+\infty} f(x)dx \le \int_a^{+\infty} g(x)dx$$

bo'ladi.

4-misol. $\int_2^{+\infty}\left(\frac{1}{x^2-1}+\frac{2}{(x+1)^2}\right)dx$ integralni hisoblang.

Yechish. $f(x)=\frac{1}{x^2-1}$ va $g(x)=\frac{2}{(x+1)^2}$ funksiyalar uchun quyidagi funksiyalar boshlang'ich funksiya bo'ladi.

$$F(x)=\frac{1}{2}\ln\frac{x-1}{x+1} \quad \text{va} \quad C(x)=-\frac{2}{(x+1)}$$

Nyuton-Leybnis formulasiga ko'ra topamiz:

$$\int_2^{+\infty}\frac{dx}{x^2-1} = -\frac{1}{2}\ln\frac{x-1}{x+1}\Big|_2^{+\infty} = -\frac{1}{2}\ln\frac{1}{3} = \frac{1}{2}\ln 3,$$

$$\int_2^{+\infty}\frac{2dx}{(x+1)^2} = -\frac{2}{x+1}\Big|_2^{+\infty} = \frac{2}{3}$$

demak, 1- xossaga asosan, berilgan integral $\frac{2}{3}+\frac{1}{2}\ln 3$ ga teng bo'ladi.

5-misol. $\int_1^{+\infty}\frac{arcctgx}{x^2}dx$ integralni hisoblang.

Yechish. Berilgan xosmas integralga bo'laklab integrallash formulasini qo'llasak, $u = arcctgx,\quad dv=\frac{1}{x^2}dx$ deyilsa, u holda

$$du=-\frac{dx}{1+x^2},\quad v=-\frac{1}{x}$$

bo'lib, quyidagiga ega bo'lamiz:

$$\int_1^{+\infty}\frac{arcctgx}{x^2}dx=-\frac{arcctgx}{x}\Bigg|_1^{+\infty}-\int_1^{\infty}\left(-\frac{1}{x}\right)\left(-\frac{1}{1+x^2}dx\right)=$$

$$=\frac{\pi}{4}-\int_1^{\infty}\frac{dx}{x\left(x^2+1\right)}=\frac{\pi}{4}-\int_1^{\infty}\left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx=\frac{\pi}{4}-\left(\ln x-\frac{1}{2}\ln\left(x^2+1\right)\right)\Bigg|_1^{+\infty}=$$

$$=\frac{\pi}{4}-\ln\frac{x}{\sqrt{x^2+1}}\Bigg|_1^{+\infty}=\frac{\pi}{4}+\ln\frac{1}{\sqrt{2}}=\frac{\pi}{4}-\frac{\ln 2}{2}.$$

3.2 uchun misollar.

Quyidagi xosmas integrallarni hisoblang:

1. $\int_1^{+\infty}\frac{dx}{\sqrt{x}(1+x)}$;
2. $\int_{-\infty}^{-2}\frac{dx}{x\sqrt{x^2-1}}$;
3. $\int_1^{+\infty}\frac{x^4dx}{\left(x^5+1\right)^4}$;
4. $\int_2^{+\infty}\frac{dx}{x\sqrt{x^2+x-1}}$;
5. $\int_{\sqrt{2}}^{+\infty}\frac{xdx}{\left(x^2+1\right)^3}$;
6. $\int_{-\infty}^{+\infty}\frac{dx}{\left(x^2+x+1\right)^2}$;
7. $\int_0^{+\infty}\frac{\ln x}{1+x^2}dx$;
8. $\int_0^{+\infty}e^{-\alpha x}dx, \quad (\alpha>0)$;
9. $\int_0^{+\infty}e^{-4x}\cos 5x dx$;
10. $\int_2^{+\infty}\frac{dx}{\left(x^2-1\right)^2}$;
11. $\int_1^{+\infty}\frac{x^2+1}{x^4+1}dx$;
12. $\int_0^{+\infty}\frac{1}{1+x^3}dx$;
13. $\int_0^{+\infty}\frac{dx}{e^x+\sqrt{e^x}}$;
14. $\int_2^{+\infty}\frac{xdx}{x^3-1}$;
15. $\int_0^{+\infty}e^{-\sqrt[3]{x}}dx$;
16. $\int_0^{+\infty}e^{-2x}\sin^2 x dx$;
17. $\int_0^{\infty}x^4\cdot e^{-x}dx$;
18. $\int_{\sqrt{2}}^{+\infty}\frac{dx}{(x-1)\sqrt{x^2-2}}$;
19. $\int_0^{+\infty}\frac{x^2+12}{\left(x^2+1\right)^2}dx$;
20. $\int_0^{+\infty}\frac{dx}{\left(4x^2+1\right)\sqrt{x^2+1}}$;
21. $\int_0^{+\infty}\frac{x\ln x}{(1+x^2)^2}dx$;
22. $\int_0^{+\infty}\frac{arctg(1-x)}{\sqrt[3]{(x-1)^4}}dx$;
23. $\int_0^{+\infty}\frac{arctgx}{(1+x^2)^{\frac{3}{2}}}dx$;
24. $\int_0^{+\infty}x^2e^{-x}dx$.

§3.3. Xosmas integrallarning yaqinlashuvchiligi haqidagi teoremalar. Integralning absolyut yaqinlashuvchiligi

$f(x)$ va $g(x)$ funksiyalar $[a,+\infty)$ da aniqlangan, $\forall x\in[a,+\infty)$ uchun $f(x)\geq 0,\quad g(x)\geq 0$ bo'lsin.

1-teorema. $\forall x\in[a,+\infty)$ uchun $0\leq f(x)\leq g(x)$ bo'lsa, u holda

a) $\int\limits_a^{+\infty} g(x)dx$ integralning yaqinlashuvchiligidan $\int\limits_a^{+\infty} f(x)dx$ ning yaqinlashuvchigi kelib chiqadi;

b) $\int\limits_a^{+\infty} f(x)dx$ integralning uzoqlashuvchiligidan $\int\limits_a^{+\infty} g(x)dx$ ning uzoqlashuvchiligi kelib chiqadi.

2-teorema. a) Agar $\forall x\in[a,+\infty)$ uchun $g(x)>0$ va

$$\lim_{x\to+\infty}\frac{f(x)}{g(x)}=R,\quad 0<R<\infty$$

mavjud bo'lsa, u holda

$$\int\limits_a^{+\infty} f(x)dx \quad \text{va} \quad \int\limits_a^{+\infty} g(x)dx$$

integrallar bir vaqtda yaqinlashuvchi yoki uzoqlashuvchi bo'ladi;

b) xususan, $x\to+\infty$ da $f(x)\sim g(x)$ bo'lsa, u holda $f(x)$ va $g(x)$ lar $[a,+\infty)$ da yo bir vaqtda integrallanuvchi, yo integrallanmovchi bo'ladi.

6-misol. $\int\limits_1^{\infty}\frac{\cos^2 3x}{\sqrt[3]{x^5+1}}dx$ integralni yaqinlashuvchilikka tekshiring.

Yechish. $[1,\ +\infty]$ da quyidagi tengsizlik o'rinli

$$0\leq\frac{\cos^2 3x}{\sqrt[3]{x^5+1}}<\frac{1}{\sqrt[3]{x^5}},$$

va

$$\int\limits_1^{\infty}\frac{dx}{\sqrt[3]{x^5}}=\left.\frac{x^{-\frac{5}{3}+1}}{-\frac{5}{3}+1}\right|_1^{\infty}=-\left.\frac{3}{2x^{\frac{2}{3}}}\right|_1^{\infty}=\frac{3}{2},$$

u holda 1-teoremaga asosan, berilgan integral yaqinlashuvchi bo'ladi.

3-teorema. (Koshi teoremasi)

$\int_a^{+\infty} f(x)dx$ xosmas integralning yaqinlashuvchi bo'lishligi uchun, $\forall\varepsilon > 0$ son olinganda ham, shunday $b = b(\varepsilon) \geq a$ son topilib, $b_1 > b$ va $b_2 > b$ bo'lgan ixtiyoriy $b_1,\ b_2$ lar uchun

$$\left|\int_{b_1}^{b_2} f(x)dx\right| < \varepsilon$$

tengsizlikning bajarilishi zarur va yetarli.

4-teorema. Agar $\int_a^{+\infty} |f(x)|dx$ integral yaqinlashuvchi bo'lsa, u holda $\int_a^{+\infty} f(x)dx$ integral ham yaqinlashuvchi bo'ladi.

3-ta'rif. Agar $\int_a^{+\infty} |f(x)|dx$ integral yaqinlashuvchi bo'lsa, u holda $\int_a^{+\infty} f(x)dx$ integral absolyut yaqinlashuvchi integral deyiladi. $f(x)$ funksiya $[a,\ +\infty)$ da absolyut integrallanuvchi deyiladi.

4-ta'rif. Agar $\int_a^{+\infty} f(x)dx$ integral yaqinlashuvchi bo'lib, $\int_a^{+\infty} |f(x)|dx$ integral uzoqlashuvchi bo'lsa, u holda $\int_a^{+\infty} f(x)dx$ integral shartli yaqinlashuvchi integral deyiladi.

7-misol. $\int_1^{+\infty} \frac{\cos x}{x^2}dx$ xosmas integralning absolyut yaqinlashuvchiligini isbotlang.

Yechish. Ushbu $\int_1^{+\infty} \left|\frac{\cos x}{x^2}\right|dx$ integralni qaraymiz. Ravshanki,

$\left|\frac{\cos x}{x^2}\right| \leq \frac{1}{x^2},\quad \forall x \in [1,\ +\infty)$ va $\int_1^{+\infty} \frac{1}{x^2}dx = \left(-\frac{1}{x}\right)\Bigg|_1^{+\infty} = 1$ ekanligidan, 1-taqqoslash teoremasiga ko'ra, $\int_1^{+\infty} \left|\frac{\cos x}{x^2}\right|dx$ integral yaqinlashuvchi bo'ladi. Demak, berilgan integral absolyut yaqinlashuvchi ekan.

5-teorema. (Dirixle alomati) $f(x)$ va $g(x)$ funksiyalar $[a, +\infty)$ oraliqda berilgan bo'lib, quyidagi shartlarni qanoatlantirsin:

a) $f(x)$ funksiya $[a, +\infty)$ da uzluksiz va shu oraliqdagi boshlang'ich funksiyasi $F(x)$ chegaralangan;

b) $g(x)$ funksiya $[a, +\infty)$ da uzluksiz differensiallanuvchi va monoton, bunda $\lim\limits_{x\to+\infty} g(x)=0$, u holda

$$\int_1^{+\infty} f(x)g(x)dx$$

integral yaqinlashuvchi bo'ladi.

6-teorema. (Abel alomati)

$f(x)$ va $g(x)$ funksiyalar $[a, +\infty)$ da berilgan bo'lib, quyidagi shartlarni qanoatlantirsin:

a) $f(x)$ funksiya $[a, +\infty)$ da uzluksiz va $\int_1^{+\infty} f(x)dx$ integral yaqinlashuvchi;

b) $g(x)$ funksiya $[a, +\infty)$ da uzluksiz differensiallanuvchi, monoton va chegaralangan,
u holda

$$\int_1^{+\infty} f(x)g(x)dx$$

integral yaqinlashuvchi bo'ladi.

8-misol. $\int_1^{+\infty} \frac{\sin x}{x^2} arctgx dx$ integralning yaqinlashuvchi ekanligini isbotlang.

Yechish. Abel alomatidan foydalanamiz, bunda $f(x)=\frac{\sin x}{x^2}$ va $g(x)=arctgx$ deb olamiz. $\frac{\sin x}{x^2}$ funksiya $[1, +\infty)$ da uzluksiz va yuqoridagi 7-misoldagi kabi ko'rsatish mumkinki,

$\int_1^{+\infty} \frac{\sin x}{x^2}$ integral yaqinlashuvchi bo'ladi. b) shart ham bajariladi, chunki $g'(x)=\frac{1}{1+x^2}>0$ (shuning uchun $g(x)$ monoton) va $|g(x)|=|arctgx|<\frac{\pi}{2}$.

Teoremaning barcha shartlari bajarilayapti, shuning uchun berilgan integral yaqinlashuvchi bo'ladi.

3.3. uchun misollar.

Quyidagi integrallarni yaqinlashuvchilikka tekshiring:

1. $\int_0^{+\infty} \frac{x^2}{1+x^4}dx$; 2. $\int_e^{+\infty} \frac{dx}{x\cdot \ln^3 x}$;

3. $\int_0^{+\infty} \frac{\sin^2 x}{x^2}dx$; 4. $\int_0^{+\infty} \frac{xdx}{\sqrt{x^3+x}}$;

5. $\int_0^{+\infty} \frac{xdx}{\sqrt[3]{1+x^7}}$; 6. $\int_0^{+\infty} \frac{arctgx}{x\sqrt{x}}dx$;

7. $\int\limits_{0}^{+\infty}\frac{xdx}{\sqrt[3]{x^5+2}}$; 8. $\int\limits_{0}^{+\infty}\frac{dx}{x^{\frac{1}{2}}+x^2}$;

9. $\int\limits_{0}^{+\infty}\frac{\sin^2 3x}{\sqrt[3]{x^4+2}}dx$; 10. $\int\limits_{1}^{+\infty}\frac{dx}{x\cdot\sqrt[4]{x^3+2}}$;

11. $\int\limits_{0}^{+\infty}\frac{1}{\sqrt{x}}arctg\frac{x}{2+x}dx$; 12. $\int\limits_{0}^{+\infty}\frac{x}{1+x^3}dx$.

Quyidagi integrallarni absolyut va shartli yaqinlashishga tekshiring:

1. $\int\limits_{1}^{+\infty}\frac{\sin x}{x^2}dx$; 2. $\int\limits_{1}^{+\infty}\frac{\sin x}{\sqrt{x}}dx$;

3. $\int\limits_{1}^{+\infty}\frac{\sin(x+x^2)}{x\sqrt{x}}$; 4. $\int\limits_{1}^{+\infty}\frac{1+x}{x^3}\sin x^3dx$;

5. $\int\limits_{0}^{+\infty}\frac{x\cdot\cos 7x}{x^2+2x+2}dx$; 6. $\int\limits_{0}^{+\infty}x\cdot\cos x^4dx$;

7. $\int\limits_{0}^{+\infty}\sin x^2dx$; 8. $\int\limits_{0}^{+\infty}\frac{\sin\ln x}{\sqrt{x}}dx$.

§3.4. Chegaralanmagan funksiyaning xosmas integrallari va ularning yaqinlashuvchiligi tushunchalari

$f(x)$ funksiya $[a,b)$ oraliqda aniqlangan bo'lib, bu oraliqning istalgan $[a,\xi]$, $\xi<b$ qismida integrallanuvchi bo'lsin.

5-ta'rif. Agar

$$\lim_{\varsigma\to b-0}\int\limits_{a}^{\xi}f(x)dx, \qquad (6)$$

mavjud bo'lsa, bu limitga $f(x)$ funksiyaning $[a,b)$ oraliq bo'yicha xosmas integrali deyiladi va kabi belgilanadi.

6-ta'rif. Agar (6) limit mavjud bo'lib, u chekli bo'lsa, $\int\limits_{a}^{b}f(x)dx$ xosmas integral yaqinlashuvchi deyiladi, $f(x)$ esa $[a,b)$ oraliqda xosmas ma'noda integrallanuvchi deyiladi.

Agar (6) limit cheksiz bo'lsa yoki mavjud bo'lmasa, $\int\limits_{a}^{b}f(x)dx$ xosmas integral uzoqlashuvchi deyiladi.

$f(x)$ funksiyaning $(a,b]$ va (a,b) oraliqlar bo'yicha xosmas integrallari analogik ravishda quyidagicha aniqlanadi, ya'ni

$$\int_a^b f(x)dx = \lim_{\xi\to a+0}\int_\xi^b f(x)dx,$$

$$\int_a^b f(x)dx = \lim_{\substack{\xi\to b-0\\ r\to a+0}}\int_r^\xi f(x)dx.$$

9-misol. $\int_0^1 \frac{dx}{x^\alpha}$, $\alpha\in R$ integralni yaqinlashishga tekshiring.

Yechish. $\alpha\neq 1$ bo'lsin, u holda

$$\int_0^1 \frac{dx}{x^\alpha} = \lim_{\xi\to+0}\int_\xi^1 \frac{dx}{x^\alpha} = \lim_{\xi\to+0}\frac{x^{-\alpha+1}}{1-\alpha}\Bigg|_\xi^1 = \lim_{\xi\to+0}\frac{1-\xi^{1-\alpha}}{1-\alpha} = \begin{cases}\frac{1}{1-\alpha}, & агар \quad \alpha<1 \quad булса,\\ +\infty, & агар \quad \alpha>1 \quad булса.\end{cases}$$

Endi $\alpha=1$ bo'lsin, u holda

$$\int_0^1 \frac{dx}{x} = \lim_{\xi\to+0}\int_\xi^1 \frac{dx}{x} = \lim_{\xi\to+0}\ln|x|\,\Big|_\xi^1 = -\lim_{\xi\to+0}\ln\xi = +\infty.$$

Ravshanki, $\int_0^1 \frac{dx}{x^\alpha}$ integral $\alpha<1$ bo'lsa yaqinlashuvchi va $\alpha\geq 1$ bo'lsa uzoqlashuvchi bo'ladi.

10-misol. $\int_1^2 \frac{dx}{x\ln x}$ xosmas integralning uzoqlashuvchi ekanligini ko'rsating.

Yechish. $\int_1^2 \frac{dx}{x\ln x} = \lim_{\xi\to 1+0}\int_\xi^2 \frac{dx}{x\ln x} = \lim_{\xi\to 1+0}(\ln(\ln x))\Big|_\xi^2 = \lim_{\xi\to 1+0}\left(\ln(\ln 2)-\ln(\ln\xi)\right) = +\infty.$

Demak, berilgan integral uzoqlashuvchi ekan.

3.4. uchun misollar.

Quyidagi xosmas integrallarni hisoblang yoki uzoqlashuvchi ekanligini ko'rsating:

1. $\int_0^1 \frac{dx}{2\sqrt{x}}$;
2. $\int_{-1}^1 e^{\frac{1}{x}}\frac{dx}{x^3}$;
3. $\int_0^4 \frac{dx}{\sqrt{x}+x}$;
4. $\int_{-1}^0 \frac{dx}{x}$;
5. $\int_0^1 \frac{\arcsin x}{\sqrt{1-x^2}}dx$;
6. $\int_{-1}^1 \frac{\arccos x}{\sqrt{1-x^2}}dx$;
7. $\int_{-2}^0 \frac{dx}{(x+1)\sqrt[3]{x+1}}$;
8. $\int_0^1 \frac{dx}{3\sqrt{1-x^2}}$;

9. $\int_0^3 \frac{x^2dx}{\sqrt{9-x^2}}$;   10. $\int_0^{\frac{1}{2}} \frac{dx}{x\ln x}$;

11. $\int_0^{\frac{1}{2}} \frac{dx}{x\ln^2 x}$;   12. $\int_1^2 \frac{xdx}{x^2-1}$;

13. $\int_0^{\pi} tgxdx$;   14. $\int_0^1 \frac{dx}{\sqrt[6]{x}}$;

15. $\int_0^1 \ln xdx$;   16. $\int_0^1 \frac{dx}{(x-1)^2}$;

17. $\int_0^1 \frac{dx}{(2-x)\sqrt{1-x}}$;   18. $\int_1^3 \frac{dx}{(3-x)^{\frac{1}{2}}}$;

19. $\int_{-1}^1 \frac{dx}{\sqrt{1-x^2}}$.

§3.5. Yaqinlashuvchi integrallarning xossalari

1. Agar $\int_a^b f(x)dx$ va $\int_a^b g(x)dx$ xosmas integrallar yaqinlashuvchi bo'lsa, u holda ixtiyoriy α va β o'zgarmas sonlar uchun

$$\int_a^b (\alpha f(x)+\beta g(x))dx$$

xosmas integral ham yaqinlashuvchi bo'lib quyidagi tenglik o'rinli bo'ladi.

$$\int_a^b (\alpha f(x)\pm \beta g(x))dx = \alpha\int_a^b f(x)dx \pm\beta\int_a^b g(x)dx . \qquad (7)$$

2. Nyuton-Leybnis formulasi.

$f(x)$ funksiya $[a,b)$ oraliqda uzluksiz va $F(x)$ esa uning shu oraliqdagi boshlang'ich funksiyasi bo'lsin, u holda

$$\int_a^b f(x)dx = F(x)\Big|_a^{b-0} = F(b-0)-F(a), \qquad (8)$$

bo'ladi, bu yerda $F(b-0)=\lim\limits_{x\to b-0} F(x)$.

3. O'zgaruvchini almashtirish formulasi.

$f(x)$ funksiya $[a,b)$ oraliqda uzluksiz, $\varphi(t)$ funksiya $t\in[\alpha,\beta)$ da uzluksiz differensiallanuvchi va

$$a=\varphi(\alpha)\le\varphi(t)<\lim\limits_{t\to\beta-0}\varphi(t)=b ,$$

bo'lsa, u holda

$$\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt \qquad (9)$$

bo'ladi.

4. Bo'laklab integrallash usuli.

Agar $u(x)$ va $v(x)$ funksiyalar $[a,b)$ oraliqda uzluksiz differensiallanuvchi bo'lib, $\lim\limits_{x\to b-0}(u\cdot v)$ mavjud bo'lsa, u holda

$$\int_a^b udv = u\cdot v\,|_a^b - \int_a^b vdu\,, \qquad (10)$$

bu yerda

$$(u\cdot v)\big|_a^b = \lim_{x\to b-0}(u\cdot v) - u(a)\cdot v(a).$$

5.Agar $\forall x\in[a,b)$ uchun $f(x)\le g(x)$ bo'lib, $\int_a^b f(x)dx$ va $\int_a^b g(x)dx$ integrallar yaqinlashuvchi bo'lsa, u holda

$$\int_a^b f(x)dx \le \int_a^b g(x)dx$$

bo'ladi.

11-misol. $\int_0^1 \frac{2-\sqrt[3]{x}-x^3}{\sqrt[5]{x^3}}dx$ integralni hisoblang.

Yechish. 1-xossaga asosan,

$$\int_0^1 \frac{2-\sqrt[3]{x}-x^3}{\sqrt[5]{x^3}}dx = \int_0^1 \frac{2}{\sqrt[5]{x^3}}dx - \int_0^1 \frac{1}{x^{\frac{4}{15}}}dx - \int_0^1 x^{\frac{12}{5}}dx =$$

$$= 2\cdot \left.\frac{x^{-\frac{3}{5}+1}}{-\frac{3}{5}+1}\right|_{+0}^1 - \left.\frac{x^{-\frac{4}{15}+1}}{-\frac{4}{15}+1}\right|_{+0}^1 - \left.\frac{x^{\frac{12}{5}+1}}{\frac{12}{5}+1}\right| = \frac{2}{\frac{2}{5}}\cdot 1 - \frac{15}{11} - \frac{5}{17} =$$

$$= 5 - \frac{15}{11} - \frac{5}{17} = \frac{935-255-55}{187} = \frac{625}{187}.$$

12-misol. $\int_0^1 \frac{2\ln x}{\sqrt{x}}dx$ integralni hisoblang.

Yechish. Berilgan xosmas integralga bo'laklab integrallash formulasini qo'llaymiz:

$$u = \ln x, \quad dv = \frac{2dx}{\sqrt{x}}\,,$$

u holda

$$du = \frac{dx}{x}, \quad v = 4\sqrt{x}\,.$$

$$\int_0^1 \frac{2\ln x}{\sqrt{x}}dx = 4\sqrt{x}\ln x\Big|_{+0}^1 - 4\int_0^1 \frac{dx}{\sqrt{x}} = -4\lim_{x\to+0}\sqrt{x}\cdot\ln x - 8\sqrt{x}\Big|_{+0}^1 = -8.$$

3.5. uchun misollar.

Quyidagi xosmas integrallarni hisoblang:

1. $\int\limits_{\sqrt{2}}^{2} \frac{dx}{(x-1)\sqrt{x^2-2}}$; 2. $\int\limits_{-1}^{1} \frac{x^4 dx}{(1+x^2)\sqrt{1-x^2}}$;

3. $\int\limits_0^1 \frac{\left(\sqrt[6]{x}+1\right)^2}{\sqrt{x}}dx$; 4. $\int\limits_0^1 \frac{1-x}{\sqrt[3]{x}}dx$;

5. $\int\limits_0^1 \frac{dx}{\sqrt{x\sqrt{x}}}$; 6. $\int\limits_0^2 \left(\frac{2}{\sqrt{2-x}}+\frac{1}{\sqrt{x}}\right)dx$;

7. $\int\limits_2^3 \frac{2-x}{\sqrt{3-x}}dx$; 8. $\int\limits_0^1 \frac{x}{\sqrt{1-x^2}}dx$;

9. $\int\limits_1^3 \frac{dx}{4x\sqrt{\ln x}}$; 10. $\int\limits_{-0,5}^{-0,25} \frac{dx}{x\sqrt{2x+1}}$;

11. $\int\limits_0^2 \frac{5x^3 dx}{\sqrt{4-x^2}}$; 12. $\int\limits_0^3 \left(\frac{3}{\sqrt[3]{x}}+\frac{8}{(3-x)^{\frac{1}{4}}}\right)dx$;

13. $\int\limits_0^{\frac{\pi}{2}} \sqrt{tgx}\,dx$; 14. $\int\limits_{-1}^{1} \frac{dx}{\sqrt{1-x^2}}$;

15. $\int\limits_0^1 \frac{x+1}{\sqrt[3]{(x-1)^2}}dx$; 16. $\int\limits_0^1 x\ln x\,dx$;

17. $\int\limits_0^1 \frac{x^3 \arcsin x}{\sqrt{1-x^2}}dx$; 18. $\int\limits_1^2 \frac{dx}{x\sqrt{3x^2-2x-1}}$;

19. $\int\limits_0^{\frac{\pi}{4}} \sqrt{ctgx}\,dx$; 20. $\int\limits_0^1 \frac{x^2}{\sqrt{1-x^2}}dx$.

§3.6. Xosmas integrallarning yaqinlashuvchiligi hakidagi teoremalar. Integralning absolyut yaqinlashuvchiligi

$f(x)$ va $g(x)$ funksiyalar $[a,b)$ da aniqlangan, $\forall x\in[a,b)$ uchun $f(x)\geq 0$, $g(x)\geq 0$ bo'lsin.

7-teorema. $\forall x\in[a,b)$ uchun $0\leq f(x)\leq g(x)$ bo'lsa, u holda

a) $\int_a^b g(x)dx$ integralning yaqinlashuvchiligidan $\int_a^b f(x)dx$ integralning yaqinlashuvchiligi kelib chiqadi;

b) $\int_a^b f(x)dx$ integralning uzoqlashuvchiligidan $\int_a^b g(x)dx$ integralning uzoqlashuvchiligi kelib chiqadi.

8-teorema. a) Agar $\forall x \in [a,b)$ uchun $g(x) > 0$ va

$$\lim_{x \to b-0} \frac{f(x)}{g(x)} = R, \qquad 0 < R < \infty$$

mavjud bo'lsa, u holda

$$\int_a^b f(x)dx \quad \text{va} \quad \int_a^b g(x)dx$$

integrallar bir vaqtda yaqinlashuvchi yoki uzoqlashuvchi bo'ladi;

b) Xususan, $x \to b-0$ da $f(x) \sim g(x)$ bo'lsa, u holda $f(x)$ va $g(x)$ lar $[a,b)$ da yo bir vaqtda integrallanuvchi, yo integrallanmovchi bo'ladi.

13-misol. $\int_0^1 \frac{\sin^2\left(\frac{1}{x}\right)}{2\sqrt{x}} dx$ integralni yaqinlashuvchilikka tekshiring.

Yechish. (0, 1) oraliqda quyidagi tengsizlik o'rinli

$$0 \le \frac{\sin^2\left(\frac{1}{x}\right)}{2\sqrt{x}} \le \frac{1}{2\sqrt{x}},$$

va $\int_0^1 \frac{dx}{2\sqrt{x}}$ integral yaqinlashuvchi ekanligidan, 7-teoremaga asosan, berilgan integral yaqinlashuvchi bo'ladi.

$f(x)$ funksiya $[a,b)$ oraliqda aniqlangan va $[a,\xi]$, $\xi < b$ kesmada xos ma'noda integrallanuvchi bo'lsin va $x = b$ nuqtaning chap atrofida chegaralanmagan bo'lsin.

9-teorema (Koshi teoremasi)

$\int_a^b f(x)dx$ xosmas integralning yaqinlashuvchi bo'lishligi uchun, $\forall \varepsilon > 0$ son olinganda ham, shunday $r \in [a,b)$ son topilib, $\forall r_1, r_2 \in (r,b)$ lar uchun

$$\left| \int_{r_1}^{r_2} f(x)dx \right| < \varepsilon$$

tengsizlikning bajarilishi zarur va yetarli.

10-teorema. Agar $\int_a^b |f(x)|\,dx$ integral yaqinlashuvchi bo'lsa, u holda $\int_a^b f(x)dx$ integral ham yaqinlashuvchi bo'ladi.

7-ta'rif. Agar $\int_a^b |f(x)|\,dx$ integral yaqinlashuvchi bo'lsa, u holda $\int_a^b f(x)dx$ integral absolyut yaqinlashuvchi integral deyiladi.

8-ta'rif. Agar $\int_a^b f(x)dx$ integral yaqinlashuvchi bo'lib, $\int_a^b |f(x)|\,dx$ integral uzoqlashuvchi bo'lsa, u holda $\int_a^b f(x)dx$ integral shartli yaqinlashuvchi integral deyiladi.

14-misol. $\int_0^1 \frac{1}{\sqrt{1-x}}\sin\frac{\pi}{1-x}dx$ xosmas integralning absolyut yaqinlashuvchiligini isbotlang.

Yechish. Avvalo, $\left|\frac{1}{\sqrt{1-x}}\sin\frac{\pi}{1-x}\right| \le \frac{1}{\sqrt{1-x}}$

bo'lishini aniqlaymiz. Ravshanki,

$$\int_0^1 \frac{dx}{\sqrt{1-x}} = -2\sqrt{1-x}\Big|_0^1 = 2$$

Unda 7-teoremaga asosan,

$$\int_0^1 \left|\frac{1}{\sqrt{1-x}}\cdot\sin\frac{\pi}{1-x}\right|dx$$

xosmas integralning yaqinlashuvchiligi kelib chiqadi, demak, berilgan xosmas integral absolyut yaqinlashuvchi.

11-teorema (Dirixle alomati)

$f(x)$ va $g(x)$ funksiyalar $[a,b)$ oraliqda berilgan bo'lib, quyidagi shartlarni qanoatlantirsin:

a) $f(x)$ funksiya $[a,b)$ da uzluksiz va shu oraliqdagi boshlang'ich funksiyasi $F(x)$ chegaralangan;

b) $g(x)$ funksiya $[a,b)$ da uzluksiz differensiallanuvchi va monoton, bunda $\lim\limits_{x\to b-0} g(x) = 0$,

u holda

$$\int_a^b f(x)\cdot g(x)dx$$

integral yaqinlashuvchi bo'ladi.

12-teorema. (Abel alomati)

$f(x)$ va $g(x)$ funksiyalar $[a,b)$ da berilgan bo'lib, quyidagi shartlarni qanoatlantirsin:

a) $f(x)$ funksiya $[a,b)$ da uzluksiz va $\int_a^b f(x)dx$ integral yaqinlashuvchi;

b) $g(x)$ funksiya $[a,b)$ da uzluksiz differensiallanuvchi, monoton va chegaralangan,

u holda

$$\int_a^b f(x)\cdot g(x)dx$$

integral yaqinlashuvchi bo'ladi.

15-misol. $\int_0^1 \cos\left(\frac{1}{1-x}\right)\frac{dx}{1-x}$ integralning yaqinlashuvchi ekanligini isbotlang.

Yechish. Dirixle alomatidan foylanamiz,

$$f(x)=\frac{1}{(1-x)^2}\cos\left(\frac{1}{1-x}\right) \text{ va } g(x)=1-x$$

deb olamiz.

$f(x)$ funksiya $[0,1)$ oraliqda uzluksiz va uning boshlangich funksiyasi $\left(\sin\frac{1}{1-x}\right)$ chegaralangan. $g(x)$ funksiya $[0,1)$ da uzluksiz differensiallanuvchi va monoton, bunda $\lim\limits_{x\to 1-0} g(x)=0$. Dirixle alomatining ikkala sharti bajarilayapti. Ravshanki, berilgan integral yaqinlashuvchi bo'ladi.

3.6. uchun misollar.

Quyidagi integrallarni yaqinlashuvchilikka tekshiring:

1. $\int_0^1 \frac{dx}{\sqrt[3]{1-x^4}}$;
2. $\int_0^8 \frac{dx}{x^2+\sqrt[3]{x}}$;
3. $\int_0^1 \frac{dx}{\sqrt[5]{1-x^{10}}}$;
4. $\int_0^1 \frac{\ln(1+x^{2\backslash 3})}{\sqrt{x}\sin\sqrt{x}}dx$;
5. $\int_0^1 x\cdot\ln\frac{1}{x}dx$;
6. $\int_0^1 x^3\ln^3\frac{1}{x}dx$;
7. $\int_0^2 \frac{dx}{\ln x}$;
8. $\int_0^2 \sqrt{\frac{16+x^4}{16-x^4}}dx$;
9. $\int_0^\pi \frac{1-\cos x}{x\sqrt{x}}dx$;
10. $\int_0^2 \frac{dx}{\sqrt{x}+arctgx}$;
11. $\int_0^\pi \frac{\sin x}{x^2}dx$.

Quyidagi integrallarni absolyut va shartli yaqinlashishga tekshiring:

1. $\int_0^1 \frac{1}{\sqrt[4]{1-x}}\cos\frac{\pi}{1-x}dx$;

2. $\int_0^1 \frac{\sin(1/x)dx}{x^2+\sqrt{x^3}+x^2\cos(1/x)}$;

3. $\int_0^1 \cos\left(\frac{1}{\sqrt{x}}-1\right)\frac{dx}{\sqrt[3]{x}}$;

4. $\int_0^1 \frac{1}{x\sqrt{x}}\cos\left(\frac{\sqrt{x}-1}{\sqrt{x}}\right)dx$;

5. $\int_0^1 \frac{\sin\frac{1}{x^2}}{x^2}dx$;

6. $\int_0^1 \frac{\sin(1/x)}{\left(\sqrt{x}-x\right)^{\frac{1}{2}}}dx$;

7. $\int_0^1 \frac{x}{x^2+1}\sin\frac{1}{x}dx$;

8. $\int_0^1 xarctgx\cdot\cos\frac{1}{x}dx$.

PARAMETRGA BOG'LIQ INTEGRALLAR

§4.1. Parametrga bog'liq integral tushunchasi

f(x,y) funksiya R^2 fazodagi biror $P=\left\{(x,y)\in R^2;\ x\in[a,b],\ y\in E\subset R\right\}$ to'plamda berilgan bo'lib, y o'zgaruvchining E to'plamidan olingan har bir tayinlangan qiymatida *f(x,y)* funksiya x o'zgaruvchining funksiyasi sifatida [a,b] oraliqda integrallanuvchi bo'lsin, ya'ni

$$\int_a^b f(x,y)dx, \qquad (1)$$

integral mavjud bo'lsin. Ravshanki, bu integralning qiymati y parametrga bog'liq bo'ladi:

$$I(y)=\int_a^b f(x,y)dx$$

Bu (1) integral parametrga bog'liq integral deb ataladi, y o'zgaruvchi esa parametr deyiladi. Bu paragrafda parametrga bog'liq (1) integralning funksional xossalari o'rganiladi. Bu xossalarni o'rganishda *f(x,y)* funksiyaning y bo'yicha limiti va unga intilish xarakteri muhim rol o'ynaydi.

Misol. Ushbu *f(x)=sinxy* funksiyaning x o'zgaruvchisi bo'yicha [a,b] segmentdagi integrali(bu yerda $y\neq 0$)

$$\int_a^b f(x,y)dx=\int_a^b \sin ydx=\frac{1}{y}\int_a^b \sin xyd(xy)=\left.\frac{\cos xy}{y}\right|_a^b=\frac{\cos ay-\cos by}{y}$$

bo'lib, $E = R\setminus\{0\}$ to'plamda berilgan $I(y)=\frac{1}{y}(\cos ay-\cos by)$ funksiyadan iboratdir.

§4.2. Limit funksiya. Tekis yaqinlashish. Limit funksiyaning uzluksizligi

f(x,y) funksiya $P=\{(x,y)\in R^2;\ a\le x\le b,\ \ y\in E\subset R\}$ to'plamda berilgan, y_o esa $E(E\subset R)$ to'plamning limit nuqtasi bo'lsin.

1-ta'rif. Agar $\forall\varepsilon>0$ olinganda ham, $\forall x\in[a,b]$ uchun shunday $\delta=\delta(\varepsilon,x)>0$ topilsaki $|y-y_0|<\delta$ tengsizlikni qanoatlantiruvchi $\forall y\in E$ uchun $|f(x,y)-\varphi(x)|<\varepsilon,\quad (a\le x\le b)$ bo'lsa, u holda $\varphi(x)$ funksiya *f(x,y)* funksiyaning $y\to y_0$ dagi limit funksiyasi deyiladi.

f(x,y) funksiya $P=\{(x,y)\in R^2;\ a\le x\le b,\ \ y\in E\subset R\}$ to'plamda berilgan bo'lib ∞ E to'plamning limit nuqtasi bo'lsin.

2-ta'rif. Agar $\forall\varepsilon>0$ olinganda ham $\forall x\in[a,b]$ uchun shunday $\Delta=\Delta(\varepsilon,x)>0$ topilsaki, $|y|>\Delta$ tengsizlikni qanoatlantiruvchi $\forall y\in E$ uchun $|f(x,y)-\varphi(x)|<\varepsilon$ bo'lsa, u holda $\varphi(x)$ funksiya *f(x,y)* funksiyaning $y\to\infty$ dagi limit funksiyasi deyiladi.

1-misol. Ushbu $f(x,y)=x^2\sin y$ funksiyaning $P=\{(x,y)\in R^2:\ 0\le x\le 1,\ \ y\in R\}$ to'plamda berilgan bo'lsin. $y\to\frac{\pi}{2}$ dagi limit funksiya x^2 ekanini ko'rsating.

Yechish. Agar $\forall\varepsilon>0$ ga ko'ra, $\delta=\varepsilon$ deb olinsa, unda $|y-y_0|=\left|y-\frac{\pi}{2}\right|<\delta$ tengsizlikni qanoatlantiruvchi $\forall y\in R$ va $\forall x\in[0,1]$ uchun

$$|f(x,y)-\varphi(x)|=\left|x^2\cdot\sin y-x^2\right|=\left|x^2\right|\left|\sin y-1\right|=x^2\left|\sin y-\sin\frac{\pi}{2}\right|=x^2\left|2\sin\frac{y-\frac{\pi}{2}}{2}\cdot\cos\frac{y+\frac{\pi}{2}}{2}\right|\le$$

$\le\left|y-\frac{\pi}{2}\right|<\varepsilon$ bo'ladi. Demak, $y\to\frac{\pi}{2}$ da $f(x,y)=x^2\sin y$ funksiyaning limit funksiyasi

$$\varphi(x)=\lim_{y\to\frac{\pi}{2}}f(x,y)=\lim_{y\to\frac{\pi}{2}}x^2\sin y=x^2 \quad \text{bo'ladi.}$$

2-misol. Ushbu $f(x,y)=\frac{xy}{1+x^2y^2}$ funksiya $P=\{(x,y)\in R^2:\ 0\le x\le 1,\ \ y\in R\}$ to'plamda berilgan bo'lsin. $y\to\infty$ dagi limit funksiyasini toping.

Yechish. $\varphi(x)=\lim\limits_{y\to\infty}\frac{xy}{1+x^2y^2}=0$ ekanligini ko'rsatamiz.

Agar $\forall\varepsilon>0$ ga ko'ra, $\Delta=\frac{1}{x\varepsilon}$ deb olinsa, unda $|y|>\Delta$ ni qanoatlantiruvchi $\forall y\in R$ uchun $|f(x,y)-\varphi(x)|=\left|\frac{xy}{1+x^2y^2}-0\right|=\frac{yx}{1+y^2x^2}<\frac{1}{yx}<\varepsilon$ bo'ladi. Demak, $0<x\le 1$ uchun $y\to\infty$ da $f(x,y)=\frac{xy}{1+x^2y^2}$ funksiyaning limit funksiyasi $\varphi(x)=0$ bo'ladi.

Yuqorida keltirilgan misollarning birinchisida, limit funksiya ta'rifidagi $\delta=\varepsilon$ bo'lib, u faqat ε gagina bog'liq, ikkinchisida esa $\Delta=\frac{1}{x\varepsilon}$ bo'lib, u berilgan $\varepsilon>0$ bilan birga qaralayotgan *x* nuqtaga ham bog'liq ekanini ko'ramiz.

Limit funksiya ta'rifidagi $\delta>0$ ning qaralayotgan *x* nuqtalarga bog'liq bo'lmay faqat ε gagina bog'liq qilib tanlab olinishi mumkin bo'lgan hol muhimdir.

3-ta'rif. R to'plamda berilgan *f(x,y)* funksiyaning $y\to y'$ dagi limit funksiyasi $\varphi(x)$ bo'lsin. $\forall\varepsilon>0$ olinganda ham shunday $\delta=\delta(\varepsilon)>0$ topilsaki, $|y-y'|<\delta$ tengsizlikni qanoatlantiruvchi $\forall y\in E$ va $\forall x\in[a,b]$ uchun

$$|f(x,y)-\varphi(x)|<\varepsilon$$

bo'lsa, *f(x,y)* funksiya o'z limit funksiyasi $\varphi(x)$ ga [a,b] da tekis yaqinlashadi deyiladi va quyidagicha belgilandi: $f(x,y)\overrightarrow{\rightarrow}\varphi(x),\ x\in[a,b]$.

4-ta'rif. R to'plamda berilgan *f(x,y)* funksiyaning $y\to y'$ dagi limit funksiyasi $\varphi(x)$ bo'lsin. $\forall\delta>0$ olinganda ham shunday $\varepsilon_0>0,\ x_0\in[a,b]$ va $|y-y'|<\delta$ tengsizlikni qanoatlantiruvchi $y_1\in E$ topilsaki, ushbu $|f(x_0,y_1)-\varphi(x)|\ge\varepsilon_0$ tengsizlik o'rinli bo'lsa, u holda *f(x,y)* funksiya $\varphi(x)$ ga [a,b] notekis yaqinlashadi deyiladi va quyidagicha belgilanadi $f(x,y)\overrightarrow{\rightarrow}\varphi(x),\ x\in[a,b]$.

3-misol. Ushbu *f(x,y)=xsiny* funksiyaning

$$P=\{(x,y)\in R^2:\ 0\le x\le 1,\ 0<y<\pi\}$$

to'plamda limit funksiyasini toping va uni tekis yaqinlashishga tekshiring.

Yechish. Funksiyani $P=\{(x,y)\in R^2:\ 0\le x\le 1,\ 0<y<\pi\}$ to'plamda qaraylik, $y'=\frac{\pi}{4}$ bo'lsin. Ravshanki, $y\to y'=\frac{\pi}{4}$ bo'lganda *f(x,y)=x·siny* funksiyaning limiti $\frac{\sqrt{2}}{2}x$ ga teng bo'ladi. Demak, $\varphi(x)=\frac{\sqrt{2}}{2}x$. $\forall\varepsilon>0$ sonni olaylik. Agar $\delta=\varepsilon$ desak, u holda $|y-y'|=\left|y-\frac{\pi}{4}\right|<\delta$ ni qanoatlantiruvchi $\forall y$ uchun va $\forall x\in[0,1]$ uchun

$$|f(x,y)-\varphi(x)|=\left|x\sin y-\frac{\sqrt{2}}{2}x\right|=|x|\left|\sin y-\frac{\sqrt{2}}{2}\right|=|x|\left|\sin y-\sin\frac{\pi}{4}\right|<\left|y-\frac{\pi}{4}\right|<\varepsilon$$

tengsizlik bajariladi. 3-ta’rifga ko’ra, $y \to \frac{\pi}{4}$ da berilgan *f(x,y)=xsiny* funksiya o’z limit funksiyasi $\varphi(x) = \frac{\sqrt{2}}{2}x$ ga [0,1] da tekis yaqinlashadi.

Yuqorida keltirilgan 2-misolda $f(x,y) = \frac{xy}{1+x^2y^2}$ funksiya $y \to \infty$ da limit funksiya $\varphi(x) = 0$ ga notekis yaqinlashadi.

Haqiqatdan ham, $\forall \Delta > 0$ sonni olaylik. Agar $\varepsilon_0 = \frac{1}{4}$, y_1 sifatida $|y_1| > \Delta$ tengsizlikni qanoatlantiruvchi ixtiyoriy y_1 ni va $x_0 = \frac{1}{y_1}$ deb olsak, u holda

$$|f(x,y) - \varphi(x)| = \frac{y_1 \cdot \frac{1}{y_1}}{1 + y_1^2 \frac{1}{y_1^2}} = \frac{1}{2} > \varepsilon_0 = \frac{1}{4}$$

bo’lib, bu 4-ta’rifga ko’ra $y \to \infty$ da $f(x,y) = \frac{xy}{1+x^2y^2}$ funksiya o’z limit funksiyasi $\varphi(x) = 0$ ga [0,1] da notekis yaqinlashadi.

4-misol. Ushbu

$$f(x,y) = \frac{\cos\sqrt{nx}}{\sqrt{n+2x}}, \quad x \in [0,+\infty), \ n \in N, \ n_0 = \infty$$

funksiyaning berilgan to’plamda $\left(M = \{ (x,y) \in R^2; \ \ x \in [0,+\infty), \ \ y \in N \subset R\}\right)$
limit funksiyasini toping va uni tekis yaqinlashishga tekshiring.

Yechish. $f(x) = \lim\limits_{n\to\infty} f_n(x) = \lim\limits_{n\to\infty} \frac{\cos\sqrt{nx}}{\sqrt{n+2x}} = \lim\limits_{n\to\infty} \frac{\frac{\cos\sqrt{nx}}{\sqrt{n}}}{\sqrt{1+\frac{2x}{n}}} = 0$

bo’lib, u $[0, +\infty)$ da yaqinlashuvchi bo’ladi.

Endi yaqinlashish xarakterini aniqlaymiz. $\forall \varepsilon > 0$ son olinganda ham, $n_0 = \left[\frac{1}{\varepsilon}\right]$ deyilsa, unda barcha *n>n₀* va $\forall x \in [0,+\infty)$ uchun

$$|f(x,y) - f(x)| = \left|\frac{\cos\sqrt{nx}}{\sqrt{n+2x}} - 0\right| = \left|\frac{\cos\sqrt{nx}}{\sqrt{n+2x}}\right| \le \frac{1}{\sqrt{n+2x}} < \frac{1}{n} \le \frac{1}{n_0+1} < \varepsilon$$

(Yuqorida aytilganlardan ko’rinadiki, n_0 natural son faqat ε gagina bog’liq: $n_0 = \left[\frac{1}{\varepsilon}\right]$).

$$f(x,y) = \frac{\cos\sqrt{nx}}{\sqrt{n+2x}}$$

limit funksiya *f(x)=0* ga tekis yaqinlashadi: $\frac{\cos\sqrt{nx}}{\sqrt{n+2x}} \rightrightarrows 0, \ x \in [0,+\infty)$.

5-misol. Ushbu $f(x,y)=\sin\frac{x}{y}$ funksiya

$$P=\{(x,y)\in R^2;\ x\in R;\ 0<y<+\infty\}$$

to'plamda berilgan bo'lsin. $y\to+\infty$ da limit funksiyasini toping va yaqinlashish xarakterini tekshiring.

Yechish. $f(x)=\lim\limits_{y\to\infty} f(x,y)=\lim\limits_{y\to+\infty}\sin\frac{x}{y}=0.$ Agar $\forall\varepsilon>0$ ga ko'ra $\Delta=\frac{|x|}{\varepsilon}$ deb olsak, unda $|y|>\Delta$ tengsizlikni qanoatlantiruvchi $\forall y\in(0,+\infty)$ uchun

$$|f(x,y)-f(x)|=\left|\sin\frac{x}{y}-0\right|=\left|\sin\frac{x}{y}\right|\le\left|\frac{x}{y}\right|<\varepsilon$$

bo'ladi. Bu yerda $\Delta=\frac{|x|}{\varepsilon}$ faqatgina ε ga bog'liq bo'lmay x ga ham bog'liqdir. Demak, qaralayotgan funksiya o'z limit funksiyasiga 4-ta'rifga ko'ra notekis yaqinlashadi.

Endi *f(x,y)* funksiyaning limit funksiyaga ega bo'lishi va unga tekis yaqinlashishi haqidagi teoremani keltiramiz.

f(x,y) funksiya $P=\{(x,y)\in R^2;\ a\le x\le b,\ y\in E\}$ to'plamda berilgan bo'lib, y_0 esa $E(E\subset R)$ to'plamning limit nuqtasi bo'lsin.

1-teorema. *f(x,y)* funksiya $y\to y_0$ da limit funksiya *f(x)* ga ega bo'lib, unga tekis yaqinlashishi uchun, $\forall\varepsilon>0$ olinganda ham $x(x\in[a,b])$ ga bog'liq bo'lmagan shunday $\delta=\delta(\varepsilon)>0$ topilib, $|y-y_0|<\delta,\ |y'-y_0|<\delta$ tengsizliklarni qanoatlantiruvchi $\forall y,\ y'\in E$ hamda $\forall x\in[a,b]$ uchun $|f(x,y)-f(x,y')|<\varepsilon$ tengsizlikning bajarilishi zarur va yetarlidir.

§4.3. Parametrga bog'liq integrallarning funksional xossalari

f(x,y) funksiya $P=\{(x,y)\in R^2;\ x\in[a,b],\ y\in E\subset R\}$ to'plamda berilgan bo'lib, y_0 nuqta E to'plamning limit nuqtasi bo'lsin.

2 -teorema. (Integral belgisi ostida limitga o'tish)

f(x,y) funksiya y ning E to'plamdan olingan har bir tayin qiymatida x ning funksiyasi sifatida [a,b] oraliqda uzluksiz bo'lsin. Agar *f(x,y)* funksiya $y\to y_0$ da $\varphi(x)$ limit funksiyaga ega bo'lsa va unga tekis yaqinlashsa, u holda

$$\lim_{y\to y_0}\int_a^b f(x,y)dx=\int_a^b\varphi(x)dx \qquad (2)$$

bo'ladi.

6-misol. Ushbu $P=\{(x,y)\in R^2;\ 0\le x\le 1,\ 0<y<\pi\}$ to'plamda berilgan *f(x,y)=xsiny* funksiyaning $y\to\frac{\pi}{4}$ da $\varphi(x)=\frac{\sqrt{2}}{2}x$ limit funksiyaga tekis yaqinlashishini ko'rgan edik:

$$\lim_{y\to\frac{\pi}{4}} x\sin y=\frac{\sqrt{2}}{2}x.$$

Berilgan funksiya *y* o'zgaruvchining har bir tayin qiymatida *x* o'zgaruvchining [0,1] oraliqdagi uzluksiz funksiyasi ekanligi ravshan. Demak, 2-teoremaga ko'ra

$$\lim_{y\to\frac{\pi}{4}}\int_0^1 x\sin y\,dx=\int_0^1\left[\lim_{y\to\frac{\pi}{4}} x\sin y\right]dx=\int_0^1\frac{\sqrt{2}}{2}x\,dx=\frac{\sqrt{2}}{2}\cdot\frac{x^2}{2}\Bigg|_0^1=\frac{\sqrt{2}}{4}=\frac{1}{2\sqrt{2}}$$

bo'ladi.

7-misol. Ushbu $\lim_{n\to\infty}\int_0^1\frac{dx}{1+\left(1+\frac{x}{n}\right)^n}$ ni toping.

Yechish. Integral ostidagi $f(x,y)=\frac{1}{1+\left(1+\frac{x}{y}\right)^y}$ funksiya

$$P=\{(x,y)\in R^2;\ x\in R;\ 0<y<+\infty\}$$

to'plamda uzluksiz ekanligi ravshandir.

$$\varphi(x)=\lim_{y\to+\infty} f(x,y)=\lim_{y\to+\infty}\frac{1}{1+\left(1+\frac{x}{y}\right)^y}=\frac{1}{1+e^x},$$

$$|f(x,y)-\varphi(x)|=\left|\frac{1}{1+\left(1+\frac{x}{y}\right)^y}-\frac{1}{1+e^x}\right|=\frac{\left|e^x-\left(1+\frac{x}{y}\right)^y\right|}{\left[1+\left(1+\frac{x}{y}\right)^y\right](1+e^x)}.$$

Agar $a=e^{ha}$ va *x>0* larda *ln(1+x)<x* ekanligini e'tiborga olsak, u holda

$$|f(x,y)-\varphi(x)|<\frac{\left|e^x-e^{y\left(\frac{x}{y}-\frac{x^2}{2y^2}\right)}\right|}{4}=\frac{\left|e^x-e^{x-\frac{x^2}{2y}}\right|}{4}=\frac{e^x\left(1-e^{-\frac{x^2}{2y}}\right)}{4}<\frac{e}{4}\left(1-e^{-\frac{1}{2y}}\right)$$

$\frac{e}{4}\left(1-e^{-\frac{1}{2y}}\right)<\varepsilon$ tengsizlikni yechib topamiz:

$$y>\frac{1}{2\ln\left(1-\frac{4\varepsilon}{e}\right)}.$$

Agar $\Delta=\dfrac{1}{2\ln\left(1-\frac{4\varepsilon}{e}\right)}$ desak, u holda $\forall\varepsilon>0$ ga ko'ra $\Delta=\dfrac{1}{2\ln\left(1-\frac{4\varepsilon}{e}\right)}$, $y>\Delta$ tengsizlikni qanoatlantiruvchi $\forall y$ lar uchun va $\forall x\in[0,1]$ uchun

$$|f(x,y)-\varphi(x)|=\left|\frac{1}{1+\left(1+\frac{x}{y}\right)^y}-\frac{1}{1+e^x}\right|<\varepsilon$$

tengsizlik o'rinli bo'ladi. Bu esa ta'rifga ko'ra berilgan *f(x,y)* funksiyaning $y\to+\infty$ da limit funksiya $\varphi(x)$ ga tekis yaqinlashishini bildiradi. Demak, 2-teoremaga ko'ra integral belgisi ostida limitga o'tish mumkin, ya'ni

$$\lim_{n\to\infty}\int_0^1\frac{dx}{1+\left(1+\frac{x}{n}\right)^n}=\int_0^1\lim_{n\to\infty}\left(\frac{1}{\left(1+\frac{x}{n}\right)^n}\right)dx=\int_0^1\frac{dx}{1+e^x}=\int_0^1\frac{e^{-x}dx}{1+e^{-x}}=-\int_0^1\frac{d\left(e^{-x}+1\right)}{e^{-x}+1}=$$

$$=-\ln\left|e^{-x}+1\right|\Big|_0^1=-\ln\left|e^{-1}+1\right|+\ln\left|e^{-0}+1\right|=-\ln\left|\frac{1}{e}+1\right|+\ln 2=\ln\frac{2}{\left(1+\frac{1}{e}\right)}.$$

3-teorema. (Integralning parametr bo'yicha uzluksizligi) Agar *f(x,y)* funksiya

$$P=\{(x,y)\in R^2;\ x\in[a,b],\ \ y\in[c,d]\}$$

to'plamda uzluksiz bo'lsa, u holda

$$I(y)=\int_a^b f(x,y)dx$$

funksiya [c,d] oraliqda uzluksiz bo'ladi.

8-misol. Ushbu $f(x,y)=\dfrac{x}{x^2+y^2+1}$ funksiya

$$P=\{(x,y)\in R^2;\ x\in[0,1],\ \ y\in[0,1]\}$$

to'plamda qaralayotgan bo'lsin. Ravshanki, *f(x,y)* funksiya *P* to'plamda uzluksizdir. 3-teoremaga ko'ra *I(y)* funksiya ham, ya'ni $I(y)=\int_0^1\dfrac{xdx}{x^2+y^2+1}$ funksiya [0,1] da uzluksiz bo'ladi.

$$I(y)=\int_0^1\frac{xdx}{x^2+y^2+1}=\frac{1}{2}\ln\left(1+x^2+y^2\right)\Big|_0^1=\frac{1}{2}\ln\frac{2+y^2}{1+y^2}.$$

4-teorema.(Integralni parametr bo'yicha differensiallash)

f(x,y) funksiya $P=\{(x,y)\in R^2;\ x\in[a,b],\ \ y\in[c,d]\}$ to'plamda berilgan va *y* o'zgaruvchining [c,d] oraliqdan olingan har bir tayin qiymatida *x* o'zgaruvchining funksiyasi sifatida [a,b] oraliqda uzluksiz bo'lsin. Agar *f(x,y)* funksiya *P* to'plamda $f'_y(f,y)$ xususiy hosilaga ega bo'lib, u *P* da uzluksiz bo'lsa, u holda *I(y)* funksiya ham [c,d] oraliqda $I'(y)$ hosilaga ega va ushbu

$$I'(y)=\int_a^b f'_y(x,y)dx \qquad (3)$$

munosabat o'rinlidir.

(3) munosabatni quyidagicha ham yozish mumkin:

$$\frac{d}{dy}\int_a^b f(x,y)dx=\int_a^b \frac{d}{dy}f(x,y)dx.$$

Bu esa differensiallash amalini integral belgisi ostiga o'tkazish mumkinligini ko'rsatadi.

9-misol. Ushbu *f(x,y)=ln(y²sin²x)* funksiya $P=\left\{(x,y)\in R^2;\ x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right],\ 0<y_0\le y\le y_1<+\infty\right\}$ to'plamda uzluksiz hamda $f'_y(x,y)=\frac{2}{y}$ hosilaga ega va u ham uzluksiz. Undan olingan $I(y)=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\ln\left(y^2\cdot\sin^2 x\right)dx$ integralni qaraylik.

4-teoremaga ko'ra, $I(y)$ funksiya hosilaga ega bo'lib,

$$I'(y)=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\ln\left(y^2\sin^2 x\right)\right)'_y dx=\left.\frac{2}{y}\right|_{\frac{\pi}{4}}^{\frac{\pi}{2}}=\frac{\pi}{2y} \quad \text{bo'ladi.}$$

10-misol. Ushbu

$$I(a)=\int_0^{\frac{\pi}{2}}\ln\frac{1+a\cos x}{1-a\cos x}\cdot\frac{dx}{\cos x}$$

integralni hisoblang.

Yechish. $f(x,a)=\begin{cases}\frac{1}{\cos x}\ln\frac{1+a\cos x}{1-a\cos x}, & агар\ x\ne\frac{\pi}{2}\ булса\\ 2a, & агар\ \ x=\frac{\pi}{2}\ \ булса\end{cases}$

funksiya hamda

$$f'_a(x,a)=\frac{2}{1-a^2\cos^2 x}$$

funksiyalar $P=\left\{(x,a)\in R^2;\ |a|\le 1-\varepsilon<1,\ \ 0\le x\le\frac{\pi}{2}\right\}$ to'plamda uzluksizdir, shuning uchun (3) formulani qo'llash mumkin. Natijada

$$I'(a)=2\int_0^{\frac{\pi}{2}}\frac{dx}{1-a^2\cos^2 x}=2\int_0^{+\infty}\frac{dt}{1-a^2+t^2}=\frac{\pi}{\sqrt{1-a^2}}$$

Bundan $I'(a)=\frac{\pi}{\sqrt{1-a^2}}$ ifodani integrallab topamiz, $I(a)=\pi\arcsin a+c$.

I(0)=0 bo'lgani uchun $c=0$. Shuning uchun esa $I(a)=\pi\arcsin a$.

f(x,y) funksiya $P=\{(x,y)\in R^2;\ x\in[a,b],\ \ y\in[c,d]\}$ to'plamda berilgan va shu to'plamda uzluksiz bo'lsin. U holda 3-teoremaga ko'ra

$$I(y)=\int_a^b f(x,y)dx$$

funksiya [c,d] oraliqda uzluksiz bo'ladi. Binobarin, bu funksiyaning [c,d] oraliq bo'yicha integrali mavjud.

5-teorema.(Integralni parametr bo'yicha integrallash)

Agar *f(x,y)* funksiya $P=\{(x,y)\in R^2;\ x\in[a,b],\ \ y\in[c,d]\}$ to'plamda uzluksiz bo'lsa, u holda

$$\int_c^d I(y)dy=\int_c^d\left[\int_a^b f(x,y)dx\right]dy=\int_a^b\left[\int_c^d f(x,y)dy\right]dx$$

bo'ladi.

11-misol. Ushbu

$$I=\int_0^1\frac{x^b-x^a}{\ln x}dx \quad (0<a<b)$$

parametrga bog'liq integralni parametr bo'yicha integrallash haqidagi formuladan foydalanib hisoblang.

Yechish. Ravshanki, *(x>0)*

$$\frac{x^b-x^a}{\ln x}=\int_a^b x^y dy$$

bo'ladi. Demak, $I=\int_0^1 dx\int_0^b x^y dy$. *f(x,y)=x^y* funksiya $P=\{(x,y)\in R^2;\ [0,1],\ \ y\in[a,b]\}$ to'rtburchakda uzluksizdir. U holda 5-teoremaga ko'ra,

$$I=\int_a^b dy\int_0^1 x^y dx=\int_a^b\frac{dy}{y+1}=\ln\frac{b+1}{a+1}.$$

§4.4. Parametrga bog'liq integrallar (umumiy hol)

f(x,y) funksiya $P=\{(x,y)\in R^2;\ x\in[a,b],\ \ y\in[c,d]\}$ to'plamda berilgan, *y* o'zgaruvchining [c,d] oraliqdan olingan har bir tayin qiymatida *f(x,y)* funksiya *x* o'zgaruvchining funksiyasi sifatida [a,b] oraliqda integrallanuvchi bo'lsin.

$x=\alpha(y),\quad x=\beta(y)$ funksiyalarning har biri [c,d] da berilgan va $\forall y\in[c,d]$ uchun

$$a\le\alpha(y)\le\beta(y)\le b \qquad (5)$$

bo’lsin. U holda

$$I(y)=\int_{\alpha(y)}^{\beta(y)} f(x,y)dx \qquad (6)$$

integralning mavjudligi va u parametr *y* ga bog’liqligi ravshandir. Bu (6) integral (1) o’rganilgan integralga qaraganda umumiyroqdir. Haqiqatan ham, (6) da $\alpha(y)=a,\ \beta(y)=b,\ (y\in[c,d])$ bo’lganda bu integral (1) ko’rinishdagi integralga keladi. Endi (6) ko’rinishdagi integralning xossalarini keltiramiz:

6-teorema. *f(x,y)* funksiya $P=\{(x,y)\in R^2;\ x\in[a,b],\ y\in[c,d]\}$ to’plamda uzluksiz va (5) shartni qanoatlantirsin. U holda $I(y)=\int_{\alpha(y)}^{\beta(y)} f(x,y)dx$ funksiya ham *[c,d]* oraliqda uzluksiz bo’ladi.

7-teorema. *f(x,y)* funksiya $P=\{(x,y)\in R^2:\ x\in[a,b],\ y\in[c,d]\}$ to’plamda uzluksiz, $f'_y(x,y)$ xususiy hosilaga ega va *P* da uzluksiz, $\alpha(y),\ \beta(y)$ funksiyalar $\alpha'(y),\ \beta'(y)$ hosilalarga ega va ular (5) shartni qanoatlantirsin. U holda $I(y)$ funksiya ham [c,d] oraliqda hosilaga ega va

$$I(y)=\int_{\alpha(y)}^{\beta(y)} f'_y(x,y)dx+\beta'(y)f(\beta(y),y)-\alpha'(y)f(\alpha(y),y)$$

munosabat o’rinlidir.

7-teorema shartlari bajarilganda $I(y)$ funksiyaning [c,d] oraliqda integrallanuvchi ekanligini kelib chiqadi.

12-misol. Ushbu $\bar{I}(\alpha)=\int_{a+\alpha}^{b+\alpha}\frac{\sin\alpha x}{x}dx$ integral uchun $\bar{I}'(\alpha)$ ni toping.

Yechish. Yuqorida keltirilgan 7-teorema shartlarining bajarilishini tekshiramiz: $f(x,\alpha)=\frac{\sin\alpha x}{x}$ funksiya $x\ne0$ da uzluksiz, $f'(x,\alpha)=\cos\alpha x$ esa P da uzluksiz bo’ladi.

$$(a+\alpha)'=(b+\alpha')=1$$

(7) formulani qo’llab quyidagini topamiz:

$$\bar{I}'(\alpha)=\int_{a+\alpha}^{b+\alpha}\cos\alpha x dx+\frac{\sin\alpha(b+\alpha)}{b+\alpha}-\frac{\sin\alpha(a+\alpha)}{a+\alpha}=$$

$$=\frac{\sin\alpha(b+\alpha)}{\alpha}-\frac{\sin\alpha(a+\alpha)}{\alpha}+\frac{\sin\alpha(b+\alpha)}{b+\alpha}-\frac{\sin\alpha(a+\alpha)}{a+\alpha}=$$

$$=\left(\frac{1}{\alpha}+\frac{1}{b+\alpha}\right)\sin\alpha(b+\alpha)-\left(\frac{1}{\alpha}+\frac{1}{a+\alpha}\right)\sin\alpha(a+\alpha).$$

4.1. uchun misollar.

Quyidagi funksiyalarning berilgan to'plamda limit funksiyalarini toping:

1. $f(x,y)=x\sin y,\qquad P=\{(x,y)\in R^2:\ 0\le x\le 1,\ \ y\in R\},\quad y_0=\frac{\pi}{2}$

2. $f(x,y)=xy,\quad P=\{(x,y)\in R^2:\ x\in[0,1],\ \ y\in[0,1]\},\quad y_0=1$

3. $f(x,y)=x^y,\quad P=\{(x,y)\in R^2:\ x\in[0,1],\ \ y\in[0,1]\},\quad y_0=1$

4. $f(x,y)=x^3\cos\frac{1}{xy},\quad P=\{(x,y)\in R^2:\ x\in[0,+\infty),\ \ y\in[0,\infty)\},\quad y=+\infty$

5. $f(x,y)=\sqrt[3]{x^3+\frac{1}{\sqrt{y}}},\quad P=\{(x,y)\in R^2:\ x\in R,\ \ 0<y<+\infty\},\quad y_0=+\infty$

6. $f(x,y)=x^3\sin y,\qquad P=\{(x,y)\in R^2:\ x\in R,\ \ 0<y<\pi\},\quad y_0=\frac{\pi}{4}$

7. $f(x,n)=n\,arctgnx,\quad P=\{(x,y)\in R^2:\ 0\le x<+\infty,\ \ n\in N\},\quad n_0=\infty$

8. $f(x,n)=x^4\cos\frac{1}{nx}\qquad P=\{(x,n)\in R^2:\ 0\le x<+\infty,\ \ n\in N\},\quad n_0=\infty$

9. $f(x,n)=\frac{nx^2}{x+3n+2}\qquad P=\{(x,n)\in R^2:\ 0\le x<+\infty,\ \ n\in N\},\quad n_0=\infty$

10. $f(x,n)=e^{-(x-n)^2}\qquad P=\{(x,y)\in R^2:\ -1\le x\le 1,\ \ n\in N\},\quad n_0=\infty$

11. $f(x,n)=\frac{nx}{1+n^3x^2},\qquad P=\{(x,n)\in R^2:\ 1\le x<+\infty,\ \ n\in N\},\quad n_0=\infty$

12. $f(x,y)=(x-1)\,arctgx^y;\qquad P\{(x,y)\in R^2;\quad x\in(0,+\infty),\quad y\in(0,+\infty)\},\quad y_0=+\infty$

13. $f(x,n)=n\left(x^{\frac{1}{n}}-1\right),\quad P=\{(x,y)\in R^2:\ x\in[1,3],\ \ n\in N\},\quad n_0=\infty$

14. $f(x,n)=n\left(x^{\frac{1}{n}}-x^{\frac{1}{2n}}\right),\ P=\{(x,y)\in R^2:\ x\in(0,+\infty),\ \ n\in N\},\quad n_0=+\infty$

15. $f(x,n)=\sqrt{n}\sin\frac{x}{n\sqrt{n}},\qquad P=\{(x,n)\in R^2:\ x\in R,\ \ n\in N\},\quad n_0=+\infty$

16. $f(x,n)=\sqrt[n]{1+x^n},\qquad P=\{(x,n)\in R^2:\ x\in[0,2],\ \ n\in N\},\quad n_0=+\infty$

17. $f(x,n)=\frac{\cos\sqrt{nx}}{\sqrt{n+2x}},\qquad P=\{(x,n)\in R^2:\ x\in[0,+\infty],\ \ n\in N\},\quad n_0=\infty$

18. $f(x,n)=x^n-x^{n+2},\qquad P=\{(x,n)\in R^2:\ x\in[0,1],\ \ n\in N\},\quad n_0=+\infty$

19. $f(x,n)=\frac{\ln nx}{nx^2},\qquad P=\{(x,n)\in R^2:\ x\in[1,+\infty],\ \ n\in N\},\quad n_0=+\infty$

20. $f(x,y)=e^{-yx^2},\qquad P=\{(x,y)\in R^2:\ 1\le x<+\infty,\ \ 0<y<+\infty\},\quad y_0=+\infty$

4.2. uchun misollar.

Quyidagi funksiyalarning berilgan to'plamda limit funksiyalarini toping va uni tekis yaqinlashishga tekshiring:

1. $f(x,y)=e^{-yx^2},\qquad P=\{(x,y)\in R^2:\ x\in[1,+\infty),\ \ y\in(0,+\infty)\},\quad y_0=+\infty$

2. $f(x,y)=\sqrt{y}\sin\frac{x}{y\sqrt{y}}$, $P=\{(x,y)\in R^2:\ x\in R^2,\ \ y\in(0,+\infty)\}$, $y_0=+\infty$

3. $f(x,y)=\frac{1}{x^3}\cos\frac{x}{y}$, $P=\{(x,y)\in R^2:\ 0<x<1,\ \ 0<y<+\infty\}$, $y_0=\infty$

4. $f(x,y)=\sin\frac{x}{y}$, $P=\{(x,y)\in R^2:\ x\in R,\ \ 0<y<+\infty\}$, $y_0=+\infty$

5.**Ошибка! Объект не может быть создан из кодов полей редактирования.**

6.**Ошибка! Объект не может быть создан из кодов полей редактирования.**

7.**Ошибка! Объект не может быть создан из кодов полей редактирования.**

8.**Ошибка! Объект не может быть создан из кодов полей редактирования.**

9. $f(x,n)=\sin^n x$, $P=\left\{(x,n)\in R^2:\ x\in\left(0,\frac{\pi}{2}\right),\ \ n\in N\right\}$, $n_0=+\infty$

10. $f(x,n)=\frac{n}{x}\ln\left(1+\frac{x}{n}\right)$, $P=\{(x,n)\in R^2:\ x\in(0,10),\ \ n\in N\}$, $n_0=+\infty$

11. $f(x,n)=x^n+x^{2n}-2x^{3n}$, $P=\{(x,n)\in R^2:\ x\in[0,1],\ \ n\in N\}$, $n_0=+\infty$

12. $f(x,n)=\sin\frac{(1+nx)}{2n}$, $P=\{(x,n)\in R^2:\ x\in[1,+\infty],\ \ n\in N\}$, $n_0=+\infty$

13. $f(x,n)=\sin\frac{x}{n^2}$, $P=\{(x,n)\in R^2:\ x\in R,\ \ n\in N\}$, $n_0=+\infty$

14. $f(x,n)=\frac{4n\sqrt{nx}}{3+4n^2x}$, $P=\{(x,n)\in R^2:\ x\in[\delta,+\infty],\ \ R\in N\}$, $\delta>0$, $n_0=+\infty$

15. $f(x,n)=\frac{\ln nx}{nx^2}$, $P=\{(x,n)\in R^2:\ x\in[1,+\infty],\ \ n\in N\}$, $n_0=+\infty$

16. $f(x,n)=x^{2n}$, $P=\{(x,n)\in R^2:\ 0\le x\le\delta,\ \ 0<\delta<1,\ \ n\in N\}$, $n_0=+\infty$

17. $f(x,n)=\frac{n^2}{n^2+x^2}$, $P=\{(x,n)\in R^2:\ x\in[-1,1],\ \ n\in N\}$, $n_0=+\infty$

18. $f(x,n)=\frac{arctgnx}{\sqrt{n+x}}$, $P=\{(x,n)\in R^2:\ x\in[0,+\infty),\ \ n\in N\}$, $n_0=+\infty$

19. $f(x,n)=\sqrt{x+\frac{1}{n}}-\sqrt{x}$, $P=\{(x,n)\in R^2:\ x\in[0,+\infty),\ \ n\in N\}$, $n_0=+\infty$

20. $f(x,n)=n\sin\left(\frac{1}{nx}\right)$, $P=\{(x,n)\in R^2:\ x\in[1,+\infty),\ \ n\in N\}$, $n_0=+\infty$

4.3, 4.4 uchun misollar.

1.Ushbu $I(y)=\int_0^1\frac{yf(x)}{x^2+y^2}dx$, $f(x)\in C[0,1]$, $f(x)\ge 0$ funksiyani uzluksizlikka tekshiring.

2.Ushbu $I(y)=\int_0^1 \frac{xdx}{x^2+y^2+1}$ funksiyani uzluksizlikka tekshiring. Integral ostidagi $f(x,y)=\frac{x}{x^2+y^2+1}$ funksiya $P=\{(x,y)\in R^2:\ x\in[0,1],\ y\in[0,1]\}$ to'plamda berilgan.

Quyidagi integrallarni hisoblang:

3. $\lim\limits_{\alpha\to 0}\int\limits_{\alpha}^{1+\alpha}\frac{dx}{1+x^2+\alpha^2}$;

4. $\lim\limits_{\alpha\to 0}\int\limits_{-1}^{1}\sqrt{x^2+\alpha}\,dx$;

5. $\lim\limits_{\alpha\to 0}\int\limits_{0}^{2}x^2\cos\alpha x dx$;

6. $\lim\limits_{\alpha\to 0}\int\limits_{\alpha}^{1+\alpha}\frac{dx}{1+x^2+\alpha^2}$;

7. $\lim\limits_{R\to\infty}\int\limits_{0}^{\frac{\pi}{2}}e^{-R\sin\theta}d\theta$.

Quyidagi funksiyalarning hosilalarini toping:

8. $I(\alpha)=\int\limits_{0}^{\alpha}\frac{\ln(1+\alpha x)}{x}dx$;

9. $I(\alpha)=\int\limits_{\sin\alpha}^{\cos\alpha}e^{\alpha\sqrt{1-x^2}}dx$;

10. $I(\alpha)=\int\limits_{a+\alpha}^{b+\alpha}\frac{\sin\alpha x}{x}dx$;

11. $I(\alpha)=\int\limits_{0}^{a+\alpha}f(x+\alpha,x-\alpha)dx$;

12. $I(\alpha)=\int\limits_{0}^{\alpha^2}dx\int\limits_{x-\alpha}^{x+\alpha}\sin(x^2+y^2-\alpha^2)dy$;

13. $I(x)=\int\limits_{x}^{x^2}e^{-x^2y}dy$;

14. $F(x)=\int\limits_{0}^{x}(x+y)f(y)dy$, $f(x)$-differensiallanuvchi funksiya bo'lsa, $F''(x)$ ni toping.

Quyidagi integrallarni hisoblang:

15. $\int\limits_{0}^{1}\frac{x^b-x^\alpha}{\ln x}dx$, $(a>0,\ b>0)$;

16. $\int\limits_{0}^{1}\frac{arctgx}{x}\cdot\frac{dx}{\sqrt{1-x^2}}$;

Ko'rsatma: $\frac{arctgx}{x}=\int\limits_{0}^{1}\frac{dx}{1+x^2y^2}$ munosabatdan foydalaning.

17. $\int\limits_{0}^{\frac{\pi}{2}}\ln(a^2\sin^2 x+b^2\cos^2 x)dx$;

18. $\int\limits_{0}^{\pi}\ln(1-2a\cos x+a^2)dx$;

19. $\int\limits_{0}^{\frac{\pi}{2}}\frac{arctg(atgx)}{tgx}dx$;

20. $\int\limits_{0}^{\frac{\pi}{2}}\ln\frac{1+a\cos x}{1-a\sin x}\cdot\frac{dx}{\cos x}$, $(|a|<1)$;

21. $\int\limits_{0}^{1}\sin\left(\ln\frac{1}{x}\right)\frac{x^b-x^\alpha}{\ln x}dx$, $a>0,\ b>0$.

§4.5. Parametrga bog'liq xosmas integral tushunchasi

f(x,y) funksiya $P=\{(x,y)\in R^2:\ x\in[a,+\infty),\ y\in E\subset R\}$ to'plamda berilgan bo'lib, y o'zgaruvchining E to'plamdan olingan har bir tayin qiymatida x o'zgaruvchining funksiyasi sifatida $[a,+\infty)$ oraliqda integrallanuvchi, ya'ni

$$\int_a^{+\infty} f(x,y)dx \quad (y\in E\subset R)$$

xosmas integral mavjud va chekli bo'lsin. Bu integral y o'zgaruvchining qiymatiga bog'liq bo'lib,

$$I(y)=\int_a^{+\infty} f(x,y)dx \qquad (1)$$

integral parametrga bog'liq chegarasi cheksiz xosmas integral deb aytiladi.

f(x,y) funksiya $P=\{(x,y)\in R^2:\ x\in[0,1],\ y\in[0,1]\}$ $P''=\{(x,y)\in R^2:\ x\in(-\infty,+\infty),\ y\in E\subset R\}$ to'plamda berilgan va y o'zgaruvchining E dan olingan har bir tayin qiymatida *f(x,y)* – x ning funksiyasi sifatida $(-\infty,a]$ $((-\infty,+\infty))$ da integrallanuvchi bo'lsin. Bunda

$$I^*(y)=\int_{-\infty}^{a} f(x,y)dx,\ \left(I^{**}(y)=\int_{-\infty}^{+\infty} f(x,y)dx\right) \qquad (2)$$

integral ham parametrga bog'liq chegarasi cheksiz xosmas integral deb ataladi.

f(x,y) funksiya $P=\{(x,y)\in R^2:\ x\in[a,b),\ y\in E\subset R\}$ to'plamda berilgan, *b*-maxsus nuqta bo'lib, E to'plamdan olingan *u* ning har bir tayin qiymatida [a,b) oraliqda integrallanuvchi, ya'ni

$$\int_a^{b} f(x,y)dx \quad (x\in E)$$

xosmas integral mavjud bo'lsin. Bu integral ham y ning qiymatiga bog'liq bo'lib,

$$I_1(y)=\int_a^{b} f(x,y)dx \qquad (3)$$

integral parametrga bog'liq chegaralanmagan funksiyaning xosmas integrali deb ataladi.

f(x,y) funksiya $P_1'=\{(x,y)\in R^2:\ x\in(a,b],\ y\in E\subset R\}$ to'plamda berilgan va y o'zgaruvchining E dan olingan har bir tayin qiymatida *f(x,y)* – x ning funksiyasi sifatida qaralganda ($x=a$ maxsus nuqta) bu funksiya (a,b] da integrallanuvchi bo'lsin. U holda

$$I_2(y)=\int_a^{b} f(x,y)dx \qquad (4)$$

integral ham paramaterga bog'liq chegaralanmagan funksiyaning xosmas integrali deb ataladi.

Umumiy holda, parametrga bog'liq chegaralanmagan funksiyaning chegarasi cheksiz xosmas integrali tushunchasi ham yuqoridagidek kiritiladi.

f(x,y) funksiya $P_2=\{(x,y)\in R^2 :\ x\in(c,+\infty),\ \ y\in E\subset R\}$ to'plamda berilgan, y o'zgaruvchining E to'plamdan olingan har bir tayin qiymatida *f(x,y)* ni x o'zgaruvchining funksiyasi sifatida qaralganda ($x=c$ maxsus nuqta) bu funksiya $(c,+\infty)$ oraliqda integrallanuvchi, ya'ni

$$\int_c^{+\infty} f(x,y)dx$$

cheagaralanmagan funksiyaning chegarasi cheksiz xosmas integrali mavjud bo'lsin. Bu integral y ning qiymatiga bog'liqdir:

$$I_3(y)=\int_c^{+\infty} f(x,y)dx \qquad (5)$$

integral parametrga bog'liq chegaralanmagan funksiyaning chegarasi cheksiz xosmas integrali deb ataladi.

Masalan, 1) $I(\alpha)=\int_a^{+\infty}\frac{dx}{x^\alpha}\quad (a>0,\ \alpha>0)$, 2) $\int_a^b\frac{dx}{(x-a)^\alpha}$,

3) $\int_a^b\frac{dx}{(x-b)^\alpha}\quad (\alpha>0)$, 4) $\Gamma(\alpha)=\int_0^{+\infty}x^{a-1}e^{-x}dx$

integrallar parametrga bog'liq xosmas integrallardir.

f(x,y) funksiya $P=\{(x,y)\in R^2\ :\ \ x\in[a,+\infty),\ \ \ y\in E\subset R\}$ to'plamda berilgan, y o'zgaruvchining E to'plamdan olingan har bir tayin qiymatida *f(x,y)* x o'zgaruvchining funksiyasi sifatida $[a,+\infty)$ da integrallanuvchi bo'lsin. U holda chegarasi cheksiz bo'lgan xosmas integral ta'rifiga ko'ra ixtiyoriy [a,t] da $(a<t<+\infty)$

$$F(t,y)=\int_a^t f(x,y)dx \qquad (6)$$

integral mavjud va

$$I(y)=\int_a^{+\infty} f(x,y)dx=\lim_{t\to+\infty}F(t,y) \qquad (7)$$

Demak, (6) va (7) integrallar bilan aniqlangan *F(t,y)* va *I(y)* funksiyalarga ega bo'lamiz va *I(y)* funksiya *F(t,y)* funksiyaning $t\to+\infty$ dagi limit funksiyasi bo'ladi.

5-ta'rif. Agar $t\to+\infty$ da *F(t,y)* funksiya o'z limit funksiyasi *I(y)* ga E to'plamda tekis yaqinlashsa, u holda

$$I(y)=\int_a^{+\infty} f(x,y)dx$$

integral E to'plamda tekis yaqinlashuvchi deb ataladi.

6-ta'rif. Agar $t\to+\infty$ da *F(t,y)* funksiya o'z limit funksiyasi *I(y)* ga E da notekis yaqinlashsa, u holda

$$I(y)=\int_a^{+\infty} f(x,y)dx$$

integral E to'plamda notekis yaqinlashuvchi deb ataladi.

(7) integralning E to'plamda tekis yaqinlashuvchi bo'lishi quyidagini anglatadi:

1) $\int_a^{+\infty} f(x,y)dx$ xosmas integral y o'zgaruvchining E to'plamdan olingan har bir tayin qiymatida yaqinlashuvchi;

2) $\forall\varepsilon>0$ olinganda ham, shunday $\delta=\delta(\varepsilon)>0$ topiladiki, $\forall t>\delta$ va $\forall y\in E$ uchun

$$\left|\int_t^{+\infty} f(x,y)dx\right|<\varepsilon$$

bo'ladi.

$\int_a^{+\infty} f(x,y)dx$ integral E to'plamda yaqinlashuvchi, ammo u shu to'plamda notekis yaknilashuvchi degani quyidagidan iboratdir:

1) $\int_a^{+\infty} f(x,y)dx$ xosmas integral y o'zgaruvchining E to'plamdan olingan har bir tayin qiymatida yaqinlashuvchi;

2) $\forall\delta>0$ olinganda ham, shunday $\varepsilon_0>0,\ y_0\in E$ va $t_1>\delta$ tengsizlikni qanoatlantiruvchi $t_1\in[a,+\infty)$ topiladiki,

$$\left|\int_{t_1}^{+\infty} f(x,y_0)dx\right|\geq\varepsilon_0$$

bo'ladi.

13-misol. Ushbu

$$I(y)=\int_0^{+\infty} ye^{-xy}dx,\quad y\in(0,+\infty) \tag{8}$$

integralning yaqinlashish xarakterini tekshiring.

Yechish. Avvalo $F(A,y)=\int_0^A ye^{-xy}dx\ \ (0\leq A<+\infty)$ integralni qaraymiz.

$$F(A,y)=\int_0^A ye^{-xy}dx=-e^{-xy}\Big|_0^A=1-e^{-Ay}\quad(0\leq A<+\infty)$$

y o'zgaruvchining $E=(0,+\infty)$ to'plamdan olingan har bir tayin qiymatida

$$\lim_{A\to+\infty}F(A,y)=\lim_{A\to+\infty}\left(1-e^{-Ay}\right)=1$$

bo'ladi. Demak, (8) xosmas integral ta'rifga ko'ra yaqinlashuvchi.

Endi (8) integralni tekis yaqinlashuvchilikka tekshiramiz:

$\left|\int_A^{+\infty} ye^{-xy}dx\right|=e^{-Ay}$ ekanini hisobga olgan holda $\forall\Delta>0$ deb olib $\varepsilon_0=\frac{1}{3},\ A_0>\Delta$ tengsizlikni qanoatlantiruvchi $\forall A_0$ uchun $y_0=\frac{1}{A_0}$ deb olsak, u holda $\left|\int_{A_0}^{+\infty} y_0e^{-xy_0}dx\right|=e^{-A_0y_0}=e^{-1}>\frac{1}{3}=\varepsilon_0$ bo'ladi.

Bu esa (8) xosmas integral $(0,+\infty)$ oraliqda notekis yaqinlashuvchi ekanini bildiradi. E to'plam sifatida $[c,+\infty)\subset(0,+\infty)$ bo'lsin, bunda c - ixtiyoriy musbat son. U holda $\forall\varepsilon>0$ olinganda ham $(0<\varepsilon<1)$ $\delta=\frac{1}{C}\ln\frac{1}{\varepsilon}$ deyilsa, $\forall A>\Delta$ va $\forall y\in[c,+\infty)$ uchun

$$\left|\int_A^{+\infty} ye^{-xy}dx\right| = e^{-Ay} < e^{-e\frac{1}{c}\ln\frac{1}{\varepsilon}} = \varepsilon$$

bo'ladi. Demak, (8) integral $[c,+\infty)\subset(0,+\infty)$ da tekis yaqinlashuvchi.

f(x,y) funksiya $P=\{(x,y)\in R^2:\ x\in[a,+\infty),\ \ y\in E\subset R\}$ to'plamda berilgan va

$$I(y)=\int_a^{+\infty} f(x,y)dx \qquad (9)$$

integral mavjud bo'lsin.

8-teorema. (Koshi teoremasi). (9) integral E to'plamda tekis yaqinlashuvchi bo'lishi uchun $\forall\varepsilon>0$ olinganda ham, shunday $\Delta=\Delta(\varepsilon)>0$ topilsaki, $A'>\Delta,\ A''>\Delta$ tengsizliklarni qanoatlantiruvchi $A',\ A''$ va $\forall y\in E$ uchun

$$\left|\int_{A'}^{A''} f(x,y)dx\right| < \varepsilon$$

tengsizlikning bajarilishi zarur va yetarlidir.

8-teoremadan misol va masalalar yechishda foydalanish murakkabroq bo'lgani sababli tekis yaqinlashishga tekshirish uchun qulayroq alomatlarini keltiramiz.

9-teorema. (Veyershtrass alomati) *f(x,y)* funksiya

$$P=\{(x,y)\in R^2:\ x\in[a,+\infty),\ \ y\in E\subset R\}$$

to'plamda berilgan, $I(y)=\int_a^{+\infty} f(x,y)dx$ integral mavjud bo'lsin. Agar shunday $\varphi(x)$ funksiya topilib $(x\in[a,+\infty))$, $\forall x\in[a,+\infty)$ va $\forall y\in E$ uchun $|f(x,y)|\le\varphi(x)$ bo'lsa, $\int_a^{+\infty}\varphi(x)dx$ xosmas integral yaqinlashuvchi bo'lsa, u holda

$$I(y)=\int_a^{+\infty} f(x,y)dx$$

integral E to'plamda tekis yaqinlashuvchi bo'ladi.

14-misol. Ushbu

$$\int_0^{+\infty}\frac{\cos xy}{1+x^2}dx,\quad y\in R$$

integralni tekis yaqinlashishga tekshiring.

Yechish. Agar $\left|\frac{\cos xy}{1+x^2}\right| \le \frac{1}{1+x^2}$ ekanini hisobga olsak va $\varphi(x)=\frac{1}{1+x^2}$ deyilsa, u holda $\int_0^{+\infty}\varphi(x)\,dx=\frac{\pi}{4}$, yaqinlashuvchi bo'lgani uchun Veyershtrass alomatiga ko'ra berilgan integral R da tekis yaqinlashuvchi bo'ladi.

10-teorema. (Abel alomati). *f(x,y)* va *g(x,y)* funksiyalar

$$P=\{(x,y)\in R^2:\ \ x\in[a,+\infty),\ \ y\in E\subset R\}$$

to'plamda berilgan, y o'zgaruvchining to'plamdan olingan har bir tayin qiymatida *g(x,y)* funksiya *x* ning funksiyasi sifatida *[a,+∞)* da monoton funksiya bo'lsin. Agar $\int_a^{+\infty} f(x,y)dx$ integral E to'plamda tekis yaqinlashuvchi va $\forall(x,y)\in P$ uchun $|g(x,y)|\le C\ \ (C=const)$ bo'lsa, u holda $\int_a^{+\infty} f(x,y)g(x,y)dx$ integral E da tekis yaqinlashuvchi bo'ladi.

15-misol. Ushbu $\int_a^{+\infty}\frac{\sin x}{x}e^{-xy}dx\ \ \ (y\in E=[0,\infty))$ integralni tekis yaqinlashishga tekshiring.

Yechish. Agar $f(x,y)=\frac{\sin x}{x},\ \ g(x,y)=e^{-xy}$ deb olinsa, Abel teoremasi shartlari bajariladi, ya'ni

$$|g(x,y)|=e^{-xy}\le 1,\ \ \forall y\in E=[0,+\infty)\quad \text{va}\quad \int_0^{+\infty}\frac{\sin x}{x}dx=\frac{\pi}{2}.$$

Demak, berilgan integral 10-teoremaga ko'ra *E=[0,+∞)* da tekis yaqinlashuvchi bo'ladi.

11-teorema. (Dirixle alomati). *f(x,y)* va *g(x,y)* funksiyalar

$$P=\{(x,y)\in R^2:\ \ x\in[a,+\infty),\ \ y\in E\subset R\}$$

to'plamda berilgan bo'lib, $\forall A\ge a$ hamda $\forall y\in E$ uchun

$$\left|\int_a^A f(x,y)dx\right|\le C\quad (C=const)$$

bo'lsa, va *g(x,y)* x bo'yicha monoton, $x\to+\infty$ da limit funksiyasi $\varphi(y)$ ga tekis yaqinlashsa, u holda

$$\int_a^{+\infty} f(x,y)g(x,y)dx$$

integral E da tekis yaqinlashuvchi bo'ladi.

16-misol. Ushbu

$$\int_0^{+\infty}\frac{\sin xy}{x}dx\quad (y\in E=[1,2])$$

integralni tekis yaqinlashishga tekshiring.

Yechish. Agar $f(x,y)=\sin xy,\quad \varphi(x,y)=\frac{1}{x}$ deyilsa, unda $\forall t>0$, $\forall y\in[1,2]$ uchun $\left|\int_0^t f(x,y)dx\right|=\left|\int_0^t \sin xydx\right|=\left|-\frac{\cos ty}{y}\right|\le 2$ bo'ladi. $x\to+\infty$ da $g(x,y)=\frac{1}{x}$ funksiya E da nolga tekis yaqinlashadi, ya'ni $g(x,y)=\frac{1}{x}\rightrightarrows 0$. Demak, berilgan integral Dirixle alomatiga ko'ra *E=[1,2]* da tekis yaqinlashuvchi bo'ladi.

4.5. uchun misollar.

Quyidagi integrallarni ko'rsatilgan oraliqlarda tekis yaqinlashishga tekshiring:

1. $\int_0^{+\infty}\frac{dx}{(x-a)^2+1},\quad (0\le a<+\infty)$;

2. $\int_0^{+\infty}e^{-ax}\sin xdx,\quad (0<a_0\le a<+\infty)$;

3. $\int_0^{+\infty}\frac{\sin x^2}{1+p^2}dx,\quad (p\ge 0)$;

4. $\int_0^{+\infty}\sqrt{\alpha}e^{-ax^2}dx,\quad (0\le a<+\infty)$;

5. $\int_0^{+\infty}\frac{\cos axdx}{1+x^2},\quad (-\infty<a<+\infty)$;

6. $\int_{-\infty}^{+\infty}e^{-(x-a)^2}dx,\quad a\in R$;

7. $\int_1^{+\infty}e^{-ax}\frac{\cos x}{x^p}dx,\quad (0\le a<+\infty)$, bu yerda *p>0* – tayinlangan

8. $\int_0^{+\infty}e^{-x^2(1+y^2)}\sin xydx,\quad (-\infty<y<+\infty)$;

9. $\int_1^{+\infty}\frac{\ln^p x}{x\sqrt{x}}dx,\quad (0\le p\le 10)$;

10. $\int_0^{+\infty}\frac{\sin x}{x}e^{-\alpha x}dx,\quad (0\le\alpha<+\infty)$;

11. $\int_1^{+\infty}x^{\alpha}e^{-x}dx,\quad (a\le\alpha\le b)$;

12. $\int_0^{+\infty}e^{-\alpha x}\cos\alpha xdx,\quad (0<\alpha_0\le\alpha<+\infty)$;

13. $\int_{-\infty}^{+\infty}e^{-(x-a)^2}dx,\quad (-l<a<l)$;

14. $\int_{-\infty}^{+\infty}e^{-(x-a)^2}dx,\quad (-\infty<a<\infty)$

15. $\int_0^{+\infty}\frac{\sin\alpha x\sin\beta xdx}{x},\quad (\alpha\in[a,b],0<a<b)$

16. $\int_0^{+\infty}\frac{arctgxy\cdot arctgxy^2}{1+x^2}e^{-xy}dx,\quad y\in[0,+\infty)$

17. $\int_1^{+\infty}x^{a-1}e^{-x}dx,\quad (0<a_0\le a\le b_0<+\infty)$;

18. $\int_1^{+\infty}\frac{x^{a-1}}{1+x}dx,\quad (0<a_0\le a\le b_0<1)$;

19. $\int_0^{+\infty}\frac{\sin xy}{1+x^2}dx,\quad (y\in E=(-\infty,+\infty))$;

20. $\int_0^{+\infty}ye^{-xy}dx,\quad (y\in[c,\infty],c>0)$.

§4.6. Parametrga bog'liq xosmas integrallarning funksional xossalari

f(x,y) funksiya $P=\{(x,y)\in R^2:\ \ x\in[0,+\infty),\ y\in E\subset R\}$ to'plamda berilgan y_0 E to'plamning limit nuqtasi bo'lsin.

11-teorema. *f(x,y)* funksiya 1) y o'zgaruvchining E dan olingan har bir tayin qiymatida x o'zgaruvchining funksiyasi sifatida $[a,+\infty)$ da uzluksiz; 2) $y\to y_0$ da $\forall[a,A]\ (a<A<+\infty)$ oraliqda $\varphi(x)$ limit funksiyaga tekis yaqinlashuvchi bo'lsin.

Agar $I(y)=\int_a^{+\infty} f(x,y)dx$ integral E da tekis yaqinlashuvchi bo'lsa, u holda $y\to y_0$ da *I(y)* funksiya limitga ega va $\lim\limits_{y\to y_0} I(y)=\lim\limits_{y\to y_0}\int_a^{+\infty} f(x,y)dx=\int_a^{+\infty}\varphi(x)dx$ munosabat o'rinli bo'ladi.

12-teorema. *f(x,y)* funksiya $P=\{(x,y)\in R^2:\ \ x\in[a,+\infty),\ y\in[c,d]\}$ to'plamda berilgan va uzluksiz bo'lib, $I(y)=\int_a^{+\infty} f(x,y)dx$ integral [c,d] oraliqda tekis yaqinlashuvchi bo'lsa, u holda *I(y)* [c,d] oraliqda uzluksiz bo'ladi.

13-teorema. *f(x,y)* funksiya $P=\{(x,y)\in R^2:\ \ x\in[a,+\infty),\ y\in[c,d]\}$ to'plamda uzluksiz $f_y'(x,y)$ xususiy hosilaga ega va u ham *R* da uzluksiz bo'lib, $y\in[c,d]$ da

$$I(y)=\int_a^{+\infty} f(x,y)dx$$

integral tekis yaqinlashuvchi bo'lsin. Agar $\int_a^{+\infty} f_y'(x,y)dx$ integral [c,d] da tekis yaqinlashuvchi bo'lsa, u holda *I(y)* funksiya ham [c,d] oraliqda $I'(y)$ hosilaga ega bo'ladi va

$$I'(y)=\int_\phi^{+\infty} f_y'(x,y)dx$$

munosabat o'rinli bo'ladi.

14-teorema. *f(x,y)* funksiya *R* to'plamda uzluksiz va $I(y)=\int_a^{+\infty} f(x,y)dx$ integral [c,d] oraliqda tekis yaqinlashuvchi bo'lsin. U holda *I(y)* funksiya [c,d] oraliqda integrallanuvchi va

$$\int_c^d I(y)dy=\int_c^d\left[\int_a^{+\infty} f(x,y)dx\right]dy=\int_a^{+\infty}\left[\int_c^d f(x,y)dy\right]dx$$

munosabat o'rinli bo'ladi.

f(x,y) funksiya $D=\{(x,y)\in R^2:\ x\in[a,+\infty),\ y\in[c,+\infty)\}$ to'plamda berilgan bo'lsin.

15-teorema. *f(x,y)* funksiya *D* to'plamda uzluksiz. $\int_a^{+\infty} f(x,y)dx,\ \int_c^{+\infty} f(x,y)dy$ integrallar mos ravishda $[c,+\infty),\ [a,+\infty)$ da tekis yaqinlashuvchi bo'lsin.

Agar $\int_c^{+\infty}\left[\int_a^{+\infty} f(x,y)dx\right]dy$ (yoki $\int_a^{+\infty}\left[\int_c^{+\infty} f(x,y)dy\right]dx$) integral yaqinlashuvchi bo'lsa, u holda

$$\int_a^{+\infty}\left[\int_c^{+\infty} f(x,y)dy\right]dx \ \text{(yoki } \int_c^{+\infty}\left[\int_a^{+\infty} f(x,y)dx\right]dy\text{)}$$

integrallar yaqinlashuvchi va o'zaro teng bo'ladi.

17-misol. Agar *f(x)* funksiya [0,+∞) oraliqda uzluksiz va chegaralangan bo'lsa, ushbu

$$\lim_{y\to 0}\frac{2}{\pi}\int_0^{+\infty}\frac{yf(x)}{x^2+y^2}dx=f(0)$$

munosabatni isbotlang.

Isbot. $x=zy$ almashtirish olaylik *(z>0, y>0)*, u holda

$$\frac{2}{\pi}\int_0^{+\infty}\frac{yf(x)}{x^2+y^2}dx=\frac{2}{\pi}\int_0^{+\infty}\frac{f(zy)}{1+z^2}dz,$$

$$\left|\frac{f(zy)}{1+z^2}\right|\le\frac{M}{1+z^2} \quad \text{va} \quad \int_0^{+\infty}\frac{M}{1+z^2}dz=\frac{\pi}{2}M$$

bo'lgani uchun, Veyershtarss alomatiga ko'ra, $\frac{2}{\pi}\int_0^{+\infty}\frac{yf(x)}{x+y^2}dx$ integral tekis yaqinlashuvchidir.

Ravshanki, $\lim_{y\to 0}\frac{f(zy)}{1+z^2}=\frac{f(0)}{1+z^2}$ $\forall\varepsilon>0$ ga ko'ra $\delta>0,\ \forall|y|<\delta$ uchun va $\forall z\in(a,b)$ larda $\left|\frac{f(zy)}{1+z^2}-\frac{f(0)}{1+z^2}\right|=\left|\frac{f(zy)-f(0)}{1+z^2}\right|<\varepsilon$ tengsizlik o'rinlidir.

11-teoremadan foydalanib topamiz,

$$\lim_{y\to 0}\frac{2}{\pi}\int_0^{+\infty}\frac{yf(x)}{x^2+y^2}dx=\lim_{y\to 0}\frac{2}{\pi}\int_0^{+\infty}\frac{f(zy)}{1+z^2}dz=\frac{2}{\pi}\int_0^{+\infty}\lim_{y\to 0}\frac{f(zy)}{1+z^2}dz=$$

$$=\frac{2}{\pi}\int_0^{+\infty}\frac{f(0)}{1+z^2}dz=f(0)\frac{2}{\pi}\int_0^{+\infty}\frac{dz}{1+z^2}=f(0).$$

18-misol. $I(\alpha)=\int_1^{+\infty}\frac{\cos x}{x^\alpha}dx,\quad(\alpha>0)$ funksiyani uzluksizlikka tekshiring.

Yechish. $f(x,\alpha)=\frac{\cos x}{x^{\alpha}}$ funksiyaning $P_{\alpha}=\{(x,\alpha)\in R^2:\ 1\le x<+\infty,\ \alpha\ge\varepsilon>0\}$ to'plamda uzluksiz ekani ravshan. Endi integralni tekis yaqinlashishga tekshiramiz: $\int_1^A \cos xdx=\sin A-\sin 1$ bo'lib, $\left|\int_1^A \cos xdx\right|\le 2$ bo'ladi, $\forall\varepsilon>0$, uchun $\exists\Delta=\Delta(\varepsilon)>0$ topiladiki, $\forall|x|$, $\forall\alpha\ge\varepsilon>0$ lar uchun $\frac{1}{x^{\alpha}}<\varepsilon$ bo'ladi. ($\Delta(\varepsilon)=\frac{\ln\frac{1}{\varepsilon}}{\varepsilon}$ qilib olsak bo'ladi). Bu esa $\frac{1}{x^{\alpha}}$ funksiyani $x\to+\infty$ da limit funksiya nolga tekis yaqinlashishini bildiradi. Dirixle alomatiga ko'ra, berilgan integral tekis yaqinlashuvchi bo'lib, 12-teoremaga asosan $I(\alpha)$ funksiyaning uzluksizligi kelib chiqadi.

19-misol. $I(a)=\int_0^{+\infty} e^{-ax}\frac{\sin x}{x}dx$, $a\in[0,c]$ $c>0$ integral hisoblansin.

Yechish. Ravshanki, $f(x,a)=e^{-ax}\frac{\sin x}{x},(f(0,a)=1)$ funksiya $M=\{(x,a)\in R^2:\ x\in[0,+\infty),\ a\in[0,c]\},\ \ c>0$ to'plamda uzluksiz ya'ni $f\in C_M$. $f'_a(x,a)=-e^{-ax}\sin x$ xususiy hosilaga ega va u ham uzluksiz funksiya. Quyidagi $\int_0^{+\infty} f'_a(x,a)dx=-\int_0^{+\infty} e^{-ax}\sin xdx$ integral esa $a\ge a_0$ $(a_0>0)$ da tekis yaqinlashuvchi.

13-teoremaga ko'ra, $I'(a)=\int_0^{+\infty}\left(e^{-ax}\frac{\sin x}{x}\right)'_a dx=-\int_0^{+\infty} e^{-ax}\sin xdx=-\frac{1}{1+a^2}$ bo'ladi. Demak,

I(a)=-arctga+c, a=+∞ bo'lganda *I(+∞)=0* bo'lib, $-\frac{\pi}{2}+C=0,\Rightarrow C=\frac{\pi}{2}$ bo'ladi.

Demak, $I(a)=\int_0^{+\infty} e^{-ax}\frac{\sin x}{x}dx=\frac{\pi}{2}-arctga$.

4.6. uchun misollar.

1. $\lim\limits_{\alpha\to+0}\int_0^{+\infty}\alpha e^{-\alpha x}dx$, munosabatda limit belgisini integral ostiga kiritish mumkinmi?

2. $\lim\limits_{n\to\infty}\int_0^{+\infty}\frac{dx}{x^n+1}$, ni toping.

3. $F(\alpha)=\int_0^{+\infty} e^{-(x-\alpha)^2}dx$ funksiyani uzluksizlikka tekshiring.

4. $I(\alpha)=\int_0^1 \frac{\sin\frac{\alpha}{x}}{x^{\alpha}}dx$ funksiyaning $0<\alpha<1$ da uzluksizlikka tekshiring.

Quyidagi funksiyalarni uzluksizlikka tekshiring:

5. $I(\alpha)=\int\limits_0^{+\infty}\frac{xdx}{2+x^{\alpha}},\qquad (\alpha>2)$ 6. $I(\alpha)=\int\limits_0^{\pi}\frac{\sin x}{x^{\alpha}(\pi-x)^{\alpha}}dx,\qquad (0<\alpha<2)$

7. $I(\alpha)=\int\limits_0^{+\infty}\frac{e^{-x}}{|\sin x|^{\alpha}}dx,(0<\alpha<1)$ 8. $I(\alpha)=\int\limits_0^{+\infty}\alpha e^{-\alpha x^2}dx,\qquad \alpha\in R$

9. $F(\alpha)=\int\limits_0^{+\infty}\frac{\cos x}{1+(x+\alpha)^2}dx$ funksiya $-\infty<\alpha<+\infty$ da uzluksiz va differensiallanuvchiligini isbotlang.

10. $\frac{e^{-ax}-e^{-bx}}{x}=\int\limits_a^b e^{-xy}dx$ tenglikdan foydalanib quyidagi integrallarni hisoblang.

a) $\int\limits_0^{+\infty}\frac{e^{-\alpha\beta}-e^{-\beta x}}{x^{\alpha}}\cos mxdx,(\alpha>0,\ \beta>0)$;

v) $\int\limits_0^{+\infty}\frac{e^{-\alpha x}-e^{-\beta x}}{x^{\alpha}}\sin mxdx,\qquad (\alpha>0,\ \beta>0)$.

Quyidagi integrallarni hisoblang:

13. $\int\limits_0^{+\infty}\frac{\cos ax}{1+x^2}$; 14. $\int\limits_0^{+\infty}\frac{\sin ax-\sin bx}{x}dx,\quad (\alpha>0,\ \beta>0)$;

15. $\int\limits_0^{+\infty}\frac{\cos ax-\cos bx}{x^2}dx,\quad (a>0,\ b>0)$; 16. $\int\limits_0^{+\infty}\sin(x^2)dx$;

17. $\int\limits_0^{+\infty}\cos(x^2)dx$; 18. $\int\limits_0^{+\infty}\frac{\sin(x^2)}{x}dx$;

19. $\int\limits_0^{+\infty}\frac{\sin x}{x}dx$; 20. $\int\limits_0^{+\infty}\frac{x^{a-1}}{1+x}dx$.

§4.7. Eyler integrallari

I. Beta funksiya (1-tur Eyler integrali). Ushbu $B(a,b)=\int\limits_0^1 x^{a-1}(1-x)^{b-1}dx\ (a>0,\ b>0)$ integral Beta funksiya yoki 1-tur Eyler integrali deb ataladi.

Beta funksiyaning xossalari:

1. $B(a,b)=B(b,a)$;
2. $B(a,b)=\frac{b-1}{a+b-1}\quad B(a,b-1),(b>1)$;

3. $B(a,n)=\frac{n-1}{a+n-1}\cdot\frac{n-2}{a+n-2}\cdots\frac{1}{a+1}B(a,1), \quad n\in N$;

4. $B(a,1-a)=\frac{\pi}{\sin a\pi}, \quad (0<a<1)$;

5. $B\left(\frac{1}{2},\frac{1}{2}\right)=\pi$.

II. Gamma funksiya (2-tur Eyler integrali). Ushbu $\Gamma(a)=\int_0^{+\infty}x^{a-1}e^{-x}dx, \ (a>0)$ integral Gamma funksiya yoki 2-tur Eyler integrali deb ataladi.

Gamma funksiyaning xossalari:

1. $\Gamma(a)=\lim_{n\to\infty}n^a\frac{1\cdot2\cdot3\cdots(n-1)}{a(a+1)\cdots(a+n-1)}$;

2. $\Gamma(a+1)=a\Gamma(a)$;

3. $\Gamma(n+1)=n!$;

4. $\Gamma(a)$ $(0,+\infty)$ da uzluksiz va barcha tartibdagi uzluksiz hosilalarga ega va

$$\Gamma^{(n)}(a)=\int_0^{+\infty}x^{a-1}e^{-x}(\ln x)^n dx \qquad (n=1,2,...);$$

5. $B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$;

6. $\Gamma(a)\Gamma(1-a)=B(a,1-a)=\frac{\pi}{\sin a\pi}; \quad \left(\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}\right)$

7. $\Gamma(a)\Gamma\left(a+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2a-1}}\Gamma(2a)$ (Lejandr formulasi).

20-misol. Ushbu $I=\int_0^{+\infty}e^{-x^2}dx$ integralni hisoblang.

Yechish. $x^2=z$ almashtirishni olsak, $2xdx=dz$

$$I=\frac{1}{2}\int_0^{+\infty}\frac{e^{-t}}{\sqrt{t}}dt=\frac{1}{2}\int_0^{+\infty}e^{-t}\cdot t^{-\frac{1}{2}}dt=\frac{1}{2}\int_0^{+\infty}t^{\frac{1}{2}-1}e^{-t}dt-\frac{1}{2}\Gamma\left(\frac{1}{2}\right)=\frac{1}{2}\sqrt{\pi}.$$

Demak , $\int_0^{+\infty}e^{-x^2dx}=\frac{\sqrt{\pi}}{2}$.

21-misol. Ushbu $I=\int_0^{\frac{\pi}{2}}\sin^4 x\cos^2 xdx$ integralni hisoblang.

Yechish. $\sin x=\sqrt{t}$ $(t>0)$ almashtirish olsak, $d(\sin x)=d\left(\sqrt{t}\right)$;

$\cos xdx=\frac{1}{2\sqrt{t}}dt$,

$$dx=\frac{1}{2\sqrt{1-\sin^2 x}}\cdot\frac{1}{\sqrt{t}}dt,$$

$$dx=\frac{1}{2\sqrt{(1-t)}}\cdot\frac{1}{\sqrt{t}},\quad \text{bu yerdan}$$

$$\int_0^{\frac{\pi}{2}}\left(\sin^2 x\right)^2\left(1-\sin^2 x\right)dx=\frac{1}{2}\int_0^1 t^2\cdot(1-t)\cdot\frac{1}{(1-t)^{\frac{1}{2}}}\cdot\frac{1}{t^{\frac{1}{2}}}dt=\frac{1}{2}\int_0^1(1-t)^{\frac{1}{2}}t^{\frac{3}{2}}dt=$$

$$=\frac{1}{2}\int_0^1(1-t)^{\frac{3}{2}-1}t^{\frac{5}{2}-1}dt=\frac{1}{2}B\left(\frac{3}{2},\frac{5}{2}\right)=\frac{1}{2}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{5}{2}\right)}{\Gamma(4)}=\frac{1}{2}\frac{\Gamma\left(1+\frac{1}{2}\right)\Gamma\left(1+\frac{3}{2}\right)}{3!}=$$

$$=\frac{1}{12}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)\cdot\frac{3}{2}\Gamma\left(\frac{3}{2}\right)=\frac{3}{48}\cdot\Gamma\left(\frac{1}{2}\right)\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)=\frac{1}{32}\Gamma^2\left(\frac{1}{3}\right)=\frac{\sqrt{\pi}}{32}.$$

4.7. uchun misollar.

Eyler integrallaridan foydalanib quyidagi integrallarni hisoblang:

1. $\int_0^{+\infty}\frac{x^2dx}{1+x^4}$;
2. $\int_0^1\frac{x^{a-1}-x^{-a}}{1-x}dx,\quad (0<a<1)$;
3. $\int_0^1\sqrt{x-x^2}dx$;
4. $\int_0^b x^2\sqrt{b^2-x^2}dx,\quad (b>0)$;
5. $\int_0^1\frac{dx}{\sqrt[n]{1-x^n}},\quad (n>1)$;
6. $\int\frac{\sqrt[4]{x}}{(1+x)^2}dx$;
7. $\int_0^{+\infty}\frac{dx}{1+x^3}$;
8. $\int_0^{\frac{\pi}{2}}\sin^6 x\cos^4 xdx$;
9. $\int_0^{+\infty}x^{2n}e^{-x^2}dx,$ (n - butun musbat);
10. $\int_0^{+\infty}\frac{dx}{\left(1+x^2\right)^n}$.

Quyidagi integrallarni Eyler integrallari orqali ifodalang va mavjudlik sohasini aniqlang.

11. $\int_0^{+\infty}\frac{x^{m-1}}{1+x^n}dx,\quad (n>0)$;
12. $\int_0^{+\infty}\frac{x^{m-1}}{(1+x)^n}dx$;
13. $\int_0^1\frac{dx}{\sqrt[n]{1-x^m}},\quad (m>0)$;
14. $\int_0^{\frac{\pi}{2}}\sin^m x\cos^n xdx$;
15. $\int_0^{\frac{\pi}{2}}tg^n xdx$;
16. $\int_0^{+\infty}e^{-x^n}dx,\quad (n>0)$;
17. $\int_0^{+\infty}x^m e^{-x^n}dx$;
18. $\int_0^{+\infty}x^p e^{-ax}\ln xdx,\quad (a>0)$;

19. $\int\limits_0^{+\infty} \frac{x \ln x}{1+x^3} dx$;

20. $\int\limits_0^1 \frac{x^{p-1} - x^{-p}}{1-x} dx$, $(0 < p < 1)$.

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